sQGP – A theorist's point of view

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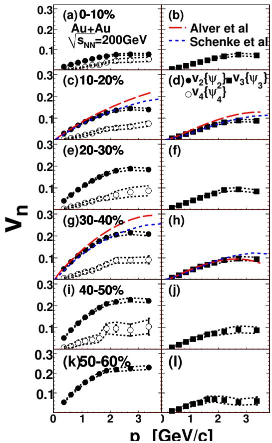
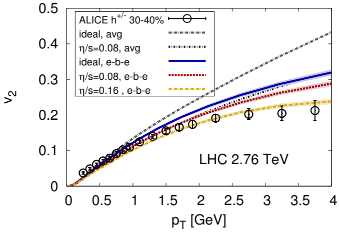
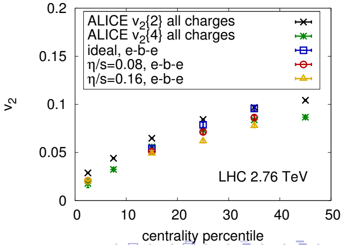
What is sQGP?

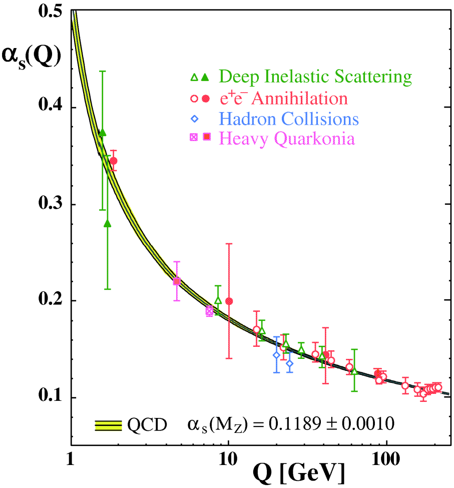
- Conventional wisdom: strongly coupled QGP
- Best Evidence: $\eta/s \sim 1/4\pi$ (Calc. by Schenke, Jeon and Gale)







Running Coupling constant



- S. Bethke Prog.Part.Nucl.Phys. 58 (2007) 351-386. 4-loop β function.
- $\alpha_S \approx 0.5$ when $Q = O(1\,\text{GeV})$
- $\alpha_S \approx 0.1$ when $Q = O(200\,\text{GeV})$
- For thermal QCD, relevant coupling constant range is $0.2 \lesssim g/2\pi \lesssim 0.4$

Where do g's appear? (perturbatively)

It's not easy to cover all relevant topics.

Where do g's appear? (perturbatively)

I'll stick with what I am able to talk about.

Where do g's appear? (perturbatively)

- CGC: Strong color field $A_\mu = O(1/g)$
- $\epsilon/\epsilon_{SB} = 1 - \#(g/2\pi)^2 + \#(g/2\pi)^3 + \cdots$: Equation of state
- Thermal QCD - Debye mass: $m_D = \#gT$
- Elastic scattering mean-free-path: $l_{\rm mfp} \propto 1/\alpha_S T$
- Jet radiational loss rate: $\Gamma \propto \alpha_S^2$
- Viscosity $\sim O(1/[\alpha_S^2 \ln(1/\alpha_S)])$
- CGC (Glasma) and thermal QCD: Power counting in g or $\sqrt{\alpha_S/\pi} = g/(2\pi)$ *not* in α_S
- $\alpha_S \approx 0.1 \to g/2\pi \approx 0.16$
- $\alpha_S \approx 0.3 \to g/2\pi \approx 0.32$
- $\alpha_S \approx 0.5 \to g/2\pi \approx 0.4$

How well (or badly) does the perturbative QCD work?

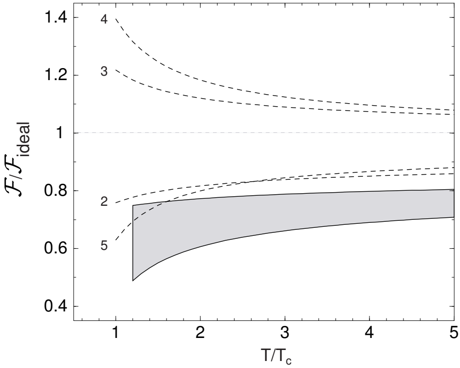
Some theoretical test possible for

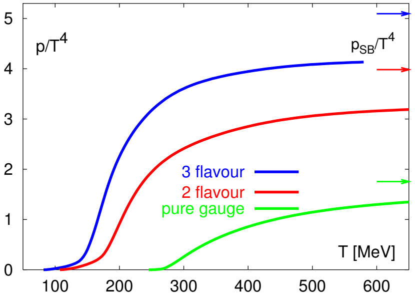
- Equation of state (AdS/QCD vs. Lattice vs. pQCD)
- Viscosity (AdS/QCD vs. Lattice vs. pQCD)

Experimental tests available for

- Viscosity, EoS via flow coefficients
- Scattering rates via Jet Quenching

Pressure in thermal QCD

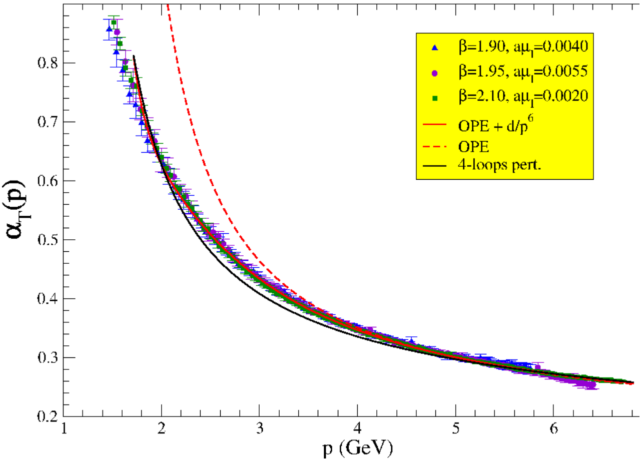




- J.O. Andersen, E. Braaten and M. Strickland, PRD 61, 074016
- Perturbative F and HTL F
- At $T/T_c = 5$, $F/F_{\text{ideal}} \approx 0.8$
- With $Q = 2\pi T$

- F. Karsch, J.Phys.Conf.Ser. 46 (2006) 122-131
- $\mathcal{P}_{LQCD}/\mathcal{P}_{SB} \approx 0.8$
- AdS/CFT: $F = (3/4)F_{SB}$

LQCD α_S



Cnsistent with pQCD running coupling.
[Blossier, Boucaud, Brinet, De Soto, Du, Morénas, Pène, Petrov, and Rodríguez-Quintero, arXiv:1210.1053]

Perturbative ϵ, $\mathcal{P}$

- Both pQCD and AdS/CFT comparable to LQCD for $T \geq 2T_c$
- Can't really say large α_S (or $g/2\pi$) is necessary.
  - Caveat: HTL calculations need $T \gg gT \gg g^2T$

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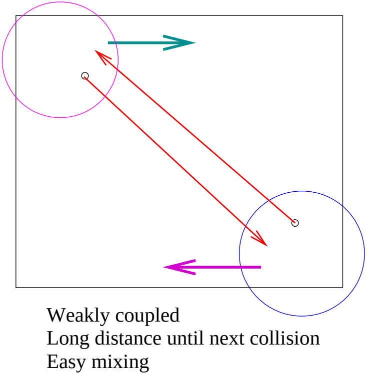
- Moving on to η/s ...

Interaction Strength and Viscosity

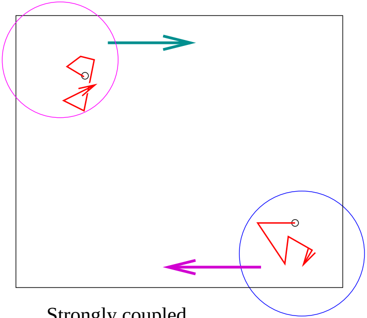
Weak coupling allows rapid *momentum diffusion*



Large η/s: $u_\mu(x)$ changes due to pressure gradient and diffusion

Interaction Strength and Viscosity

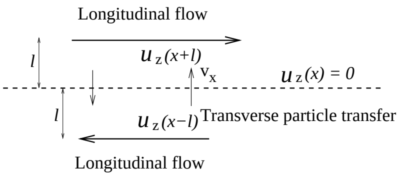
Strong coupling *does not* allow momentum diffusion



Strongly coupled
Very short distance until next collision
Mixing takes very long time

Small η/s: $u_\mu(x)$ changes due to pressure gradient only

Kinetic Theory estimate



u_z: Flow velocity
v_x: Average speed of microscopic particles

- Rough estimate (fluid rest frame, or $u_z(x) = 0$)
  - The momentum density: $T_{0z} = (\epsilon + \mathcal{P})u_0 u_z$ diffuses in the x direction with $v_x = u_x/u_0$. Net change:

$$\begin{aligned}&\langle\epsilon + \mathcal{P}\rangle |v_x| u_0 (u_z(x - l_{\rm mfp}) - u_z(x + l_{\rm mfp}))\\ &\approx -2\langle\epsilon + \mathcal{P}\rangle |v_x|\, u_0\, l_{\rm mfp}\partial_x u_z(x)\\ &\sim -\eta u_0 \partial_x u_z\end{aligned}$$

  Here $l_{\rm mfp}$: Mean free path

  - Recall thermo. id.: $\langle\epsilon + \mathcal{P}\rangle = sT$

$$\eta \sim \langle\epsilon + \mathcal{P}\rangle\ l_{\rm mfp}\ \langle|v_x|\rangle \sim s\,T\,l_{\rm mfp}\ \langle|v_x|\rangle$$

Perturbative estimate

High Temperature limit: $\langle |v_x| \rangle = O(1)$

- $\eta/s \approx T l_{\rm mfp} \approx \dfrac{T}{n\sigma} \sim \dfrac{1}{T^2\sigma}$
- The only energy scale: T

$$\sigma \sim \frac{(\text{coupling constant})^{\#}}{T^2}$$

Hence

$$\frac{\eta}{s} \sim \frac{1}{(\text{coupling constant})^{\#}}$$

- Perturbative QCD partonic 2-2 cross-section

$$\frac{d\sigma_{\rm el}}{dt} = C\frac{2\pi\alpha_S^2}{t^2}\left(1 + \frac{u^2}{s^2}\right)$$

Perturbative estimate - Cont.

