Bottomonium and QCD phase diagram

Seyong Kim

Sejong University

based on PRL106, (2011) 061602 arXiv:1010.3752, JHEP1111, (2011) 103 arXiv:1109.4496, and arXiv:1210.2903
and PLB 711, (2012) 199 arXiv:1202.4353

Outline

Lattice Gauge Theory

- Through lattice gauge theory, we would like to calculate low energy non-perturbative quantities in QCD reliably and quantitatively.

- with the systematic errors (from finite lattice spacing, finite spacetime volume, finite quark mass) under control

- cf. K.G. Wilson, PRD10 (1974) 2445

Lattice Gauge Theory

2012 PDG summary on QCD

28 *9. Quantum chromodynamics*

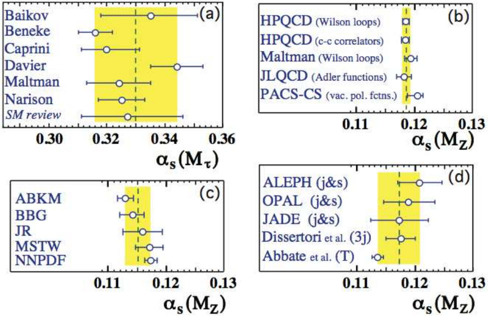


Figure 9.2: Summary of determinations of α_s from hadronic τ-decays (a), from lattice calculations (b), from DIS structure functions (c) and from event shapes and jet production in e^+e^--annihilation (d). The shaded bands indicate the average values chosen to be included in the determination of the new world average of α_s.

Basics

• lattice theory is the only theoretical tool for systematic and quantitative study for non-perturbative physics

• discretization of spacetime into hypercubic lattice

• quantum physics is “a numerical integral problem” according to Feynman’s path integral formulation

• Monte Carlo method for the numerical integral

Basics

- Schroedinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = \mathcal{H}\psi \tag{1}$$

- path integral

$$\langle O\rangle\frac{\int [dq]\; O\; e^{\frac{i}{\hbar}\int dt\,\mathcal{L}}}{\int [dq]\, e^{\frac{i}{\hbar}\int dt\,\mathcal{L}}} \tag{2}$$

Basics

- path integral formulation of quantum field theory

$$\langle O \rangle = \frac{\int dA_\mu d\psi d\overline{\psi} e^{-S_E} O[A_\mu, \psi, \overline{\psi}]}{\int dA_\mu d\psi d\overline{\psi} e^{-S_E}} \tag{1}$$

- infinite dimensional integral problem
- Monte Carlo method for numerical integral
- importance sampling

$$\langle O \rangle \sim \frac{1}{N} \sum_i O[A_i] \tag{2}$$

Basics

Lattice NRQCD

• relativity + quantum mechanics = quantum field theory

$$\mathcal{L} = \overline{\psi}(\not{D} + m)\psi - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} \tag{1}$$

• but we are familiar with quantum mechanics !

$$i\,\hbar\frac{\partial\phi}{\partial t} = \mathcal{H}\phi \tag{2}$$

• how do we go from a relativistic quantum field theory to quantum mechanics ?

Lattice NRQCD

• do you remember relativistic quantum mechanics? and how to derive hyperfine interaction and etc?

• in Coulomb gauge, Foldy-Wouthuysen-Tani transform

• in field theory language, NRQCD is an effective theory in which the momentum mode higher than the heavy quark mass, M, is integrated away

• need to show power-counting and factorization etc (cf. Braaten, Bodwin, Lepage, Phys. Rev. D51 (1995) 1125)

• inclusive decay rate = partonic decay rate $\times$ the probability for heavy quark to meet anti-heavy quark

