Heavy-Ion Meeting (HIM 2013)

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ASYMMETRIC MATTER PROPERTIES IN A CHIRAL SOLITON MODEL

Ulugbek Yakhshiev
Inha University

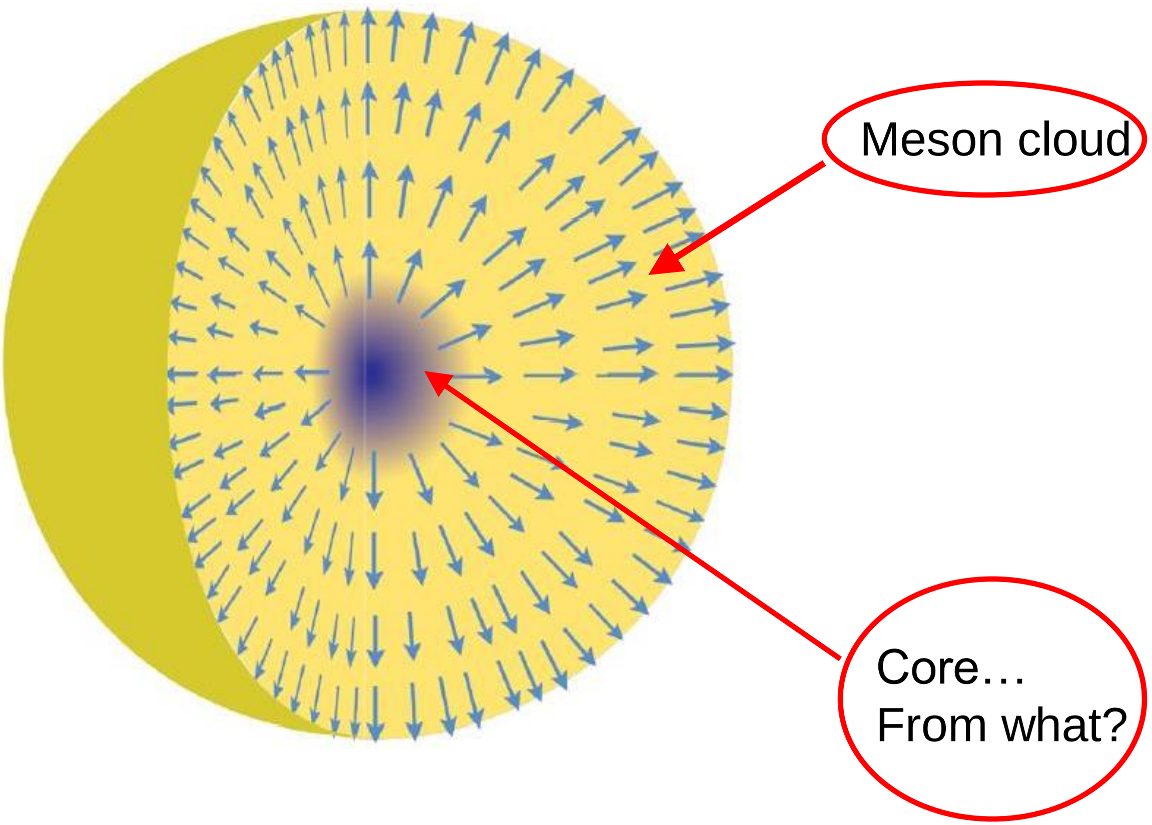
Content

- Topological models and soliton
- Medium modifications of nucleons
  - "Outer shell" modifications
  - "Inner core" modifications
- Nuclear matter
  - Symmetric matter
  - Asymmetric matter
- Structure analysis of in-medium nucleons
- Summary
- Outlook

Topological models and soliton

Structure

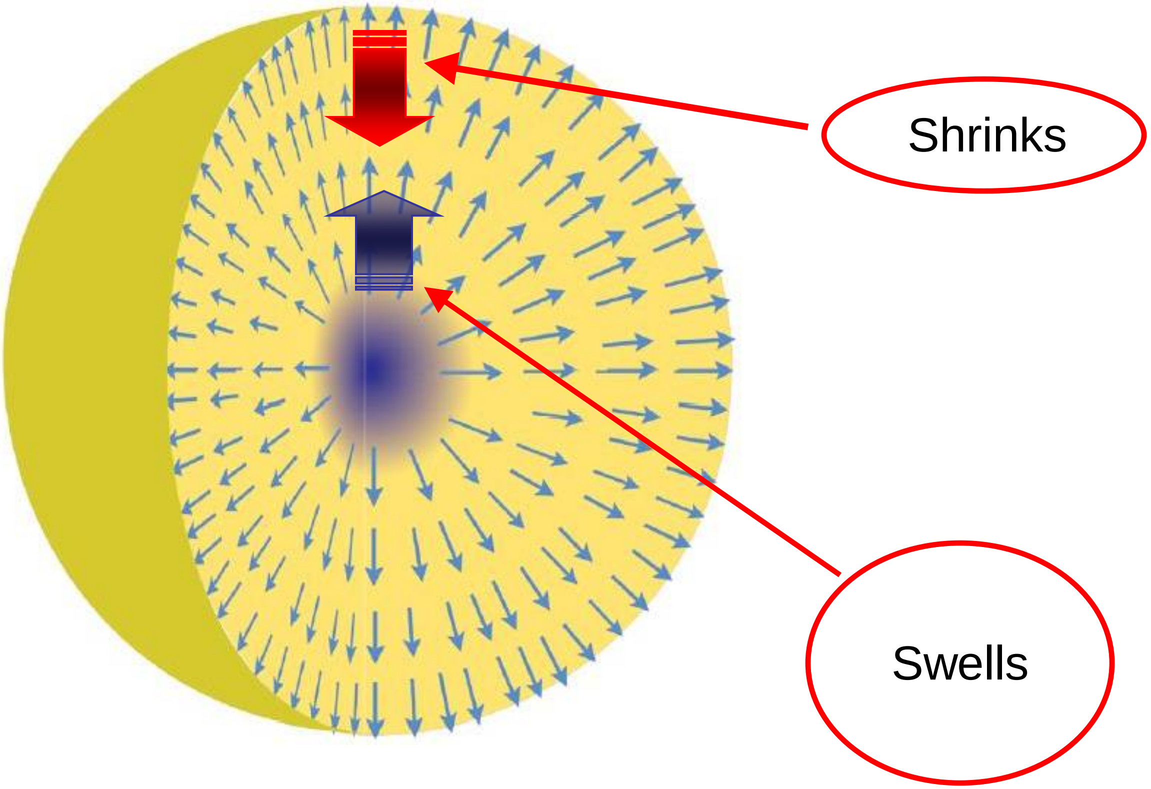
- What is a nucleon and, in particular, its core?
- At large number of colors it still has the mesonic content