- Naively expect

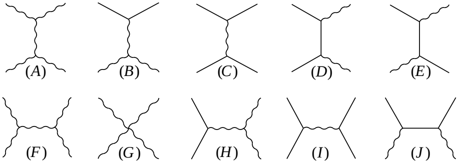
$$\eta/s \sim \frac{1}{\alpha_s^2}$$

- Coulomb enhancement (cut-off by m_D) leads to

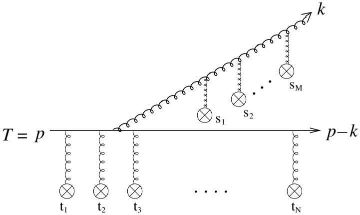
$$\eta/s \sim \frac{1}{\alpha_s^2 \ln(1/\alpha_s)}$$

QCD η calc

Relevant processes



($\sim$ 80 %)



($\sim$ 20 %)

Use kinetic theory

$$\frac{df}{dt} = \mathcal{C}_{2\leftrightarrow 2} + \mathcal{C}_{1\leftrightarrow 2}$$

Complication: $1 \leftrightarrow 2$ process needs resummation (LPM effect, AMY)

QCD Estimates of η/s

- Danielewicz and Gyulassy [PRD **31**, 53 (1985)]:
  - η/s bound from the kinetic theory: Recall: $\eta \sim s\, T\, l_{\rm mfp}\, \langle |v_x| \rangle$ Use $l_{\rm mfp}\, \langle |v_x| \rangle \sim \Delta x \Delta p/m$ to get

$$\frac{\eta}{s} \gtrsim \frac{1}{12} \approx 0.08 \approx (1/4\pi)$$

  - QCD estimate in the small α_S limit with $N_f = 2$ and $2 \to 2$ only (min. at $\alpha_S = 0.6$):

$$\eta \approx \frac{T}{\sigma_\eta} \approx \frac{0.57\, T^3}{\alpha_S^2 \ln(1/\alpha_S)} \gtrsim 0.2s \approx (2.5/4\pi)s$$

- Baym, Monien, Pethick and Ravenhall [PRL 64, 1867 (1990)]

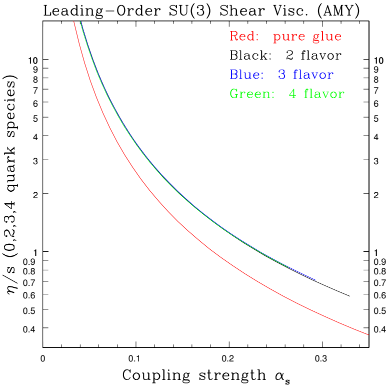
$$\eta \approx \frac{1.16\, T^3}{\alpha_S^2 \ln(1/\alpha_S)} \gtrsim 0.4s \approx (5/4\pi)s$$

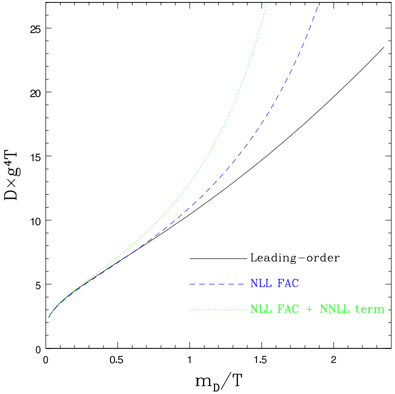
- M. Thoma [PLB 269, 144 (1991)]

$$\eta \approx \frac{1.02\, T^3}{\alpha_S^2 \ln(1/\alpha_S)} \gtrsim 0.4s \approx (5/4\pi)s$$

Full leading order calculation of η/s

- Arnold-Moore-Yaffe (JHEP **0305**, 051 (2003)) [Plots: Guy]:





Minimum $\eta/s \approx 0.6 \approx 7.5/4\pi$ for $\alpha_S \approx 0.3$

NB: Approximate formula $\eta/s \approx \dfrac{1}{15.4\alpha_S^2 \ln(0.46/\alpha_S)}$

is not good for $\alpha_S > \frac{1}{4\pi(1+N_f/6)}$

Shear viscosity in $\mathcal{N}=4$ SYM

Son, Starinets, Policastro, Kovtun, Buchel, Liu, ...

- Strong coupling limit, 4 ingredients
  - Kubo formula

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, d^3x\, e^{i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle$$

  - Gauge-Gravity duality

$$\sigma_{\rm abs}(\omega) = \frac{8\pi G}{\omega} \int dt\, d^3x\, e^{i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle$$

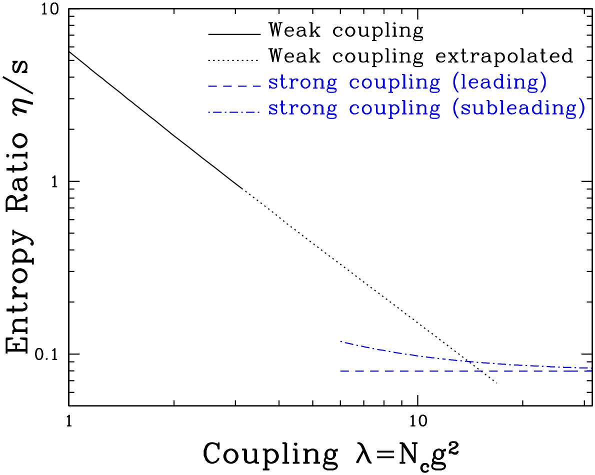
  - $\lim_{\omega\to 0} \sigma_{\rm abs}(\omega) = A_{\rm blackhole}$
  - Entropy of the BH : $s = A_{\rm blackhole}/4G$

Therefore, (including the first order correction)

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{7.12}{(g^2 N_c)^{3/2}}\right)$$

Correction is small if $g \gg 1$ (10 % at $g = 2.4$).

$N = 4$ SYM



S. Carno-Huot, S. Jeon and G. D. Moore, Phys. Rev. Lett. 98, 172303 (2007)

- Perturbative calculation and the strong coupling calculation behave very differently

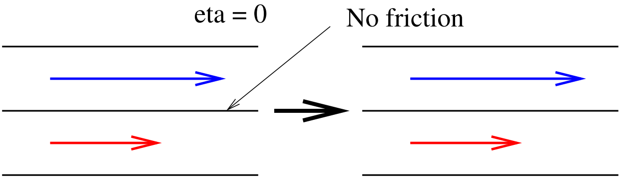
Experimental Evidence for $\eta/s \sim 1/4\pi$

- Theoretical situation:
  - Perturbative calculations: $\eta/s \geq 7.5/(4\pi)$
  - AdS/CFT in the infinite coupling limit: $\eta/s = 1/(4\pi)$
  - Roughly an order of magnitude difference $\Longrightarrow$ Testable!
- A relativistic heavy ion collision produces a complicated system $\Longrightarrow$ Need a hydrodynamics simulation suite
- We use MUSIC (3+1D e-by-e viscous hydrodynamics)
- Viscosity measurement is through the flow coefficients

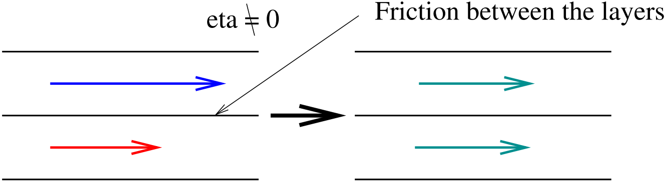
$$\frac{dN}{dyd^2p_T} = \frac{dN}{2\pi dyp_T dp_T}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos(n(\phi - \psi_n))\right)$$

- v_n is a translation of the eccentricities ϵ_n via pressure gradient

Effect of viscosity

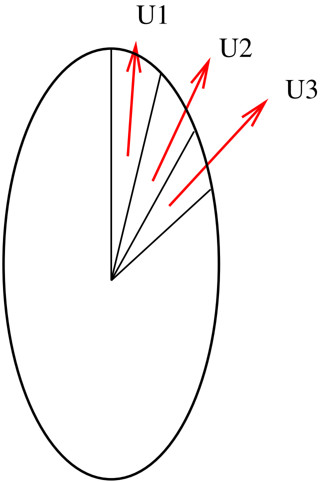


The relative velocity of the two layers does not change.



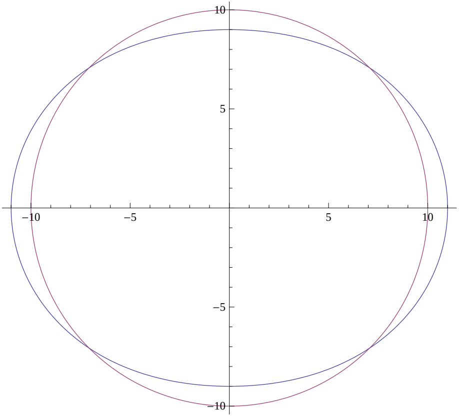
The velocities eventually become the same.

Effect of viscosity

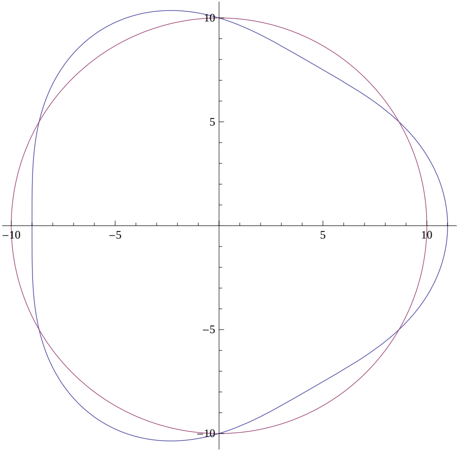


- $\eta = 0$ means $u_1 < u_2 < u_3$ is maintained for a long time
- $\eta \neq 0$ means that $u_1 \simeq u_2 \simeq u_3$ is achieved more quickly
- Shear viscosity smears out flow differences (it's a diffusion)
- Shear Viscosity reduces non-sphericity

ϵ_n

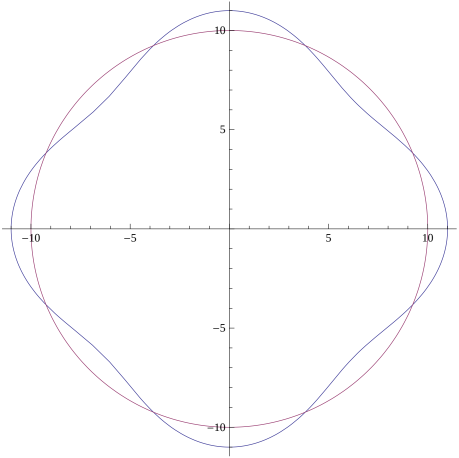


This causes elliptic flow. It is harder to destroy this than



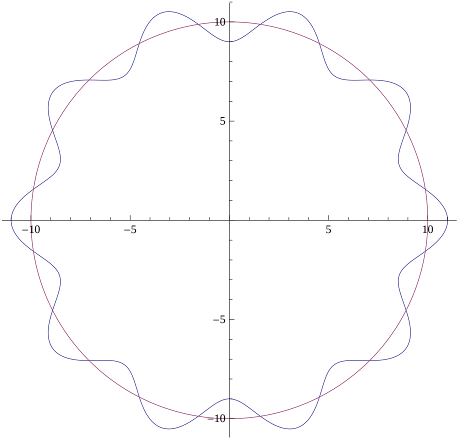
this (v_3) ...

ϵ_n



or this (v_4) ...

ϵ_n



or this (v_{10}) ...