Lattice NRQCD

$$\mathcal{L} = \mathcal{L}_0 + \delta\mathcal{L}, \tag{1}$$

with

$$\mathcal{L}_0 = \psi^\dagger \left(D_\tau - \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(D_\tau + \frac{\mathbf{D}^2}{2M} \right) \chi, \tag{2}$$

and

$$\begin{aligned} \delta\mathcal{L} = \quad & -\frac{c_1}{8M^3} \left[\psi^\dagger (\mathbf{D}^2)^2 \psi - \chi^\dagger (\mathbf{D}^2)^2 \chi \right] \\ & + c_2 \frac{ig}{8M^2} \left[\psi^\dagger \left(\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D} \right) \psi + \chi^\dagger \left(\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D} \right) \chi \right] \\ & - c_3 \frac{g}{8M^2} \left[\psi^\dagger \sigma \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D}) \psi + \chi^\dagger \sigma \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D}) \chi \right] \\ & c_4 \frac{g}{2M} \left[\psi^\dagger \sigma \cdot \mathbf{B} \psi - \chi^\dagger \sigma \cdot \mathbf{B} \chi \right]. \end{aligned} \tag{3}$$

Lattice NRQCD

- NRQCD is an effective field theory
- expansion in v, the heavy quark velocity in heavy quarkonium
- $Ma \sim 1$
- quarkonium spectrum is one of "gold plated" result from lattice QCD (PRL 92 (2004) 022001)

Lattice NRQCD

- Non-relativistic QCD

$$G(\vec{x}, t=0) = S(x) \tag{1}$$

$$G(\vec{x}, t=1) = \left[1 + \frac{1}{2n}\frac{\vec{D}^2}{2m_b^0}\right]^n U_4^\dagger(\vec{x}, t)\left[1 + \frac{1}{2n}\frac{\vec{D}^2}{2m_b^0}\right]^n G(\vec{x}, 0) \tag{2}$$

$$G(\vec{x}, t+1) = \left[1 + \frac{1}{2n}\frac{\vec{D}^2}{2m_b^0}\right]^n U_4^\dagger(\vec{x}, t)\left[1 + \frac{1}{2n}\frac{\vec{D}^2}{2m_b^0}\right]^n [1 - \delta H] G(\vec{x}, t) \tag{3}$$

Lattice NRQCD

where $S(x)$ is the source and

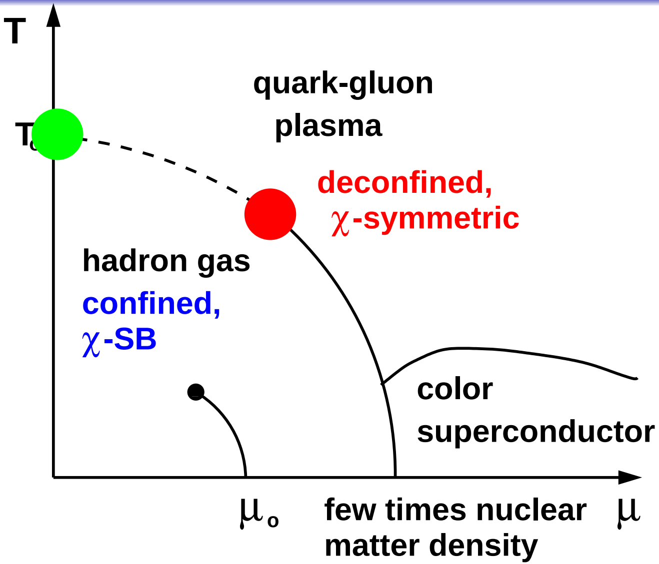
$$
\begin{aligned}
\delta H &= -\frac{(\vec{D}^{(2)})^2}{8(m_b^0)^3} + \frac{ig}{8(m_b^0)^2}(\vec{D}\cdot\vec{E} - \vec{E}\cdot\vec{D}) \\
&- \frac{g}{8(m_b^0)^2}\vec{\sigma}\cdot(\vec{D}\times\vec{E} - \vec{E}\times\vec{D}) - \frac{g}{2m_b^0}\vec{\sigma}\cdot\vec{B} \\
&+ \frac{a^2\vec{D}^{(4)}}{24m_b^0} - \frac{a(\vec{D}^{(2)})^2}{16n(m_b^0)^2} \qquad (1)
\end{aligned}
$$

• calculate NRQCD propagator using gluon field which has light quark vacuum polarization effect