Topological models and soliton

Stabilization

- Soliton has finite size and finite energy
- One needs at least two contrterms in the effective Lagrangian



Topological models and soliton

Skyrme model

[T.H.R. Skyrme, Pros.Roy.Soc.Lond. A260(1961)]

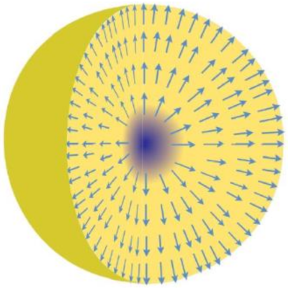
- Nonlinear chiral effective meson (pionic) theory)

$$\mathcal{L} = \frac{F_\pi^2}{16}\mathrm{Tr}\left(\partial_\alpha U\right)\left(\partial^\alpha U^+\right) + \frac{1}{32e^2}\mathrm{Tr}\left[U^+\partial_\alpha U, U^+\partial_\beta U\right]^2$$

Shrinks ←→ Swells

- Hedgehog soliton (nontrivial mapping)

$$U = \exp\left\{\frac{i\bar{\tau}\,\bar{\pi}}{2F_\pi}\right\} = \exp\{i\bar{\tau}\,\bar{n}F(r)\}$$



Topological models and soliton

Original Lagrangian in use

[G.S. Adkins *et al.* Nucl. Phys. B228 (1983)]

$$\mathcal{L}_{\text{free}} = \frac{F_\pi^2}{16}\mathrm{Tr}\left(\partial^\alpha U\right)\left(\partial_\alpha U^+\right) + \frac{1}{32e^2}\mathrm{Tr}\left[U^+\partial_\alpha U, U^+\partial_\beta U\right]^2 + \frac{F_\pi^2 m_\pi^2}{16}\mathrm{Tr}\left(U + U^+ - 2\right)$$

- Nontrivial mapping
- It has topologically nontrivial solitonic solutions in different topological sectors with corresponding conserved topological number A
- Nucleon is quantized state of the classical soliton-skyrmion

$$U = \exp\{i\bar{\tau}\,\bar{\pi}/2F_\pi\} = \exp\{i\bar{\tau}\,\bar{n}F(r)\}$$

$$B^\mu = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\alpha\beta}Tr(L_\nu L_\alpha L_\beta) \qquad L_\alpha = U^+\partial_\alpha U$$

$$A = \int d^3 r B^0$$

$$H = M_{cl} + \frac{\bar{S}^2}{2I} = M_{cl} + \frac{\bar{T}^2}{2I},$$

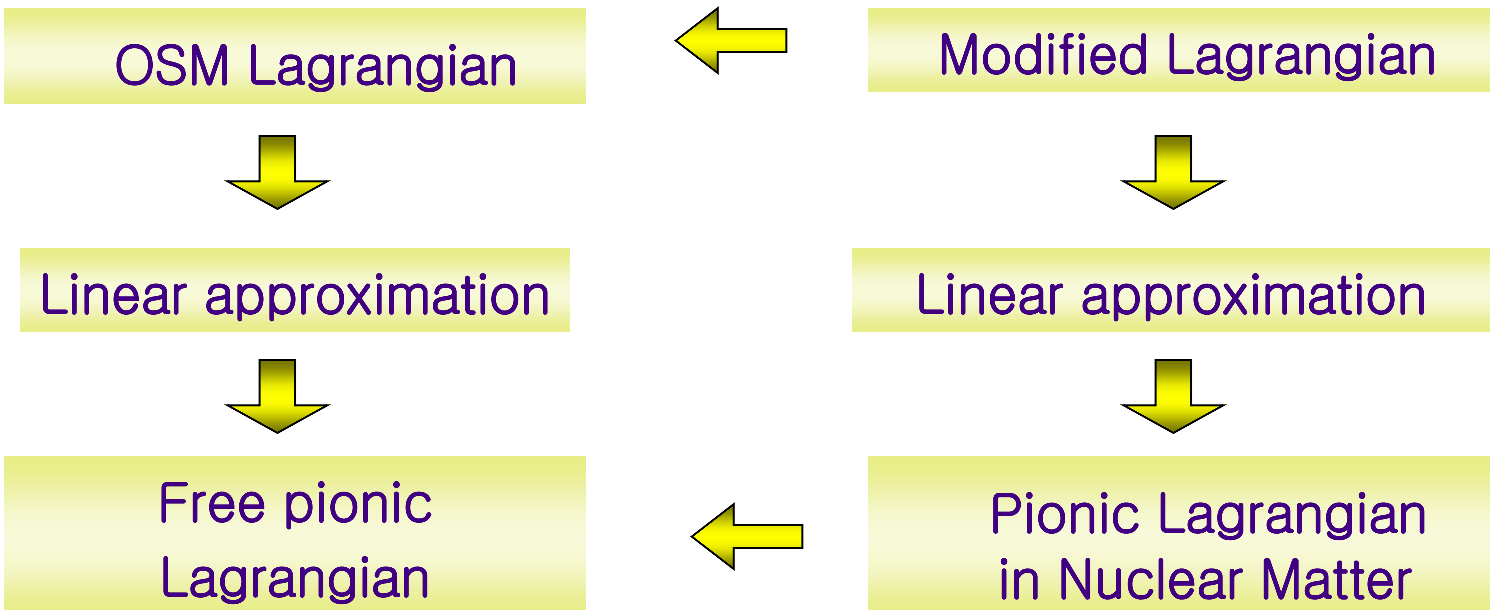
$$|\,S = T, s, t> = (-1)^{t+T}\sqrt{2T+1}D_{-t,s}^{S=T}(A)$$

Medium modifications

- What happens in a medium?
- One should be able to describe
  - Deformations
  - Mass change
  - Swelling
  - Effective NN interactions
  - Etc.