MUSIC

MUScl for Ion Collisions

MUSCL: Monotone Upstream-centered Schemes for Conservation Laws

Current MUSIC (and MARTINI) Team

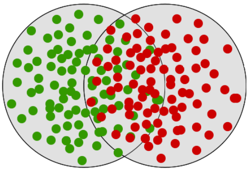
- Charles Gale (McGill)
- Sangyong Jeon (McGill)
- *Björn Schenke* (Formerly McGill, now BNL)
- *Clint Young* (Formerly McGill, now UMN)
- *Gabriel Denicol* (McGill)
- *Matt Luzum* (McGill/LBL)
- Sangwook Ryu (McGill)
- Gojko Vujanovic (McGill)
- Jean-Francois Paquet (McGill)
- Michael Richard (McGill)
- Igor Kozlov (McGill)

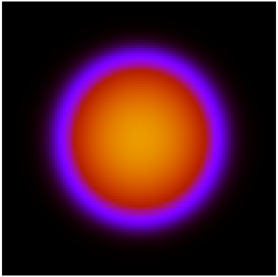
3+1D Event-by-Event Viscous Hydrodynamics

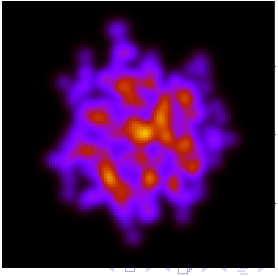
- 3+1D parallel implementation of new *Kurganov-Tadmor Scheme* in (τ, η) with an additional baryon current
  (No need for a Riemann Solver. Semi-discrete method.)
- Ideal *and* Viscous Hydro
- Event-by-Event fluctuating initial condition
- Sophisticated Freeze-out surface construction
- Full resonance decay (3+1D version of Kolb and Heinz)
- Many different equation of states including the newest from Huovinen and Petreczky
- *New Development*: Glasma Initial Conditions & UrQMD after-burner

Fluctuating Initial Condition

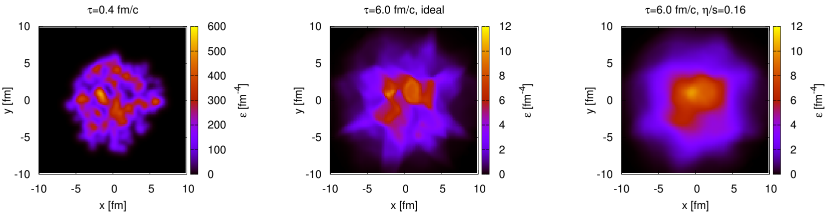
Each event is *not* symmetric: Fluctuating initial condition $\Longrightarrow$ All v_n are non-zero.







Ideal vs. Viscous

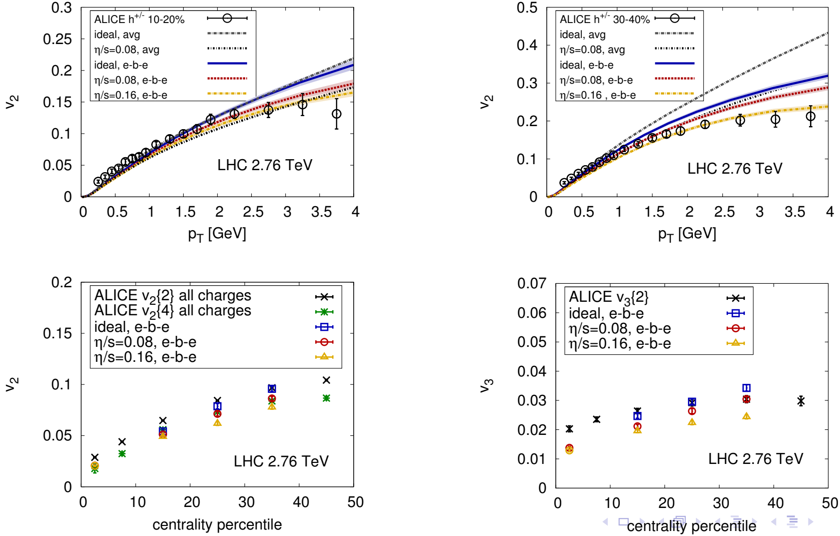


Fluctuations and Viscosity

- Magnitude of higher harmonics, $v_3, v_4, \cdots$, (almost) independent of centrality – Local fluctuations dominate
- Higher harmonics are easier to destroy that v_2 which is a *global* distortion – Viscosity effect.
- To get a good handle on flow: Both fluctuations and viscosity are essential

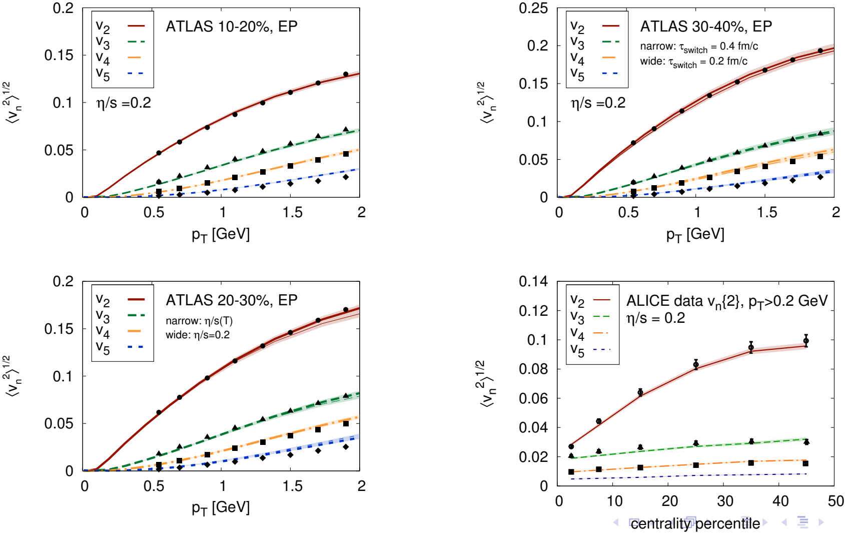
E-by-E MUSIC vs LHC Data

[Schenke, Jeon and Gale, Phys. Rev. C 85, 024901 (2012)]
Best value $\eta/s = 0.16 = 2/(4\pi)$.



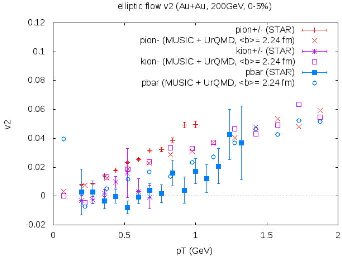
New Development 1: Glasma Initial Condition

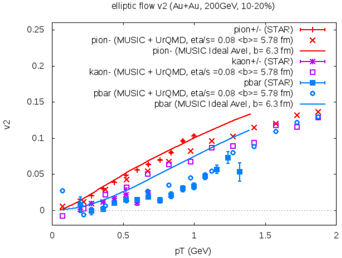
[Gale, Jeon, Schenke, Tribedy and Venugopalan, arXiv:1209.6330]
Best value $\eta/s = 0.2 = 2.5/(4\pi)$. More on this in Björn's talk.

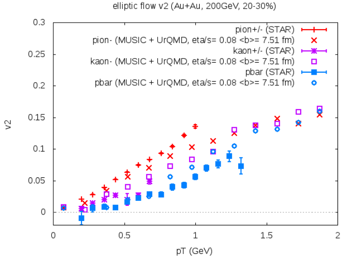


New Development 2: UrQMD Afterburner

v_2 at RHIC (Midrapidity). In each centrality class: 100 UrQMD times 100 MUSIC events. [Ryu, Jeon, Gale, Schenke and Young, arXiv:1210.4558]



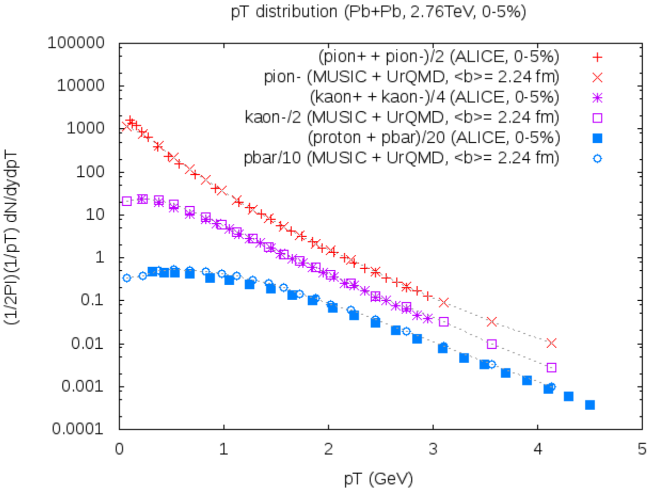




- $\eta/s = 1/4\pi$
- Using previous MUSIC parameters that were tuned to reproduce PHENIX v_n

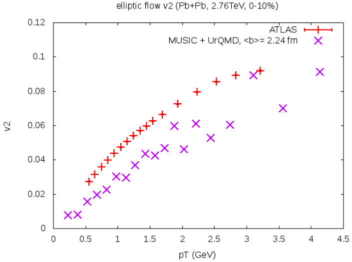
LHC Spectra

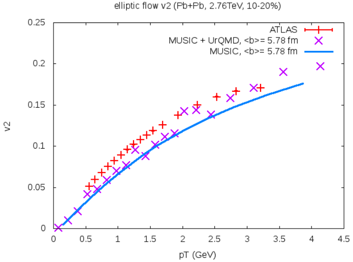
In each centrality class: 100 MUSIC times 10 UrQMD events. $\eta/s = 2/(4\pi)$. ALICE data from QM12.

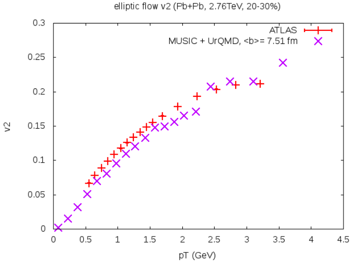


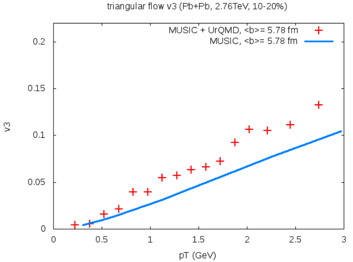
LHC Flows

In each centrality class: 100 MUSIC times 10 UrQMD events







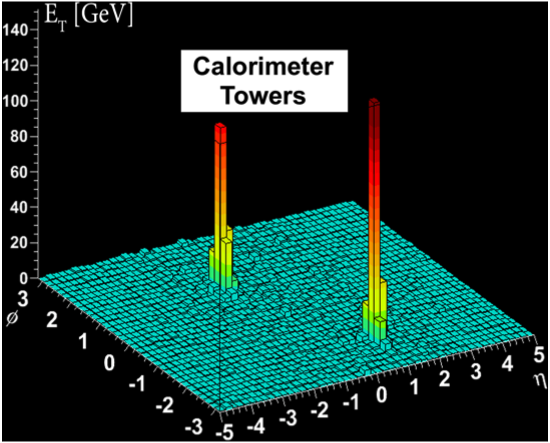


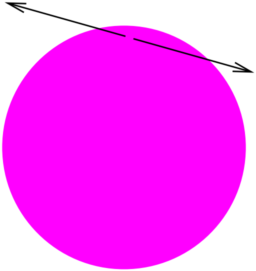
Conclusions and questions for η/s

- Strong flows: Strongest evidence that η/s has to be small
- η/s much larger than 0.2 cannot be accommodated within current understanding of the system.
- Perturbative result of $\eta/s = 0.4 - 0.6$ is out.
- Using the LQCD EoS.
- LQCD estimate $(\eta + 3\zeta/4)/s \approx 0.20 - 0.26$ between $1.58T_c - 2.32T_c$.
  [H. Meyer, Eur.Phys.J.A47:86,2011]
- Does this mean very large coupling?