Non-zero T on lattice

- $N_s^3 \times N_\tau$ lattice, $T = \frac{1}{N_\tau a}$
- boundary condition for quantum field on the time direction
- high temperature means $N_t << N_s \rightarrow$ spectrum in finite temperature environment is not feasible

Lattice study of quarkonium in non-zero T



Lattice study of quarkonium in non-zero T

- Recall Schroedinger eq.

$$i\frac{\partial\psi}{\partial t} = \mathcal{H}\psi \tag{2}$$

with $\mathcal{H} = 2M - \frac{\nabla^2}{2M} + V(r)$

- $T = 0$, e.g., Cornel potential;

$$V(r) = -\frac{\alpha}{r} + \sigma r \tag{3}$$

- $T \neq 0$, Debye screening;

$$V(r,T) = \frac{\sigma}{\mu(T)}(1 - e^{-\mu(T)r}) - \frac{\alpha}{r}e^{-\mu(T)r} \tag{4}$$

where $\mu(T) = 1/r_D(T)$

Lattice study of quarkonium in non-zero T

- F. Karsch, M.T. Mehr, and H. Satz, Z.Phys.C37 (1988) 617.

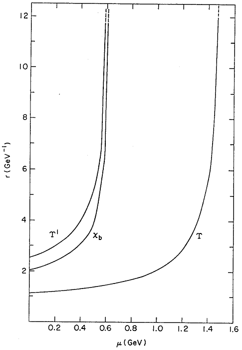


FIGURE 6

Lattice study of quarkonium in non-zero T

• separation of scale: M (heavy quark mass) and Mv (bound state momentum)

• decay rate = the probability for heavy quark and heavy anti-quark to meet $\times$ partonic cross-section for quark–anti-quark annihilation

• similar to positronium

Lattice study of quarkonium in non-zero T

• obtain finite temperature heavy quark potential by lattice calculation $\rightarrow$ solve Schroedinger equation

• obtain spectral function of heavy meson correlator by lattice calculation $\rightarrow$ observe temperature modification of spectrum

• “derive” potential from Wilson loop

• study heavy quarkonium correlator in finite temperature by lattice NRQCD on anisotropic lattice

Anisotropic lattice study of quarkoinum in non-zero T

- choose the time direction lattice spacing a_τ different from the space direction lattice spacing a_s

- more lattice points along the temperature direction (or time direction)

Anisotropic lattice study of quarkoinum in non-zero T

• Anisotropic lattice on $12^3 \times N_t$ (ref. G. Aarts et al, PRD 76 (2007) 094513)

N_s	N_t	a_τ^{-1}	T(MeV)	T/T_c	No. of Conf.
12	80	7.35GeV	90	0.42	250
12	32	7.35GeV	230	1.05	1000
12	28	7.35GeV	263	1.20	1000
12	24	7.35GeV	306	1.40	500
12	20	7.35GeV	368	1.68	1000
12	18	7.35GeV	408	1.86	1000
12	16	7.35GeV	458	2.09	1000

Table: summary for the lattice data set

• two-plaquette Symanzik improved gauge action, fine-Wilson, coarse-Hamber-Wu fermion action with stout-link smearing

Anisotropic lattice study of quarkoinum in non-zero T

- bound state $\rightarrow$ exponentially falling behavior of the propagator

$$G(\tau) \sim Ae^{-E\tau} \tag{2}$$

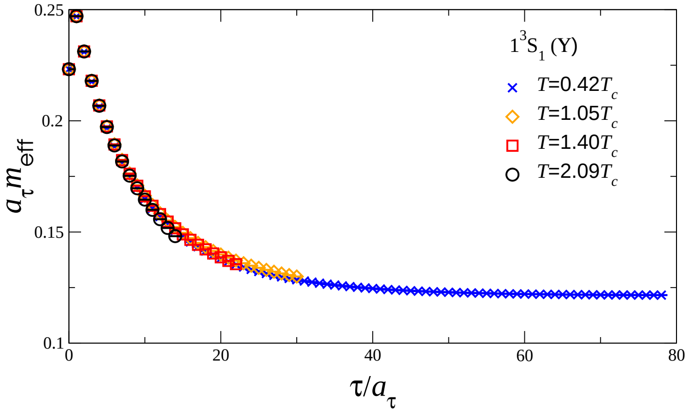
- free state $\rightarrow$ power-like falling behavior