Medium modifications

- Modification in the mesonic sector modifies the baryonic sector

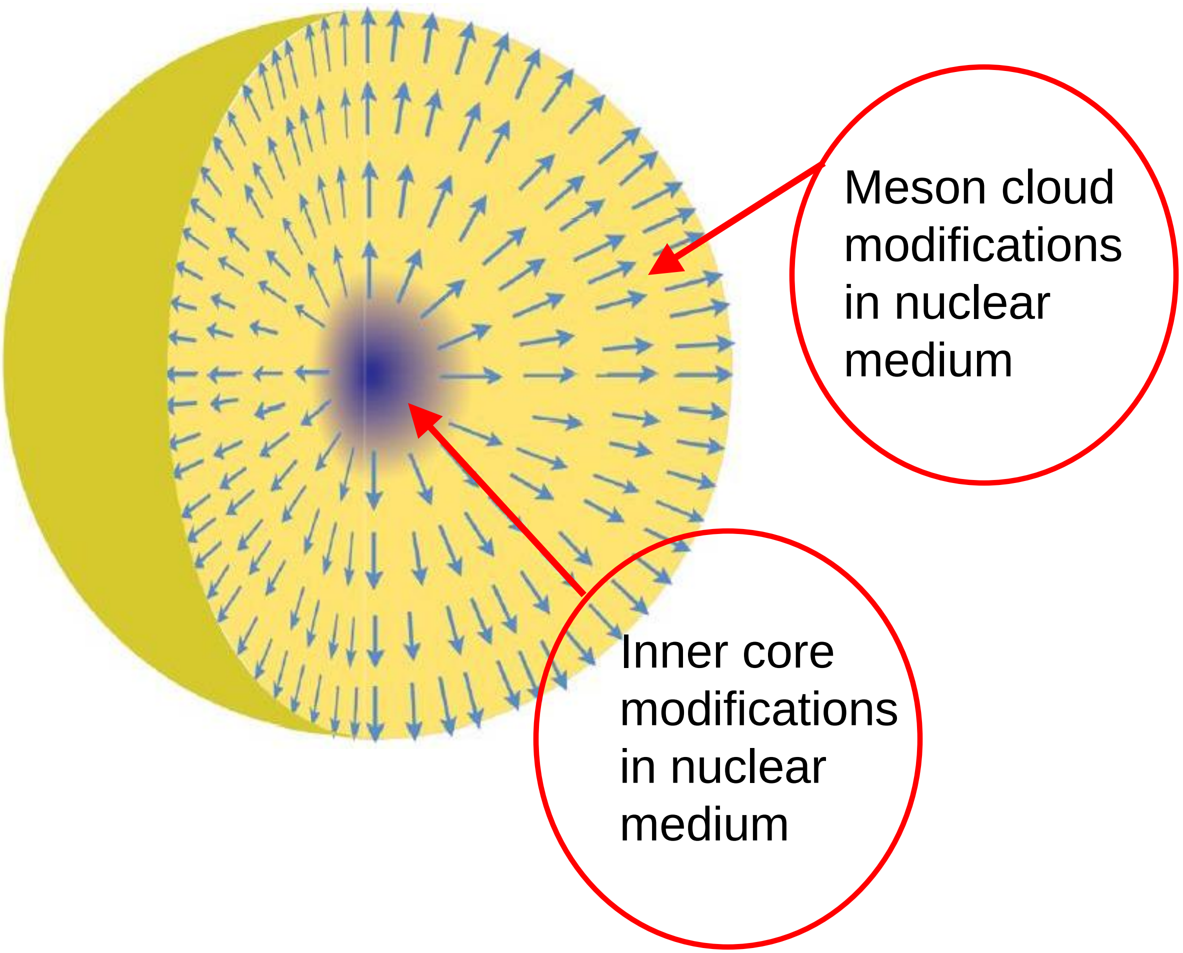


- How to modify the mesonic sector?

Medium modifications

Soliton in Nuclear Medium

- Outer shell modifications plus
- Inner core modifications (in particular at higher densities)



Medium modifications

"Outer shell" modifications

- Three types of pions treated separately
- In nuclear matter, one considers three types of polarization operators
- There will be some parameters which correspond to isospin breaking effects in the surrounding environment

$$\left(\partial^{\mu}\partial_{\mu} + m^2_{\pi^{(\pm,0)}}\right)\vec{\pi}^{(\pm,0)} = 0$$

$$\left(\partial^{\mu}\partial_{\mu} + m^2_{\pi^{(\pm,0)}} + \hat{\Pi}^{(\pm,0)}\right)\vec{\pi}^{(\pm,0)} = 0$$

	π-atom	$T_\pi = 50$ MeV
$b_0\ [m_\pi^{-1}]$	- 0.03	- 0.04
$b_1\ [m_\pi^{-1}]$	- 0.09	- 0.09
$c_0\ [m_\pi^{-3}]$	0.23	0.25
$c_1\ [m_\pi^{-3}]$	0.15	0.16
g'	0.47	0.47

Medium modifications

"Inner core" modifications

[UY & HC Kim, PRC83 (2011); UY, JKPS62 (2013)]

- May be related to
  - vector meson properties in nuclear matter
  - nuclear matter properties

$$\mathcal{L}_4^* = -\frac{1}{16e_\tau^{*2}} \mathrm{Tr}\left[L_0, L_i\right]^2 + \frac{1}{32e_s^{*2}} \mathrm{Tr}\left[L_i, L_j\right]^2$$

$$\boxed{e \to e^* = e\,\gamma^{1/2}(\rho)}$$

Medium modifications

Final Lagrangian

[U.Meissner *et al.*, EPJ A36 (2008); UY, JKPS62 (2013)]

- Separated into two parts

$$\mathcal{L}^* = \mathcal{L}^*_{\rm sym} + \mathcal{L}^*_{\rm asym}$$

  - Isoscalar part

$$\mathcal{L}^*_{\rm sym} = \mathcal{L}^*_2 + \mathcal{L}^*_4 + \mathcal{L}^*_{\chi SB}$$

  - Isovector part

$$\mathcal{L}^*_{\rm asym} = \Delta\mathcal{L}_{\rm mes} + \Delta\mathcal{L}^*_{\rm env}$$

$$\begin{aligned}
\mathcal{L}^*_2 &= \frac{F_\pi^2}{16}\Big\{\alpha_s^{02}\,\mathrm{Tr}\,(\partial_0 U \partial_0 U^\dagger) \\
&\qquad - \alpha_p^0\,\mathrm{Tr}(\vec{\nabla} U \cdot \vec{\nabla} U^\dagger)\Big\}, \\
\mathcal{L}^*_4 &= -\frac{1}{16e^2\zeta_\tau}\,\mathrm{Tr}\,[U^\dagger\partial_0 U, U^\dagger\partial_i U]^2 \\
&\quad + \frac{1}{32e^2\zeta_s}\,\mathrm{Tr}\,[U^\dagger\partial_i U, U^\dagger\partial_j U]^2, \\
\mathcal{L}^*_{\chi SB} &= \frac{F_\pi^2 m_\pi^2}{8}\,\alpha_s^{00}\,\mathrm{Tr}\,(U-1),
\end{aligned}$$

$$\begin{aligned}
\Delta\mathcal{L}_{\rm mes} &= -\frac{F_\pi^2}{32}\sum_{a=1}^{2}(m_{\pi^\pm}^2 - m_\pi^2)\mathrm{Tr}(\tau_a U)\mathrm{Tr}(\tau_a U^\dagger), \\
\Delta\mathcal{L}^*_{\rm env} &= -\frac{F_\pi^2}{32}\sum_{a,b=1}^{2}\varepsilon_{ab3}\frac{\Delta\chi}{m_\pi}\mathrm{Tr}(\tau_a U)\mathrm{Tr}(\tau_b\partial_0 U^\dagger).
\end{aligned}$$