Jet Quenching

- Fact: Jets lose energy (ATLAS images).





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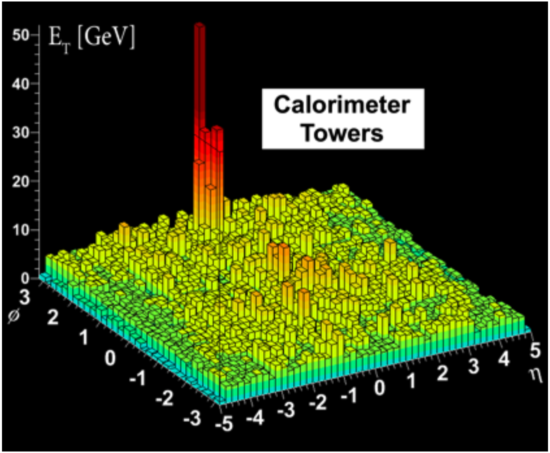
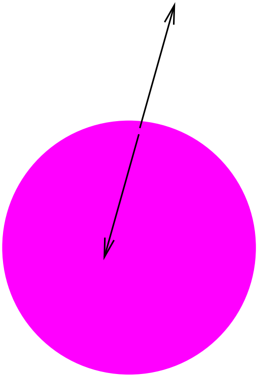
- Fact: Jets lose energy (ATLAS images).





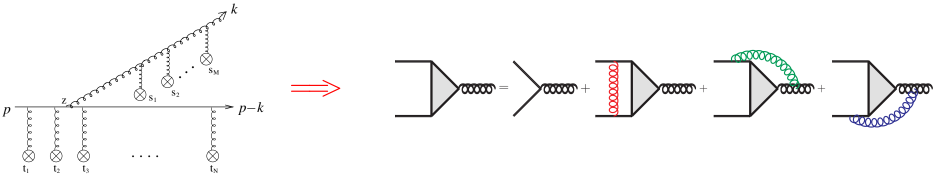
Energy Loss Mechanism

- Collisional energy loss rate [Wicks, Horowitz, Djordjevic and Gyulassy, NPA 784, 426 (2007), Qin, Gale, Moore, Jeon and Ruppert, Eur. Phys. J. C 61, 819 (2009)]:

$$\frac{dE}{dx} \approx C_1 \pi \alpha_S^2 T^2 \left[\log\left(\frac{E_p}{\alpha_S T} \right) + C_2 \right]$$

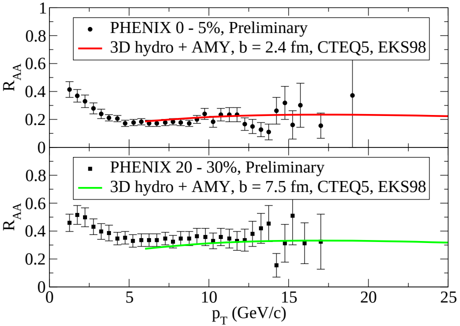
  $C_{1,2}$: Depends on the process. $O(1)$.

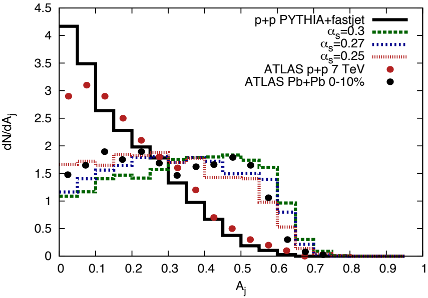
- Radiational $\propto \alpha_S^2$ (Arnold, Moore, Yaffe, JHEP 0206, 030 (2002))



What we want to get at

- What α_S do we need for these?





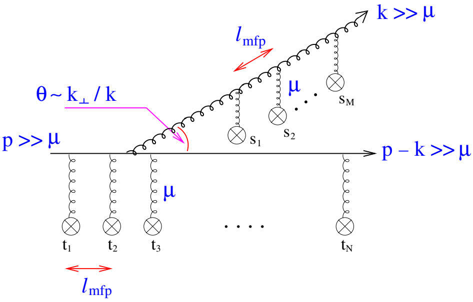
$$R_{AA} = \frac{(dN_{AA}/dp_T)}{(N_{\rm coll}\, dN_{pp}/dp_T)}$$

$$A_j = (E_> - E_<)/(E_> + E_<)$$

Radiational (Inelastic) Energy Loss – Qualitative understanding

Coherent scattering can be important

Following BDMPS

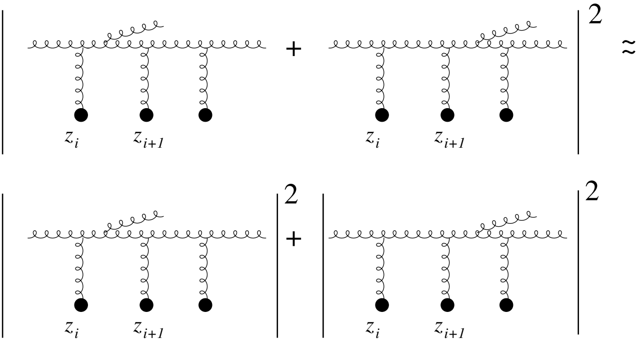


- What we need to calculate R_{AA}: Differential gluon radiation rate $\omega \dfrac{dN_g}{d\omega dz}$
  Medium dependence comes through a scattering length scale $l \approx t$

$$\omega \frac{dN_g}{d\omega dz} \approx \frac{1}{l} \omega \left. \frac{dN_g}{d\omega} \right|_{\mathrm{BH}}$$

Length Scales

Following BDMPS

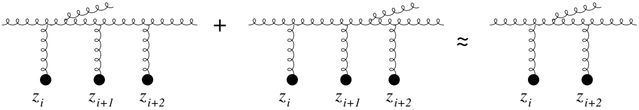


- If all scatterings are incoherent ($l_{\rm mfp} > l_{\rm coh}$),

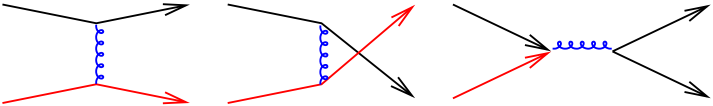
$$l = l_{\rm mfp} = 1/\rho\sigma$$

Length Scales

Following BDMPS



- If $l_{\rm coh} \geq l_{\rm mfp} \implies$ LPM effect:
  All scatterings within $l_{\rm coh}$ effectively count as a single scattering.
- $l = l_{\rm coh}$



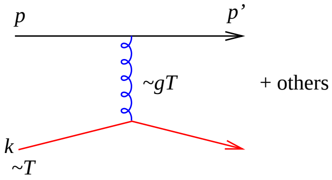
- Mean free path (textbook definition)

$$\frac{1}{l_{\rm mfp}} \equiv \int d^3k\, \rho(k) \int dq^2\, (1 - \cos\theta_{pk}) \frac{d\sigma^{\rm el}}{dq^2}$$

where

- $\rho(k)$: density, $(1 - \cos\theta_{pk})$: flux factor
- Elastic cross-section (Coulombic) $\dfrac{d\sigma}{dq^2} \approx C_R \dfrac{2\pi\alpha_s^2}{(q^2)^2}$

Estimation of l_{mfp}



- Mean free path (textbook definition)

$$\frac{1}{l_{\mathrm{mfp}}} \equiv \int d^3k\, \rho(k) \int dq^2 \,(1 - \cos\theta_{pk})\, \frac{d\sigma^{\mathrm{el}}}{dq^2}$$

where

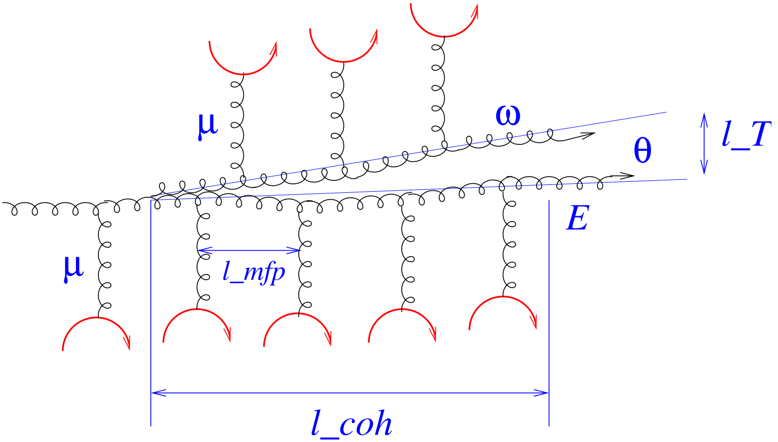
  - $\rho(k)$: density, $(1 - \cos\theta_{pk})$: flux factor
  - Elastic cross-section (Coulombic) $\dfrac{d\sigma}{dq^2} \approx C_R \dfrac{2\pi\alpha_s^2}{(q^2)^2}$

Estimation of l_{mfp}

- With thermal $\rho(k)$, this yields

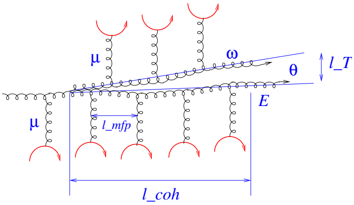
$$\frac{1}{l_{\text{mfp}}} \sim \int d^3k\,\rho(k) \int_{m_D^2}^{\infty} dq^2 \frac{\alpha_S^2}{q^4} \sim T^3 \alpha_S^2 / m_D^2 \sim \alpha_S T$$

Estimation of $l_{\rm coh}$



- $E \gg \omega_g \gg \mu$

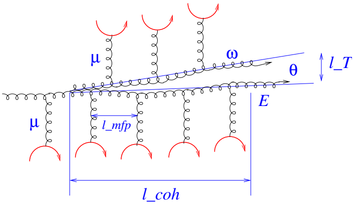
Estimation of $l_{\rm coh}$



- $\omega \ll E \implies$ The radiated gluon random walks away from the original parton. Original parton's trajectory is less affected.
- From the geometry $\frac{\omega_g}{k_T^g} \approx \frac{l_{\rm coh}}{l_T}$
- Separation condition: l_T is longer than the transverse size of the radiated gluon. $l_T \approx 1/k_T^g$
- Putting together,

$$l_{\rm coh} \approx \frac{\omega_g}{(k_T^g)^2}$$

Estimation of $l_{\rm coh}$



- Putting together,

$$l_{\rm coh} \approx \frac{\omega_g}{(k_T^g)^2}$$

- After suffering $N_{\rm coh}$ collisions (random walk),

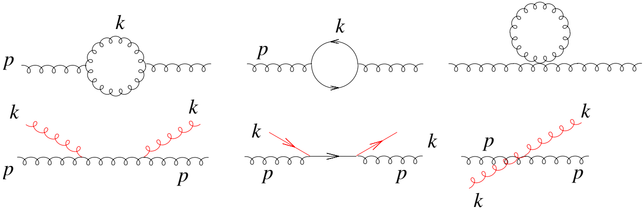
$$\left\langle (k_T^g)^2 \right\rangle = N_{\rm coh}\mu^2 = \frac{l_{\rm coh}}{l_{\rm mfp}}\mu^2$$