$$G(\tau) \sim A'\tau^{-\gamma} \tag{3}$$

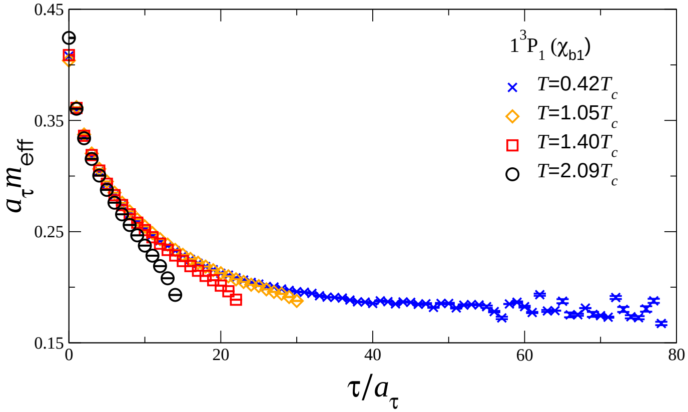
Effective mass for Υ

$$m_{\rm eff}(\tau) = -\log[G(\tau)/G(\tau - a_\tau)], \quad (4)$$

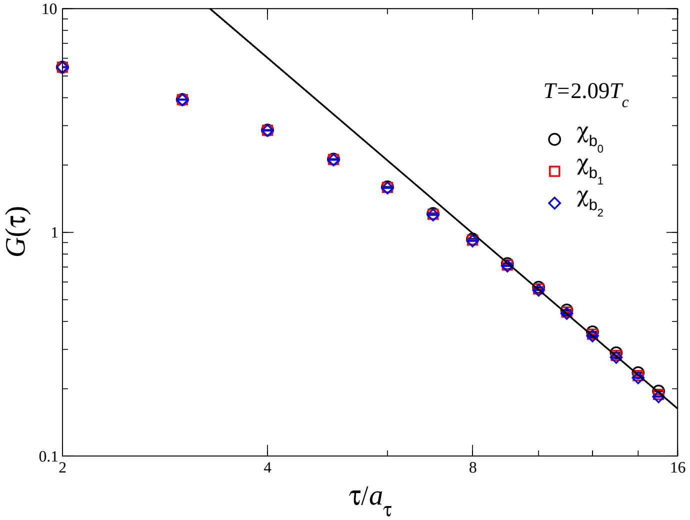
Effective mass for Υ



Effective mass for χ_{b1}



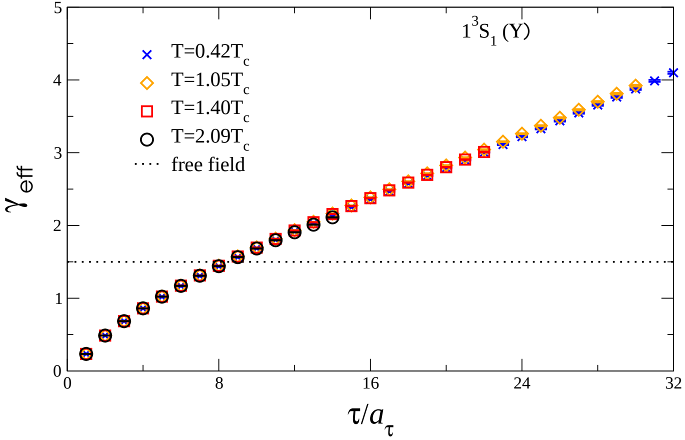
χ_b propagator



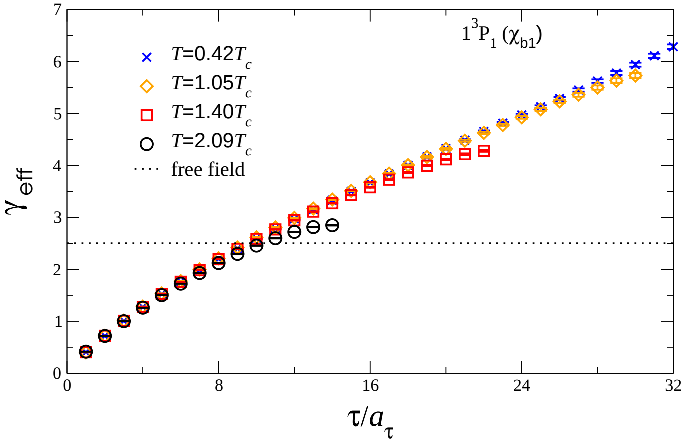
effective exponent for Υ

$$\gamma_{\rm eff}(\tau) = -\tau \frac{G'(\tau)}{G(\tau)} = -\tau \frac{G(\tau + a_\tau) - G(\tau - a_\tau)}{2a_\tau G(\tau)} \qquad (4)$$

effective exponent for Υ



effective exponent for χ_b