Medium modifications

Medium functionals and their parameters

[U.Meissner *et al.*, EPJ A36 (2008); UY, JKPS62 (2013)]

- From pionic atoms and pion-nucleon scattering

$$\alpha_s^{00} = 1 + \left(\tilde{b}_0 + \frac{3(3\pi^2\rho/2)^{1/3}}{8\pi^2\eta} \tilde{b}_0^2 \right) \frac{\rho}{m_\pi^2},$$

$$\alpha_s^{02} = 1 + \left(\tilde{b}_0 + \frac{3(3\pi^2\rho/2)^{1/3}}{4\pi^2\eta} \left(\tilde{b}_0^2 - \tilde{b}_1^2 \right) \right) \frac{\rho}{m_\pi^2},$$

$$\alpha_p^0 = 1 - \frac{2\pi\,(c_0\rho - c_1\delta\rho)}{\eta + 4\pi g'\,(c_0\rho - c_1\delta\rho)} - \frac{2\pi\,(c_0\rho + c_1\delta\rho)}{\eta + 4\pi g'\,(c_0\rho + c_1\delta\rho)},$$

$$\Delta\chi = \tilde{b}_1\delta\rho - \frac{2\pi m_\pi}{\eta m_{\rm N}} c_1 \left(\vec{\nabla}^2\delta\rho \right)$$

Nuclear matter stabilization

- From nuclear matter properties

$$\zeta_\tau = 1 + \zeta_0\rho\,,$$

$$\zeta_s = \frac{1}{2}\left\{ \exp\left(-\frac{\gamma_{{\rm n},0}\rho + \gamma_{{\rm n},1}\delta\rho}{1 + \gamma_{{\rm d},0}\rho + \gamma_{{\rm d},1}\delta\rho} \right) + \exp\left(-\frac{\gamma_{{\rm n},0}\rho - \gamma_{{\rm n},1}\delta\rho}{1 + \gamma_{{\rm d},0}\rho - \gamma_{{\rm d},1}\delta\rho} \right) \right\}$$

Medium modifications

Consistency with other approaches

- Effective values of pion decay constant and pion mass are qualitative agreement with
  - in-medium CPT [U.Meissner, J.Oller, A.Wirzba, Ann. Phys. 297 (2002)]
  - QCD sum rules [H.c.Kim, M.Oka, NPA 720 (2003)]

$$F_{\pi,t} \to F^*_{\pi,t} = F_\pi (1 + \chi_s^{02} m_\pi^{-2})$$

$$F_{\pi,s} \to F^*_{\pi,s} = F_\pi (1 - \chi_p^{0})$$

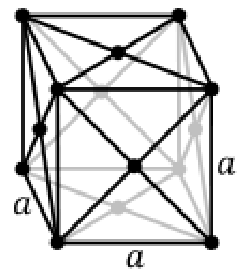
$$m_\pi \to m^*_\pi = m_\pi \left(\frac{1 + \chi_s^{00} m_\pi^{-2}}{1 - \chi_p^{0}} \right)$$

Medium modifications

Consistency with other Skyrme Model approaches

[Manton & Wood, PRD74 (2006)]

- Nuclear matter studies in framework of the Skyrme model (alternate approaches)
  - "Calibration" method
  - Crystalline structures
- Ideas behind



[H.J.Lee *et al.*,NPA723 (2003)]

$$F_\pi \to F_\pi^*, \quad m_\pi \to m_\pi^*, \quad e \to e^*$$

Medium modifications

Binding-energy-formula terms in our model

$$\varepsilon(A,Z) \;=\; -a_V + a_S\,\frac{(N-Z)^2}{A^2} + \dots,$$

- Volume term
  - Infinite and asymmetric nuclear matter
- Asymmetry term
  - Isospin asymmetric environment
- Surface and Coulomb terms
  - Nucleons in a finite volume
- Finite nuclei properties
  - Local density approximation

Medium modifications

Volume term and Symmetry energy

- At infinite nuclear matter approximation the binding energy formula takes form

$$\begin{aligned}\varepsilon(\lambda,\beta) &= -a_V(\lambda)+a_S(\lambda)\beta^2+\mathcal{O}(\beta^4)\\ &\equiv \varepsilon_V(\lambda)+\varepsilon_S(\lambda,\beta)\,,\end{aligned}$$

  - λ is a normalized nuclear matter density
  - β is an asymmetry parameter
  - a_S is the symmetry energy

Symmetric Nuclear Matter

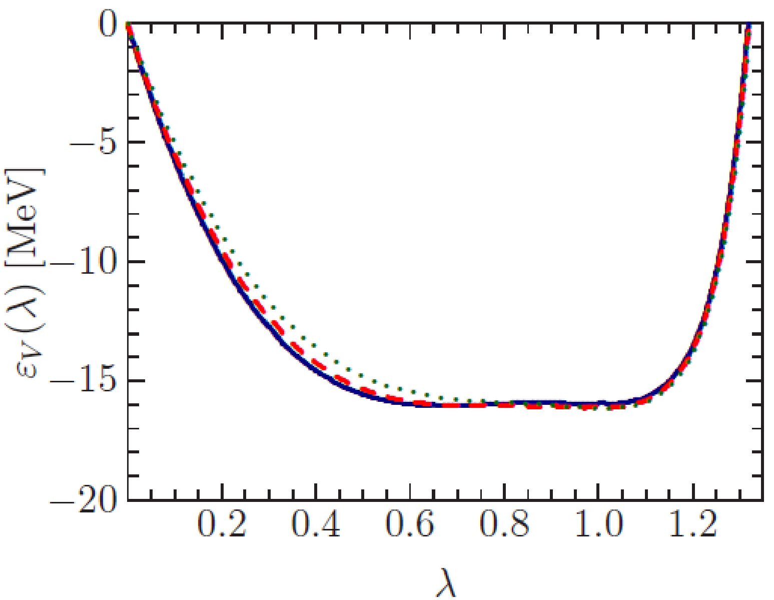
Volume term

- Volume term (binding energy per nucleon) in the binding energy formula can be defined as

$$\varepsilon_V(\lambda) = \frac{m_p^*(\lambda,0)+m_n^*(\lambda,0)}{2} - \frac{m_p+m_n}{2}$$