- Becomes, with $\hat{q} = \mu^2/l_{\rm mfp}$ and $E_{\rm LPM} = \mu^2 l_{\rm mfp}$,

$$l_{\rm coh} \approx l_{\rm mfp}\sqrt{\frac{\omega_g}{E_{\rm LPM}}} = \sqrt{\frac{\omega_g}{\hat{q}}}$$

Estimation of μ^2

- Debye mass



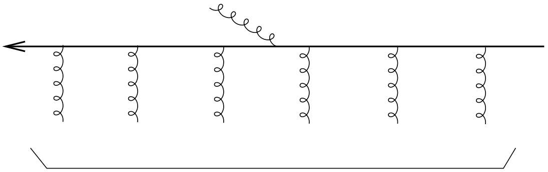
- Second row: Physical forward scattering with particles in the medium
- The last term is easiest to calculate:

$$m_D^2 \propto g^2 \int \frac{d^3k}{E_k} f(k) \propto g^2 T^2$$

- Effectively add $m_D^2 A_0^2 \Longrightarrow$ NOT gauge invariant $\Longrightarrow$ Gauge invariant formulation: Hard Thermal Loops

Rough Idea – Multiple Emission (Poisson ansatz)

After each collision, there is a finite probability to emit



Number of effective collisions

- Let the emission probability be p
- Total number of *effective* collisions N_{trial} taking into account of l_{mfp} and l_{coh}.
- Average number of emissions $\langle n \rangle = N_{\mathrm{trial}} p$
- Probability to emit n gluons

$$P(n) = \frac{N_{\mathrm{trial}}!}{n!(N_{\mathrm{trial}} - n)!} p^n (1-p)^{N_{\mathrm{trial}} - n}$$

Rough Idea – Multiple Emission (Poisson ansatz)

- Poisson probability: Limit of binary process as $\lim_{N_{\text{trial}}\to\infty} N_{\text{trial}} p \to \langle n \rangle$

$$P(n) = e^{-\langle n \rangle} \frac{\langle n \rangle^n}{n!}$$

- Average number of gluons emitted up to $t_i < t$

$$\langle n \rangle = \int_{-\infty}^{E} d\omega \int_{t_i}^{t} dz \frac{dN}{dz d\omega} = \int_{-\infty}^{E} d\omega \frac{dN}{d\omega}(t)$$

- Probability to lose ϵ amount of energy by emitting n gluons:

$$\begin{aligned} \langle n \rangle^n &\to D(\epsilon, t) \\ &= \int_{-\infty}^{E} d\omega_1 \frac{dN}{d\omega_1} \int_{-\infty}^{E} d\omega_2 \frac{dN}{d\omega_2} \cdots \int_{-\infty}^{E} d\omega_n \frac{dN}{d\omega_n} \delta(\epsilon - \sum_{k=1}^{n} \omega_k) \end{aligned}$$

Rough Idea – Multiple Emission (Poisson ansatz)

Parton spectrum at t

$$P(p,t) = \int d\epsilon\, D(\epsilon,t)\, P_0(p+\epsilon)$$

where

$$D(\epsilon,t) = e^{-\int d\omega \frac{dN}{d\omega}(\omega,t)} \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int d\omega_i\, \frac{dN}{d\omega_i}(\omega_i,t)\right] \delta\left(\epsilon - \sum_{i=1}^{n} \omega_i\right)$$

Can easily show that this Poisson ansatz solves:

$$\frac{dP(p,t)}{dt} = \int d\omega\, \frac{dN_{\text{Poiss.}}}{d\omega dt}(\omega) P(p+\omega,t) - P(p,t) \int d\omega\, \frac{dN_{\text{Poiss.}}}{d\omega dt}(\omega)$$

with the p (jet energy) independent rate

$$\frac{dN}{d\omega}(\omega,t) = \int_{t_0}^{t} dt'\, \frac{dN_{\text{Poiss.}}}{d\omega dt}(\omega,t')$$

Rough Idea - The behavior of R_{AA}

Use $P_0(p+\epsilon)/P_0(p) \approx 1/(1+\epsilon/p)^n \approx e^{-n\epsilon/p}$ when $n \gg 1$. Include gain by absoprtion or $\omega < 0$:

$$R_{AA}(p) = \frac{P(p)}{P_0(p)} \approx \exp\left(-\int_{-\infty}^{p} d\omega \int_0^t dt' \, (dN_{\rm inel+el}/d\omega dt)(1 - e^{-\omega n/p})\right)$$

For the radiation rate, use simple estimates

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi\omega}\frac{N_c}{l_{\rm mfp}} \quad \text{for} \quad 0 < \omega < l_{\rm mfp}\mu^2$$

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi\omega} N_c \sqrt{\frac{\mu^2}{l_{\rm mfp}\omega}} \quad \text{for} \quad l_{\rm mfp}\mu^2 < \omega < l_{\rm mfp}\mu^2 (L/l_{\rm mfp})^2$$

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi\omega}\frac{N_c}{L} \quad \text{for} \quad l_{\rm mfp}\mu^2 (L/l_{\rm mfp})^2 < \omega < E$$

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi|\omega|}\frac{N_c}{l_{\rm mfp}} e^{-|\omega|/T} \quad \text{for} \quad \omega < 0$$

Rough Idea - The behavior of R_{AA}

For elastic energy loss,

$$
\begin{aligned}
R_{AA}^{\rm el} &\approx \exp\left(-\int_{-\infty}^{\infty} d\omega \int_0^t dt'\,(d\Gamma_{\rm el}/d\omega dt)(1-e^{-\omega n/p})\right)\\
&\approx \exp\left(-t\left(\frac{dE}{dt}\frac{K(\omega_0)}{|\omega_0|}\right)\right)\\
&\approx \exp\left(-t\left(\frac{dE}{dt}\right)\left(\frac{n}{p}\right)\left(1-\frac{nT}{p}\right)\right)
\end{aligned}
$$

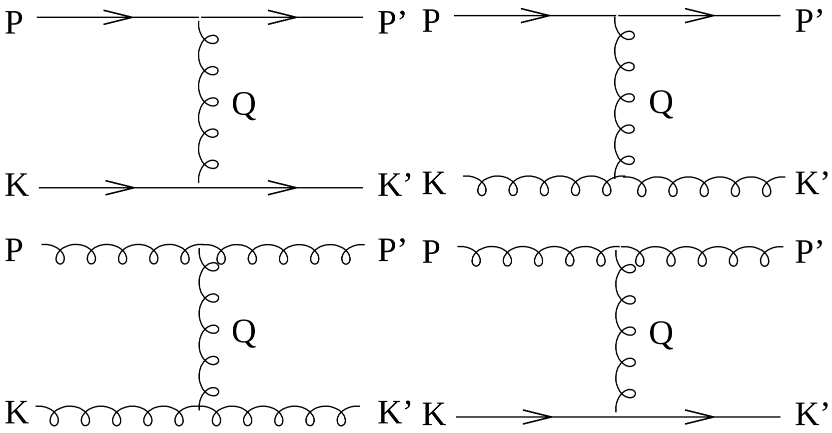
valid for $p > nT$ and we used

$$
\begin{aligned}
K(\omega_0) &= (1+n_B(|\omega_0|))(1-e^{-|\omega_0|n/p}) + n_B(|\omega_0|)(1-e^{|\omega_0|n/p})\\
&\approx |\omega_0|\left(\frac{n}{p}\right)\left(1-\frac{nT}{p}\right) \quad \text{for small } \omega_0
\end{aligned}
$$

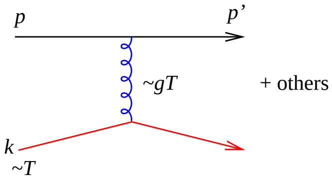
where ω_0 is the typical gluon energy

Elastic scattering rate

Coulombic *t*-channel dominates



Rough Idea - Elastic energy loss(Following Bjorken)



- Mean free path (textbook definition)

$$\frac{1}{l_{\mathrm{mfp}}} \equiv \int d^3k\, \rho(k) \int dq^2\, (1 - \cos\theta_{pk}) \frac{d\sigma^{\mathrm{el}}}{dq^2}$$

- Energy loss per unit length

$$\frac{dE}{dz} = \int d^3k\, \rho(k) \int dq^2\, (1 - \cos\theta_{pk}) \Delta E \frac{d\sigma^{\mathrm{el}}}{dq^2}$$

where

- $\rho(k)$: density, $(1 - \cos\theta_{pk})\Delta E \approx q^2/2k$: flux factor
- Elastic cross-section (Coulombic) $\frac{d\sigma}{dq^2} \approx C_R \frac{2\pi\alpha_s^2}{(q^2)^2}$

Rough Idea - Elastic energy loss(Following Bjorken)

- With thermal ρ, this yields

$$\left(\frac{dE}{dz}\right)_{\text{coll}} \sim \int d^3k\rho(k)/k \int dq^2\alpha_S^2/q^2 \sim \alpha_S^2 T^2 \ln(ET/m_D^2)$$

Upper limit determined by

$$q^2 = (p-k)^2 = p^2 + k^2 - 2pk \approx -2pk \sim ET$$

when $|\mathbf{p}| = E$ (emitter) and $|\mathbf{k}| = O(T)$ (thermal scatterer)
Lower limit determined by the Debye mass $m_D = O(gT)$.

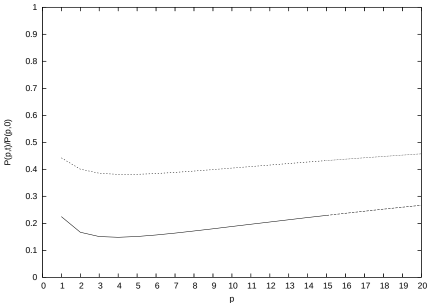
Elastic scattering rate

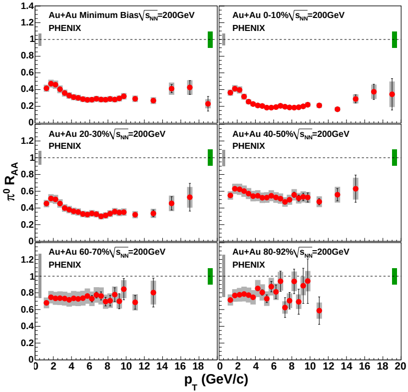
More precisely,

$$\frac{dE}{dt} = \frac{1}{2E}\int_{k,k',p'} \delta^4(p+k-p'-k')\,(E-E')|M|^2\, f(E_k)[1\pm f(E'_k)]$$
$$= C_r\pi\,\alpha_s^2 T^2\left[\ln(ET/m_g^2) + D_r\right]$$

where C_r and D_r are channel dependent $O(1)$ constants.