S-wave bottomonium spectral function

$$G_\Gamma(\tau) = \sum_{\vec{x}} \langle \overline{\psi}(\tau,\vec{x})\Gamma\psi(\tau,\vec{x})\overline{\psi}(0,\vec{0})\Gamma\psi(0,\vec{0})\rangle \tag{4}$$

$$= \int \frac{d^3p}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega)\rho_\Gamma(\omega,\vec{p}) \tag{5}$$

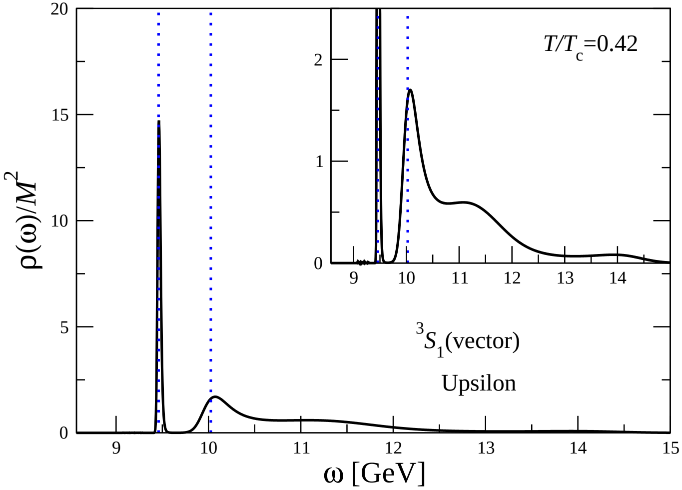
and

$$K(\tau,\omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}. \tag{6}$$

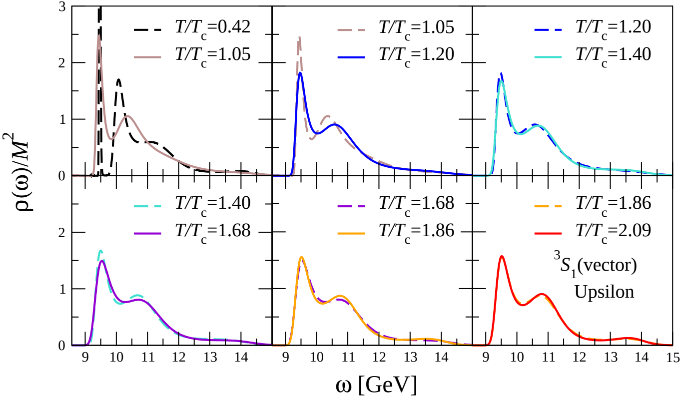
With $\omega = 2M + \omega'$ and $T/M << 1$,

$$G(\tau) = \int_{-2M}^\infty \frac{d\omega'}{2\pi} \exp(-\omega'\tau)\rho(\omega') \tag{7}$$