- Model I - solid curve
- Model II - dashed curve
- Model III – dotted curve

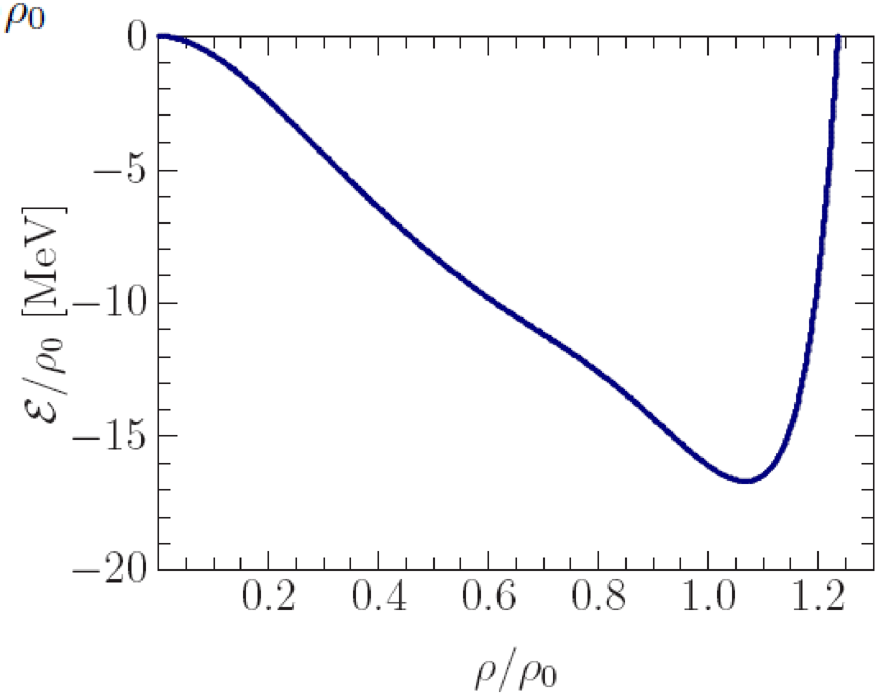
Model	γ_0 $[m_\pi^{-3}]$	$\gamma_{n,0}$ $[m_\pi^{-3}]$	$\gamma_{d,0}$ $[m_\pi^{-3}]$
I	0.0	1.901	0.070
II	0.5	1.867	0.049
III	1.0	1.840	0.031



Symmetric Nuclear Matter

- Fraction of the binding energy per unit volume to normal nuclear matter density as a function of normalized density [UY, JKPS60(2012)]

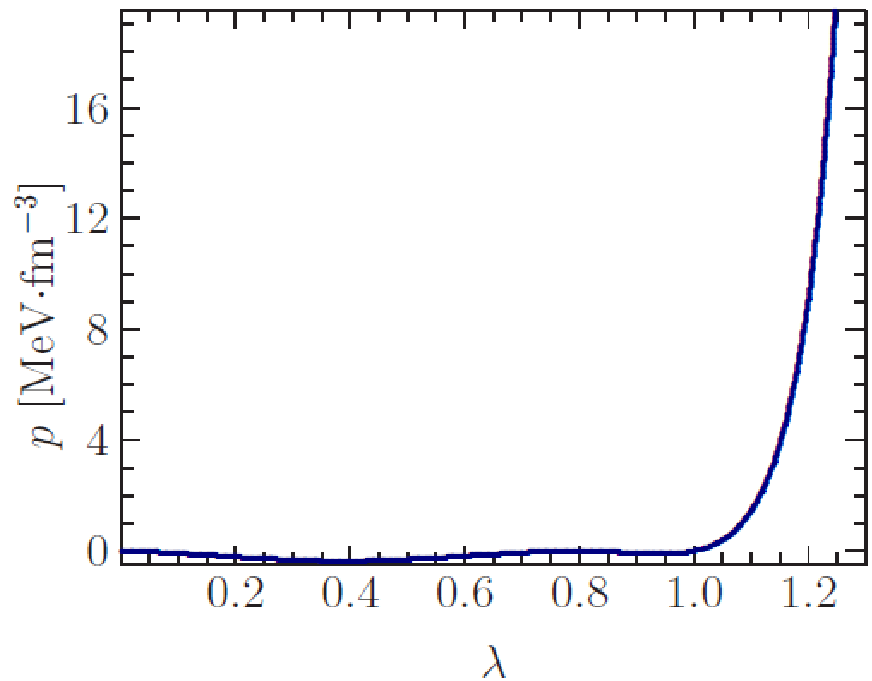
$$\tilde{\mathcal{E}}_V \equiv \frac{\mathcal{E}_V(\rho)}{V} = -a_V(\rho)\frac{A}{V} = -\lambda a_V(\lambda)\rho_0$$



Symmetric Nuclear Matter

- Pressure as a function of normalized density (Model II)

$$p = \rho \frac{\partial \tilde{\mathcal{E}}_V(\rho)}{\partial \rho} - \tilde{\mathcal{E}}_V(\rho) = -\rho_0 \lambda^2 \frac{\partial a_V(\lambda)}{\partial \lambda}$$



Symmetric Nuclear Matter

TABLE I: The volume term coefficient $a_V(1)$ at the normal nuclear matter density $\lambda = 1$ and the compression modulus K_0 of symmetric nuclear matter. Their values are given for the three different sets of parameters. The variational parameters $\gamma_{\mathrm{n},0}$ and $\gamma_{\mathrm{d},0}$ are chosen in such a way that at saturation point $\rho = \rho_0 = 0.15\ \mathrm{fm}^{-3}$ the value of volume energy per nucleon is close to its experimental value, $\varepsilon_V^{\mathrm{exp}} \simeq -16$ MeV.