Rough Idea - The Dip in R_{AA}

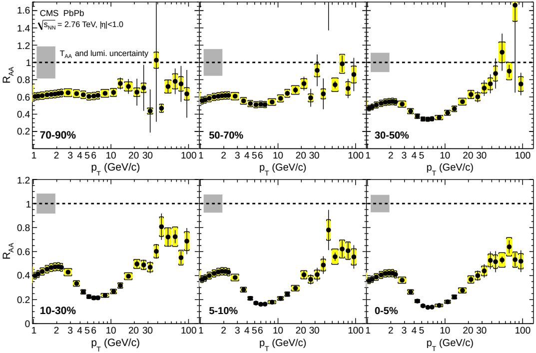




- Upper line: Without elastic
- Lower line: With elastic
- Flat R is produced in both cases up to $O(10\,T)$.
- R just not that sensitive to p in the RHIC-relevant range.

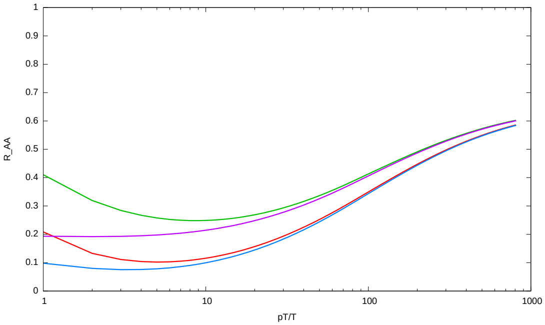
Rough Idea - The Dip in R_{AA}

CMS: Up to $p_T = 100\,\text{GeV}$



No longer flat. Logarithmic rise for $p_T \gtrsim 10\,\text{GeV}$.
Can we understand these features?

Rough Idea - The Dip in R_{AA}



- Red: Elastic on, thermal absorption on
- Blue: Elastic on, thermal absorption off
- Green: Elastic off, thermal absorption on
- Magenta: Elastic off, thermal absorption off
- Dip, rise, leveling-off roughly reproduced
- *No dip if thermal absorption is turned off*

Rising R_{AA}

Use $R_{AA} \approx 1/(1+\epsilon/p)^n \approx e^{-n\epsilon/p}$ when $n \gg 1$. Include gain by absoprtion or $\omega < 0$:

$$R_{AA}(p) = \frac{P(p)}{P_0(p)} \approx \exp\left(-\int_{-\infty}^{\infty} d\omega \int_0^t dt' \, (dN_{\mathrm{inel+el}}/d\omega dt)(1 - e^{-\omega n/p}) \right)$$

For the radiation rate, use simple estimates

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi\omega}\frac{N_c}{l_{\mathrm{mfp}}} \quad \text{for} \quad 0 < \omega < l_{\mathrm{mfp}}\mu^2$$

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi\omega} N_c \sqrt{\frac{\mu^2}{l_{\mathrm{mfp}}\omega}} \quad \text{for} \quad l_{\mathrm{mfp}}\mu^2 < \omega < l_{\mathrm{mfp}}\mu^2 (L/l_{\mathrm{mfp}})^2$$

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi\omega}\frac{N_c}{L} \quad \text{for} \quad l_{\mathrm{mfp}}\mu^2 (L/l_{\mathrm{mfp}})^2 < \omega < E$$

$$\frac{dN}{d\omega dt} \approx \frac{\alpha}{\pi|\omega|}\frac{N_c}{l_{\mathrm{mfp}}} e^{-|\omega|/T} \quad \text{for} \quad \omega < 0$$

Rising R_{AA}

With $E = p$ (original parton energy) and the system size L and $(1 - e^{-n\omega/E}) \approx n\omega/E$:

- Then $\ln R_{AA} \approx -n\Delta E/E$
- If $E < E_{\rm LPM} = \mu^2 l_{\rm mfp}$,

$$\ln R_{AA} \approx -L \int_0^E d\omega \, \frac{dN}{d\omega dt} \left(\frac{n\omega}{E} \right) \approx -\frac{nL}{E} \int_0^E d\omega\omega \left(\frac{\alpha_S}{\pi\omega} \frac{N_c}{l_{\rm mfp}} \right) \sim \text{Const.}$$

Flat R_{AA}

- If $E_{\rm LPM} < E < E_L = L^2\mu^2/l_{\rm mfp}$,

$$\begin{aligned} \ln R_{AA} &\approx -\frac{nL}{E} \int_0^{E_{\rm LPM}} d\omega\omega \left(\frac{\alpha_S}{\pi\omega} \frac{N_c}{l_{\rm mfp}} \right) - \frac{nL}{E} \int_{E_{\rm LPM}}^{E} d\omega\omega \left(\frac{\alpha_S}{\pi\omega} N_c \sqrt{\frac{\mu^2}{l_{\rm mfp}\omega}} \right) \\ &= -\frac{nL\alpha_S N_c}{\pi l_{\rm mfp}} \left(2\sqrt{\frac{E_{\rm LPM}}{E}} - \frac{E_{\rm LPM}}{E} \right) \end{aligned}$$

Slowly rising R_{AA}

Plateau at high p_T

- If $l_{\rm coh} > L$, effectively only a single scattering happens. $\Longrightarrow$ Goes back to BH

If $E > E_L = L^2\mu^2/\lambda$,

$$\begin{aligned}
\ln R_{AA} &\approx -\frac{nL}{E}\int_0^{E_{\rm LPM}} d\omega\omega\left(\frac{\alpha_S}{\pi\omega}\frac{N_c}{l_{\rm mfp}}\right) \\
&\quad -\frac{nL}{E}\int_{E_{\rm LPM}}^{E_L} d\omega\omega\left(\frac{\alpha_S}{\pi\omega}N_c\sqrt{\frac{\mu^2}{l_{\rm mfp}\omega}}\right) \\
&\quad -\frac{nL}{E}\int_{E_L}^{E} d\omega\omega\left(\frac{\alpha_S}{\pi\omega}\frac{N_c}{L}\right) \\
&\approx -n\frac{\alpha_S N_c}{\pi}\left(1+\frac{E_L}{E}(1-l_{\rm mfp}/L)\right)
\end{aligned}$$

This is approximately constant for large E.

Strongly coupled?

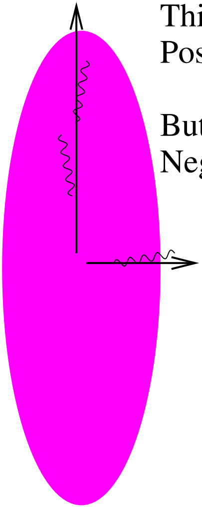
- Most models use $\alpha_S \approx 1/3$.
- The transport coefficient

$$\hat{q} = \frac{\mu^2}{l_{\rm mfp}} \sim \alpha^2 T^3$$

- Can we pin-point $\hat{q}$?

A short detour – Understanding high p_T part of v_2 with energy loss

Understanding high p_T part of v_2



This jet loses more energy:
Positive v2

But it radiates more photons:
Negative photon v2

Understanding high p_T part of v_2

- Start with an isotropic distribution of high energy particles
- After going through the almond:

$$p_x = E - \Delta E_x$$
$$p_y = E - \Delta E_y$$

That is,

$$p_x^2 \approx E^2 - 2\Delta E_x E$$

- Elliptic flow definition:

$$v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \sim \frac{2\Delta E_y E - 2\Delta E_x E}{2E^2} = \left(\frac{\Delta E_y - \Delta E_x}{E} \right)$$

Approx. relationship between R_{AA} and v_2 at high p_T

- BDMPS: If $dN/p_T dp_T \approx 1/p_T^n$, $\ln R_{AA} \approx -n\dfrac{\Delta E}{E}$
- If $E < E_{\rm LPM} = \mu^2 l_{\rm mfp}$, $\ln R_{AA} \approx -\dfrac{nL}{E}\dfrac{\alpha_S N_c}{\pi_{\rm mfp}}$

$$v_2 \sim \left(\frac{\Delta E_y - \Delta E_x}{E}\right) \propto (L_y - L_x)$$

Flat v_2

- If $E_{\rm LPM} < E < E_L = L^2\mu^2/l_{\rm mfp}$,

$$\ln R_{AA} \sim -\frac{nL\alpha_S N_c}{\pi l_{\rm mfp}}\left(2\sqrt{\frac{E_{\rm LPM}}{E}} - \frac{E_{\rm LPM}}{E}\right)$$

$$v_2 \sim \left(\frac{\Delta E_y - \Delta E_x}{E}\right) \propto (L_y - L_x)\sqrt{\frac{\hat{q}}{E}}$$

Slowly falling v_2

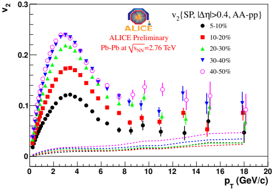
Very high p_T

- If $E > E_L = L^2\mu^2/l_{\rm mfp}$, $\ln R_{AA} \approx -n\frac{\alpha_S N_c}{\pi}\left(1 + \frac{E_L}{E}(1 - l_{\rm mfp}/L)\right)$

$$v_2 \sim \left(\frac{\Delta E_y - \Delta E_x}{E}\right) \propto (L_y^2 - L_x^2)\frac{\hat{q}}{E}$$

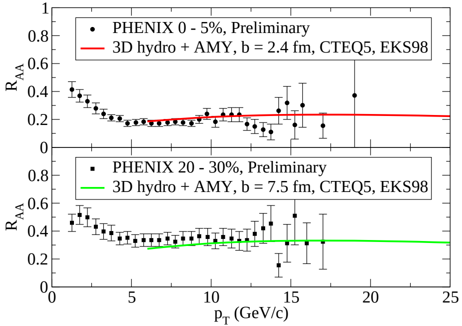
Faster falling v_2

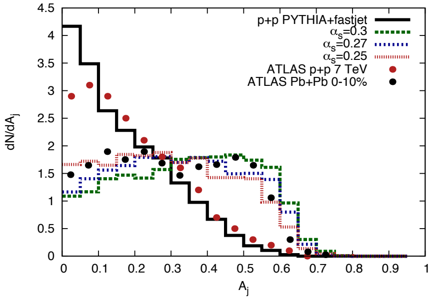
LHC Data



- Data: ALICE, 1105.3865v2
- High p_T v_2: Flat, then falls like $1/\sqrt{p_T}$ and then $1/p_T$.
- Can understand high p_T data qualitatively although $1/p_T$ behavior may not be visible since this is for $E > E_L$.
- The slope $dv_2/dp_T \propto -\sqrt{\hat{q}}$
- Of course, this is very rough: Viscosity also curves it down and $p_T \gtrsim 3\,\text{GeV}$ may not be high enough.

- What α_S do we need for these?





$$R_{AA} = \frac{(dN_{AA}/dp_T)}{(N_{\rm coll}\, dN_{pp}/dp_T)}$$

$$A_j = (E_> - E_<)/(E_> + E_<)$$

Tool

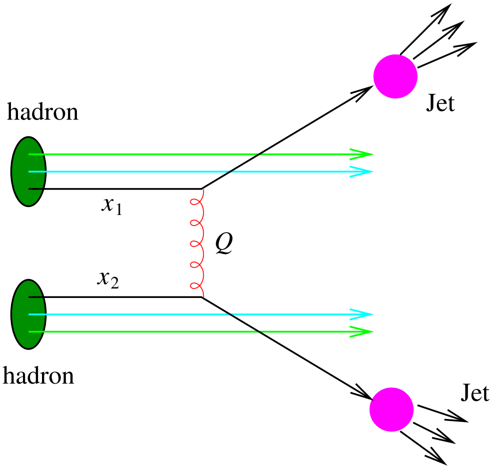
- Event generator
  – Jet propagation through evolving QGP medium.
- Several on the market. We use MARTINI.

MARTINI

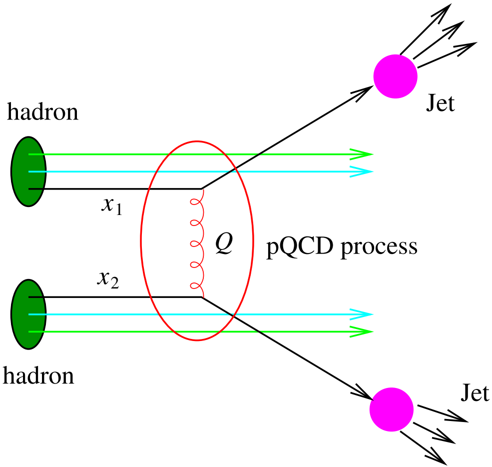
MARTINI

- Modular Alogorithm for Relativistic Treatment of Heavy IoN Interactions
- Hybrid approach
  - Calculate Hydrodynamic evolution of the soft mode (MUSIC)
  - Propagate jets in the evolving medium according to McGill-AMY

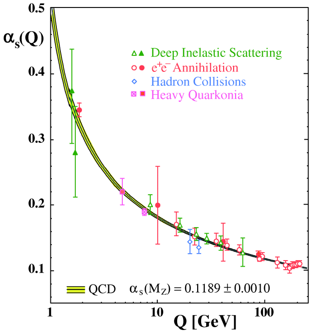
Hadronic Jet production



Hadronic Jet production

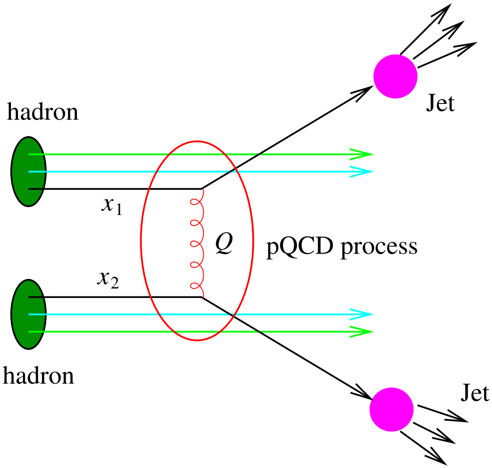


If $Q \gg \Lambda_{\rm QCD}$, $\alpha_s(Q) \ll 1$:
Jet production is perturbative.



Bethke, Prog.Part.Nucl.Phys. 58 (2007) 351-386.

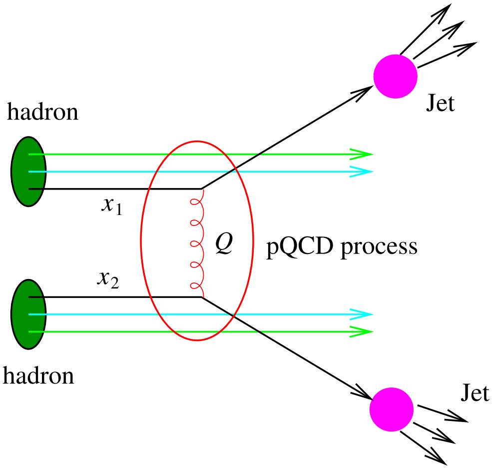
Hadronic Jet production



If $Q \gg \Lambda_{\rm QCD}$, $\alpha_s(Q) \ll 1$:
Jet production is perturbative.

➥ Calculation is possible.

Hadronic Jet production

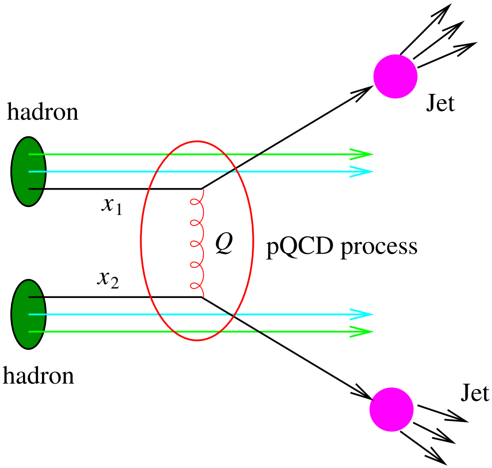


If $Q \gg \Lambda_{\text{QCD}}$, $\alpha_s(Q) \ll 1$:
Jet production is perturbative.

➥ Calculation is possible.

➥ We understand this process in hadron-hadron collisions.

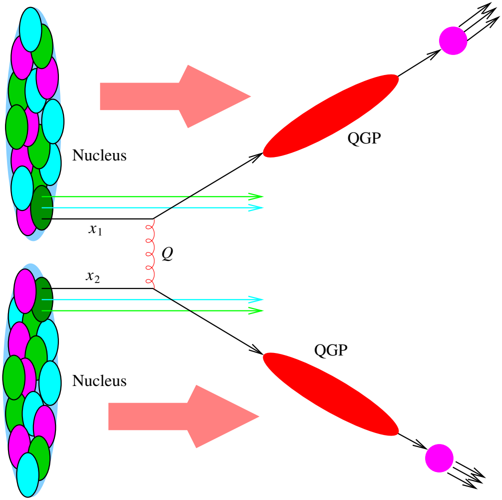
Hadronic Jet production



Hadron-Hadron Jet production scheme:

$$\frac{d\sigma}{dt} = \int_{abcd} f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \times \frac{d\sigma_{ab\to cd}}{dt} D(z_c, Q)$$

Heavy Ion Collisions

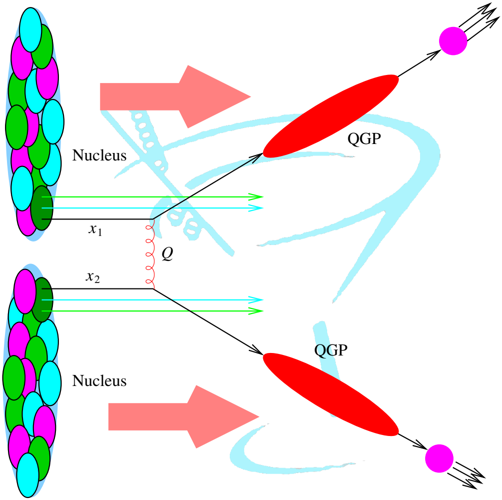


HIC Jet production scheme:

$$\frac{d\sigma_{AB}}{dt} = \int_{\text{geometry}} \int_{abcdc'} \times f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \times \frac{d\sigma_{ab\to cd}}{dt} \times \mathcal{P}(x_c \to x'_c | T, u^\mu) \times D(z'_c, Q)$$

$\mathcal{P}(x_c \to x'_c | T, u^\mu)$: Medium modification of high energy parton property

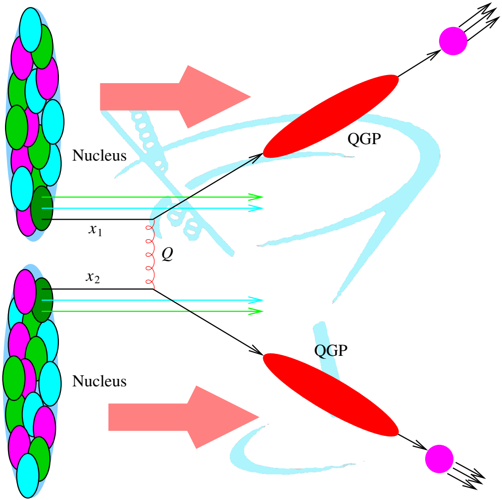
MARTINI - Basic Idea



$$\frac{d\sigma_{AB}}{dt} = \int_{\text{geometry}} \int_{abcdc'}$$
$$\times f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f)$$
$$\times \frac{d\sigma_{ab\to cd}}{dt}$$
$$\times \mathcal{P}(x_c \to x_c' | T, u^\mu)$$
$$\times D(z_c', Q)$$

- Sample collision geometry using Wood-Saxon

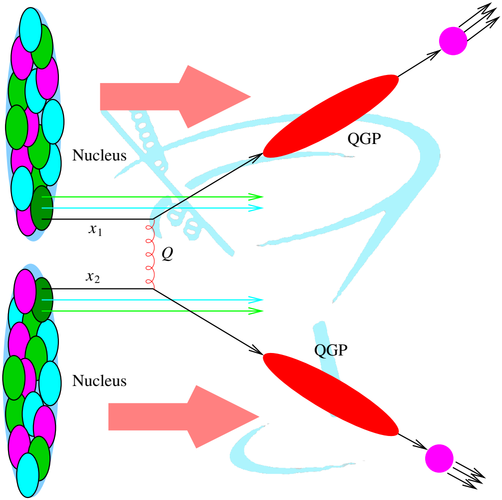
MARTINI - Basic Idea



$$\frac{d\sigma_{AB}}{dt} = \int_{\text{geometry}} \int_{abcdc'} \times f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \times \frac{d\sigma_{ab\to cd}}{dt} \times \mathcal{P}(x_c \to x'_c | T, u^\mu) \times D(z'_c, Q)$$

- PYTHIA 8.1 generates high p_T partons
- Shadowing included
- Shower (Radiation) stops at $Q = \sqrt{p_T/\tau_0}$

MARTINI - Basic Idea



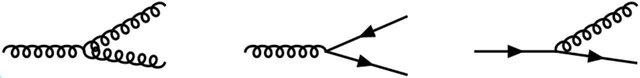
$$\frac{d\sigma_{AB}}{dt} = \int_{\text{geometry}} \int_{abcdc'} \times f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \times \frac{d\sigma_{ab\to cd}}{dt} \times \mathcal{P}(x_c \to x'_c | T, u^\mu) \times D(z'_c, Q)$$

- Hydrodynamic phase (MUSIC)
- AMY evolution – MC simulation of the rate equ's.