Model	γ_0 [m_π^{-3}]	$\gamma_{\mathrm{n},0}$ [m_π^{-3}]	$\gamma_{\mathrm{d},0}$ [m_π^{-3}]	$a_V(1)$ [MeV]	K_0 [MeV]
I	0.0	1.901	0.070	15.94	202
II	0.5	1.867	0.049	16.11	218
III	1.0	1.840	0.031	16.12	366

Asymmetric Nuclear Matter

- Asymmetry energy

$$\varepsilon_S(\lambda,\beta) = \frac{m_{\rm p}^*(\lambda,\beta)+m_{\rm n}^*(\lambda,\beta)}{2} - \frac{m_{\rm p}^*(\lambda,0)+m_{\rm n}^*(\lambda,0)}{2}$$

- Symmetry energy

$$a_S(\lambda) = \frac{1}{2}\left.\frac{\partial^2 \varepsilon_S(\lambda,\beta)}{\partial\beta^2}\right|_{\beta=0}$$

- Symmetry energy coefficients

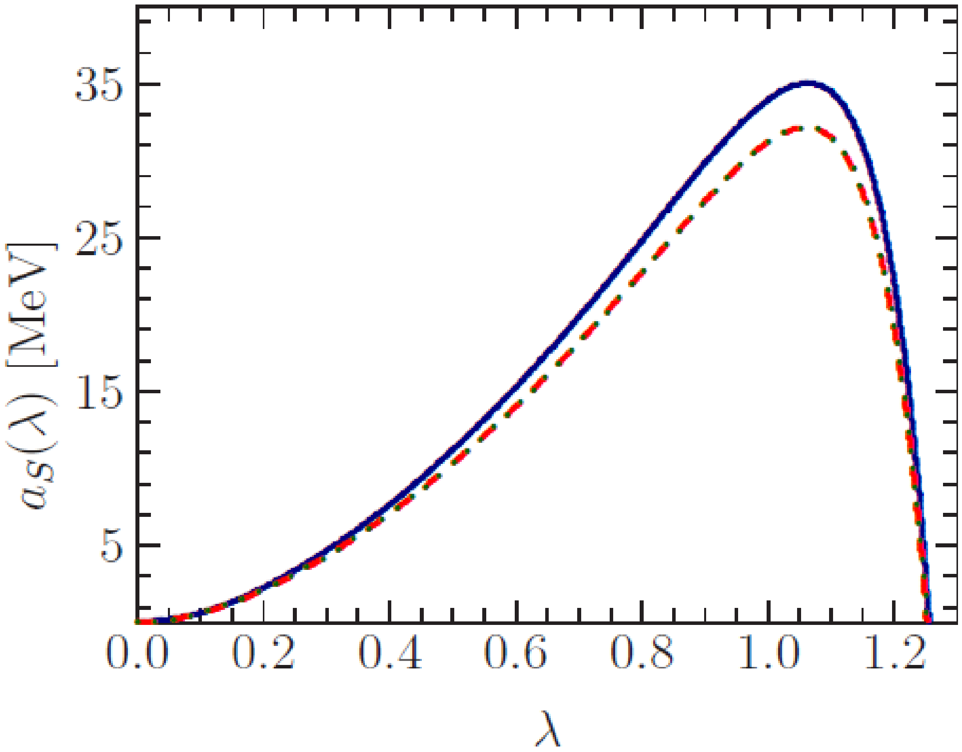
$$a_S(\lambda) = a_S(1) + \frac{L_S}{3}(\lambda-1) + \frac{K_S}{18}(\lambda-1)^2 + \ldots$$

Asymmetric Nuclear Matter

Symmetry energy

- Symmetry energy as function of normalized density
  - Model I - solid curve
  - Model II - dashed curve
  - Model III – dotted curve

Model	$\gamma_{n,1}$ $[m_\pi^{-3}]$	$\gamma_{d,1}$ $[m_\pi^{-3}]$
I	0.830	0.415
II	0.860	0.430
III	0.830	0.374



Asymmetric Nuclear Matter

TABLE II: The slope L_S and the curvature K_S of symmetry energy. The variational parameters $\gamma_{\mathrm{n},1}$ and $\gamma_{\mathrm{d},1}$ are chosen in such a way that at normal nuclear matter density $\rho = \rho_0 = 0.15\ \mathrm{fm}^{-3}$ the value of symmetry energy $a_S(1)$ is close to its experimental value, $a_S^{\mathrm{exp}} \approx 32$ MeV. Other parameters $\gamma_{\mathrm{n},0}$ and $\gamma_{\mathrm{d},0}$ are given in Table I.

Model	$\gamma_{\mathrm{n},1}$ $[m_\pi^{-3}]$	$\gamma_{\mathrm{d},1}$ $[m_\pi^{-3}]$	$a_S(1)$ [MeV]	L_S [MeV]	K_S [MeV]
I	0.830	0.415	33.99	91.75	−3428
II	0.860	0.430	31.21	85.66	−2761
III	0.830	0.374	31.21	76.41	−2800

Structure analysis of the in-medium nucleon

- Nucleon structure itself is very interesting topic
- Scalar, vector and axial-vector properties of the nucleon have been studied extensively
- From other side GPD accessible via hard exclusive reactions gives information about Energy Momentum Tensor form factors.

Structure analysis of the in-medium nucleon

- It allows to address questions like:
  - How are the total angular momentum and angular momentum of the nucleon shared among its constituents?
  - How are the strong forces experienced by its constituents distributed inside the nucleon?

- EMT form factors studied in lattice QCD, ChPT and in different models (chiral quark soliton model, Skyrme model, etc.)

- We make further step studying EMT form factors in nuclear matter

Structure analysis of the in-medium nucleon

Energy-momentum tensor form-factors

- Definition

$$\langle p'|\hat{T}_{\mu\nu}(0)|p\rangle = \bar{u}(p',\, s')\left[M_2(t)\,\frac{P_\mu P_\nu}{M_N} + J(t)\,\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N}\right]u(p,\, s)\,,$$

- These three form factors give information about momentum distribution, angular momentum distribution and about the stabilization of strong forces inside the nucleon.

$$T^*_{00}(r) \;=\; \frac{F^{*2}_{\pi,s}}{8}\left(\frac{2\sin^2F}{r^2}+F'^2\right)+\frac{\sin^2F}{2\,e^{*2}\,r^2}\left(\frac{\sin^2F}{r^2}+2F'^2\right)+\frac{m^{*2}_\pi F^{*2}_{\pi,s}}{4}\,(1-\cos F)\,,$$

$$T^*_{0k}(r,s) \;=\; \frac{\epsilon^{klm}r^l s^m}{(\mathrm{s}\times\mathrm{r})^2}\,\rho^*_J(r)\,,$$

$$T^*_{ij}(r) \;=\; s^*(r)\left(\frac{r_ir_j}{r^2}-\frac{1}{3}\,\delta_{ij}\right)+p^*(r)\,\delta_{ij}\,,$$

$$M^*_2(t)-\frac{t}{5M^{*2}_N}\,d^*_1(t) \;=\; \frac{1}{M^*_N}\int \mathrm{d}^3r\;T^*_{00}(r)\;j_0(r\sqrt{-t})\,,$$

$$d^*_1(t) \;=\; \frac{15M^*_N}{2}\int \mathrm{d}^3r\;p^*(r)\;\frac{j_0(r\sqrt{-t})}{t}\,,$$

$$M^*_2(0) \;=\; \frac{1}{M^*_N}\int \mathrm{d}^3r\;T^*_{00}(r)=1\,,\qquad J^*(0) \;=\; \int \mathrm{d}^3r\;\rho^*_J(r)=\frac{1}{2}\,.\qquad J^*(t) \;=\; 3\int \mathrm{d}^3r\;\rho^*_J(r)\;\frac{j_1(r\sqrt{-t})}{r\sqrt{-t}}\,,$$

Structure analysis of the in-medium nucleon

Properties of the in-medium nucleons

[H.C.Kim, P. Schweitzer, U.Y. PLB 718 (2012)]

Different quantities related to the nucleon EMT densities and their form factors: $T^*_{00}(0)$ denotes the energy in the center of the nucleon; $\langle r^2_{00}\rangle^*$ and $\langle r^2_J\rangle^*$ depict the mean square radii for the energy and angular momentum densities, respectively; $p^*(0)$ represents the pressure in the center of the nucleon, whereas r^*_0 designates the position where the pressure changes its sign; d^*_1 is the value of the $d^*_1(t)$ form factor at the zero momentum transfer.

ρ/ρ_0	$T^*_{00}(0)$ [GeV fm^{-3}]	$\langle r^2_{00}\rangle^*$ [fm^2]	$\langle r^2_J\rangle^*$ [fm^2]	$p^*(0)$ [GeV fm^{-3}]	r^*_0 [fm]	d^*_1
0	1.45	0.68	1.09	0.26	0.71	−3.54
0.5	0.96	0.83	1.23	0.18	0.82	−4.30
1.0	0.71	0.95	1.35	0.13	0.90	−4.85

Structure analysis of the in-medium nucleon

Pressure densities of the in-medium nucleons

[H.C.Kim, P. Schweitzer, U.Y. PLB 718 (2012)]

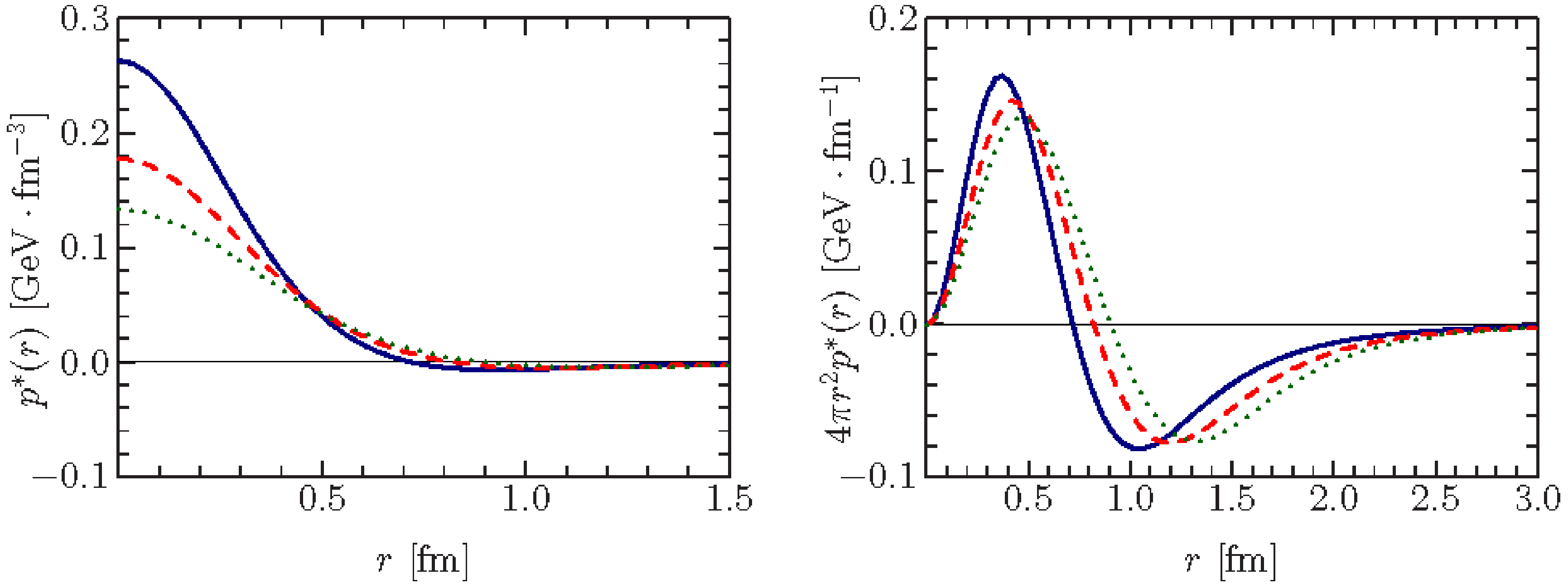


FIG. 2: (Color online) The pressure densities $p^*(r)$ and $4\pi r^2 p^*(r)$ as functions of r in the left and right panels, respectively. Notations are the same as in Fig. 1.

Structure analysis of the in-medium nucleon

Pressure densities of the in-medium nucleons

[H.C.Kim, P. Schweitzer, U.Y. PLB 718 (2012)]