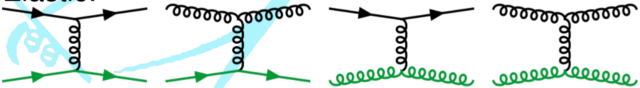
Parton propagation

Process include in MARTINI (all of them can be switched on & off):

- Inelastic:



- Elastic:

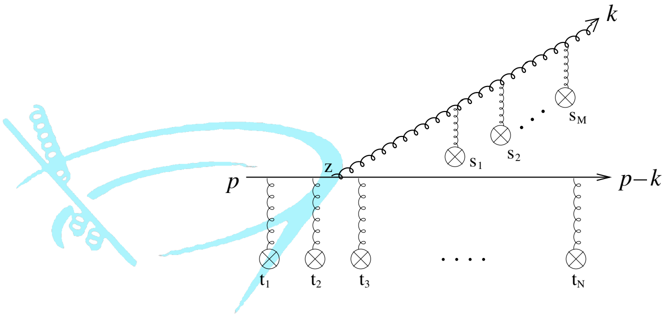


- Conversion:

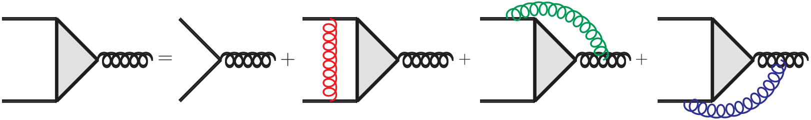
- Photon: emission & conversion

Parton propagation

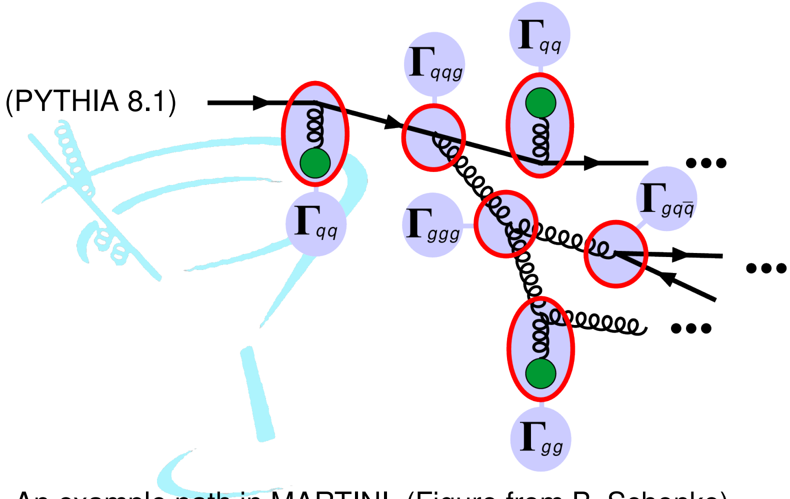
Resummation for the inelastic processes included:



- All such graphs are leading order (BDMPS)
- Full leading order SD-Eq (AMY): (Figure from G. Qin)



Parton propagation



An example path in MARTINI. (Figure from B. Schenke)

Ideal MUSIC

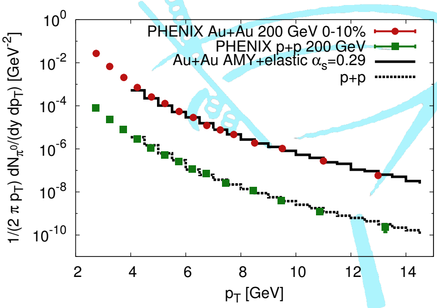
- While this is happening in the background ...

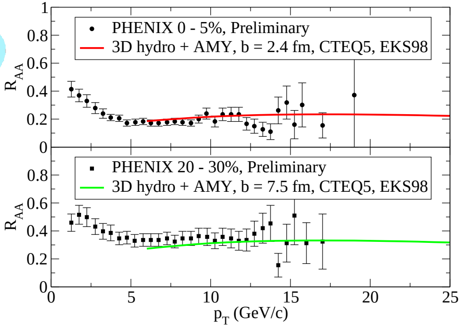
Projection on to the longitudinal plane

Projection onto the transverse plane

[Schenke, Jeon and Gale, Phys. Rev. C 80, 054913 (2009)]

- π^0 spectra and R_{AA}



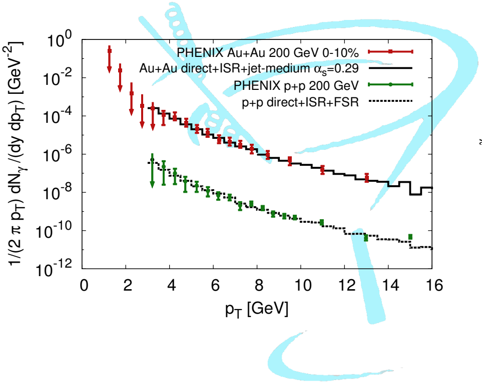


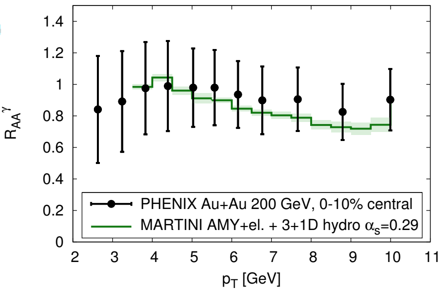
- For RHIC, $\alpha_S = 0.29$

Photon production

[Schenke, Jeon and Gale, Phys. Rev. C 80, 054913 (2009)]

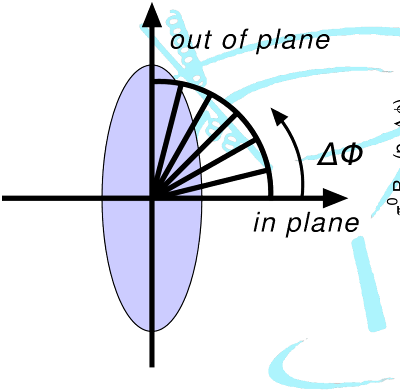
- Spectra and R_{AA}^{γ}

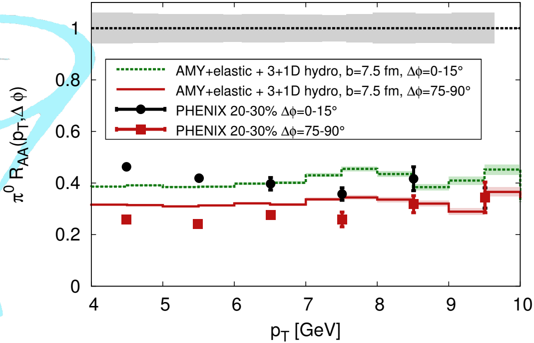




- $\alpha_S = 0.29$

- $R_{AA}(p_T, \Delta\phi)$



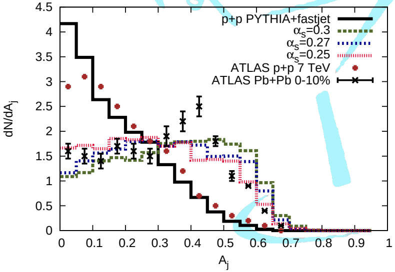


- $\alpha_S = 0.29$

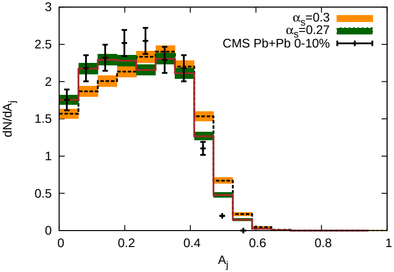
MARTINI – LHC dN/dA

[Young, Schenke, Jeon, Gale, Phys. Rev. C 84, 024907 (2011)].

- $A = (E_t - E_a)/(E_t + E_a)$
- This is with ideal hydro with a smooth initial condition
- Full jet reconstruction with FASTJET
- $\alpha_s = 0.27$ seems to work.



ATLAS, PRL 105 (2010) 252303



CMS, arXiv: 1102.1957 (2011)

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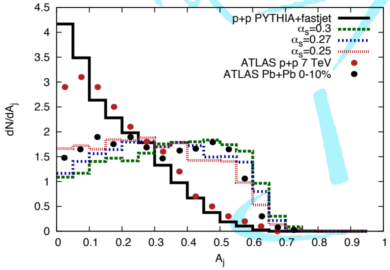
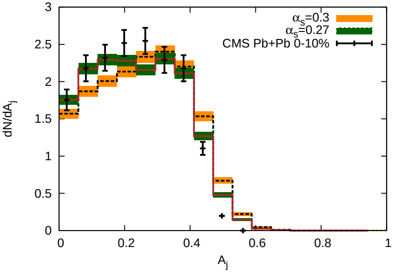
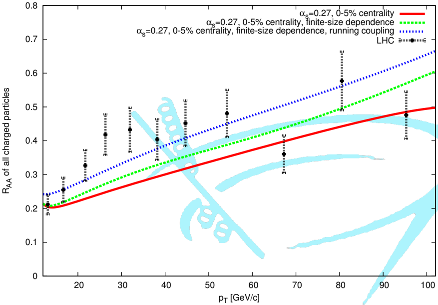


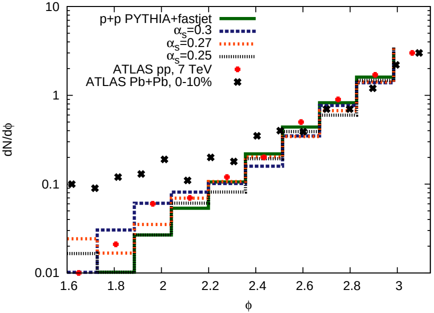
ATLAS, QM 2011



CMS, arXiv: 1102.1957 (2011)

[Clint Young's HP2012 Proceedings]





- R_{AA} – For LHC, constant α_S suppresses jets too much.
- Need to incorporate finite length effect (Caron-Huot-Gale) and running α_s. This is with maximum $\alpha_S = 0.27$.
- Don't quite get azimuthal dependence yet. $\Delta\phi$ broadening may be due to the background fluctuations $\Longrightarrow$ Need to combine UrQMD background?

Conclusions, Summary and Open questions

- Thermal QCD quantities
  - pQCD formulas seem to work for thermodynamic quantities albeit with $\alpha_S \approx 0.3 - 0.5$.
  - pQCD calculation of $\eta/s \approx 7.5/(4\pi)$ fails miserably with $\alpha_S \approx 0.3 - 0.5$
- AdS/CFT with $\lambda = \infty$ OK with both
- Jet quenching needs $\alpha_S \approx 0.3$ and running (towards smaller values) at the LHC.
  - Apples and Oranges. This is hard on soft where as the above are soft only.

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- Where do we stand?
  - Why do pQCD formulas work well when they do? Is $\alpha_S = 0.3$, or $g = 2$, or $g/2\pi = 0.3$ small enough for perturbation?
  - LQCD seems to measure small η/s $\Longrightarrow$ Is it possible that higher order corrections brings $\eta/s \sim 0.2$?

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- What else can we do?

Some (random) thoughts

- Jets pulling or (pushing) the medium?
- The cross-section defines minimum granularity $\Longrightarrow$ Big cross-section suppress higher v_n. What's the relationship?
- How can we experimentally get at the thermalization time (hydro τ_0)?