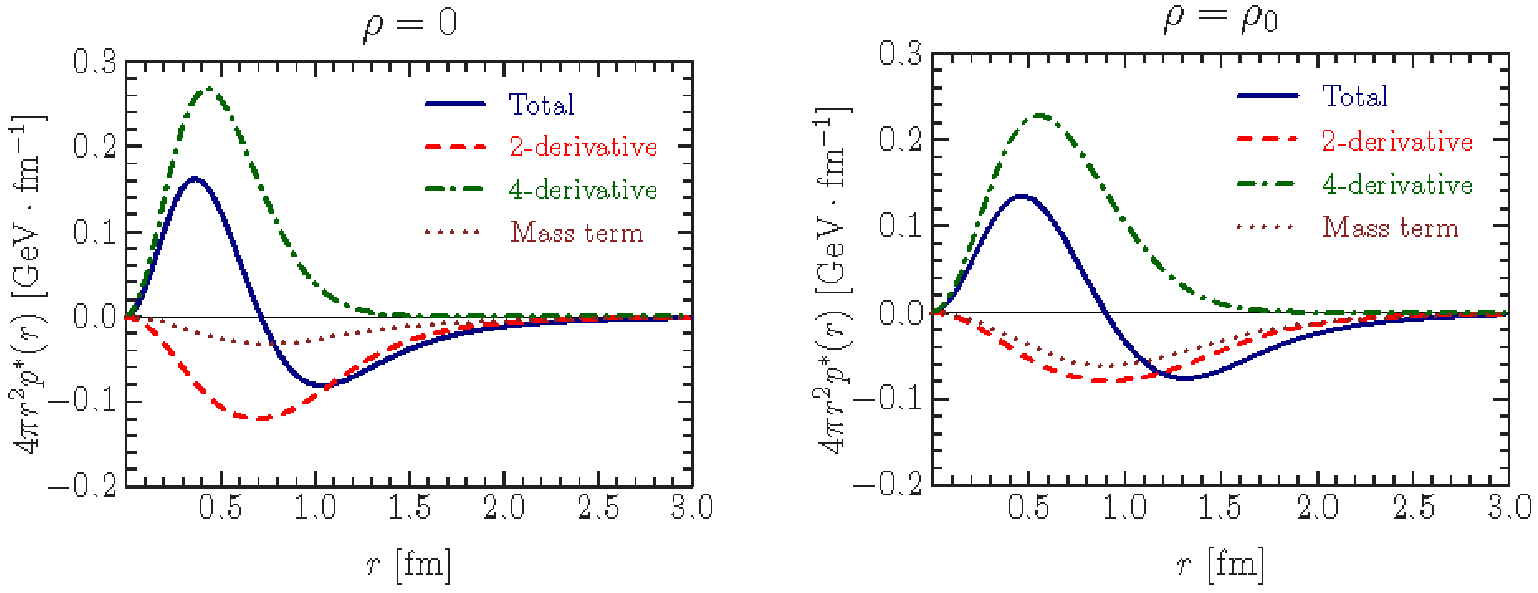


FIG. 3: (Color online) The decomposition of the pressure densities $4\pi r^2 p^*(r)$ as functions of r, in free space ($\rho = 0$) and at $\rho = \rho_0$, in the left and right panels, respectively. The solid curves denote the total pressure densities, the dashed ones represent the contributions of the 2-derivative (kinetic) term, the long-dashed ones are those of the 4-derivative (stabilizing) term, and the dotted ones stand for those of the pion mass term.

Structure analysis of the in-medium nucleon

Present applicability of the model

[H.C.Kim, P. Schweitzer, U.Y. PLB 718 (2012)]

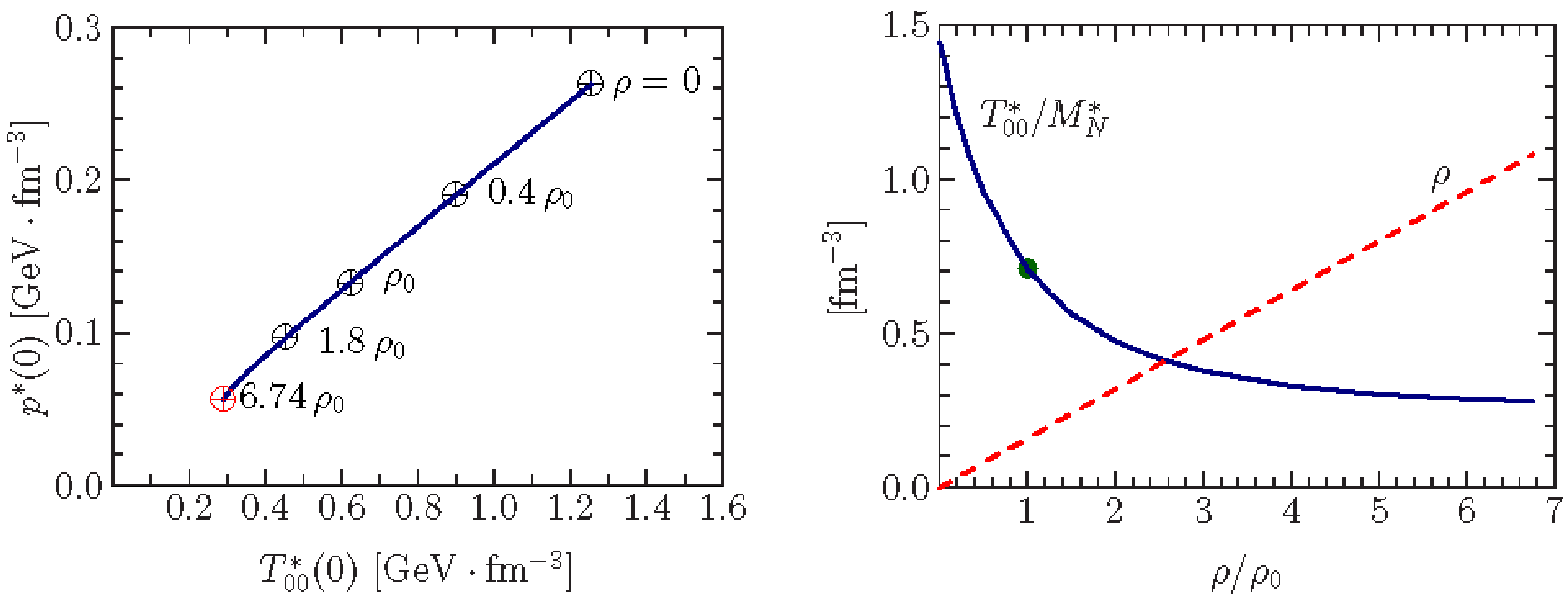


FIG. 5: (Color online) In the left panel, the correlated change of $p^*(0)$ and $T^*_{00}(0)$ drawn with ρ varied. In the right panel, the T^*_{00}/M^*_N and ρ depicted as a function of ρ/ρ_0. The maximal density is given as about $6.74\,\rho_0$, above which the Skyrmion does not exist anymore. The filled circle on the solid curve represents the value of T^*_{00}/M^*_N at normal nuclear matter density.

Summary

- Within the applicability range the model describes
  - the single hadrons properties
    - in separate state
    - in the community of their partners
  - as well as the properties of that whole community at same footing

Outlook

Extensions and applicability

- Nucleon tomography in nuclear matter
  [H.C. Kim, UY (arXiv:1304.5926)]
- NN interactions in nuclear matter
- Neutron stars
- Finite nuclei properties
  - Mirror nuclei
  - Exotic nuclei
- Nucleon-knock out reactions
- Vector mesons in nuclear matter
  [J.H.Jung, UY, H.C.Kim (arXiv:1212.4616), to appear in PLB]

Thanks for colaborators!

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Ulf-G Meissner (Bonn University)

Andreas Wirzba (Institute of Nuclear Physics, Juelich)

Hyun-Chul Kim (INHA University)

Peter Schweitzer (University of Connecticut)

Ju-Hyun Jung (Inha University)

Thank you for your attension!