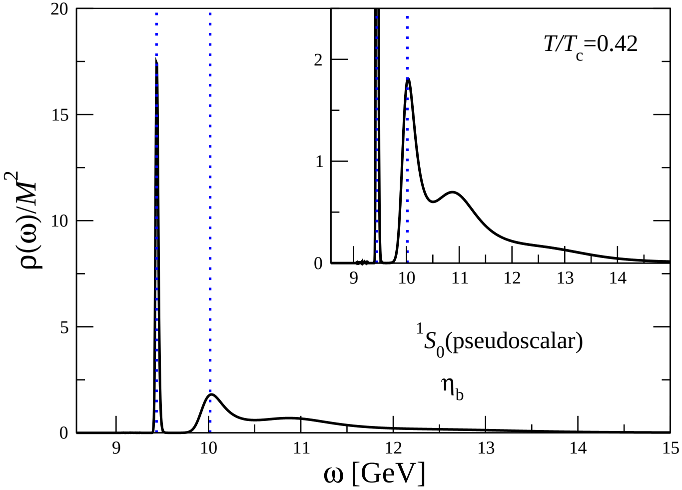
S-wave bottomonium spectral function



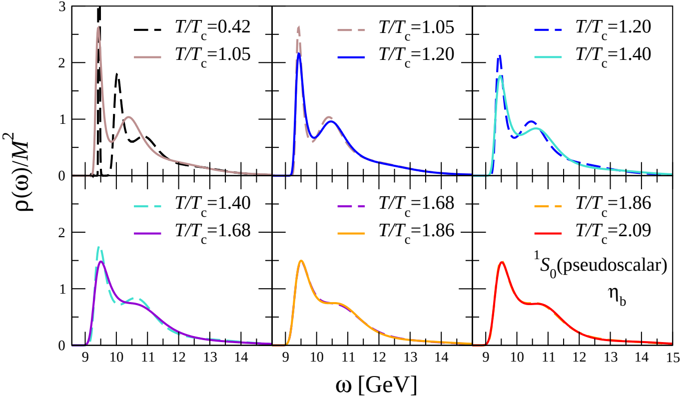
S-wave bottomonium spectral function



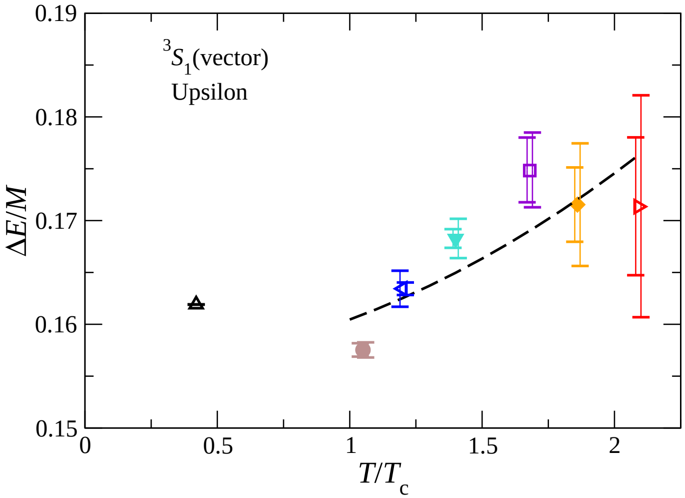
S-wave bottomonium spectral function



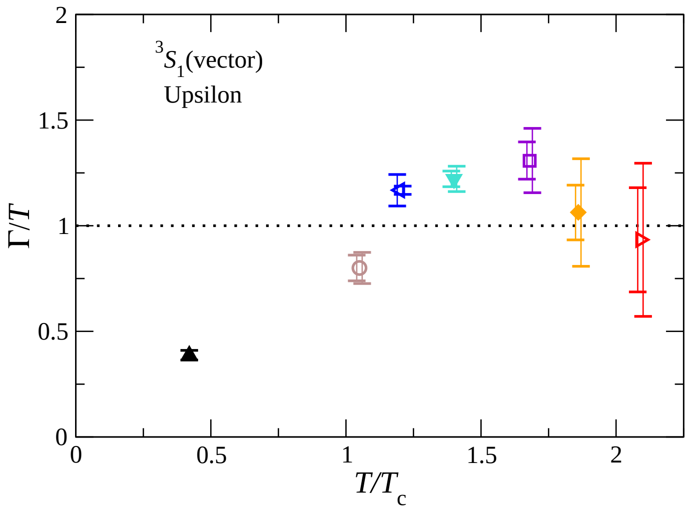
S-wave bottomonium spectral function



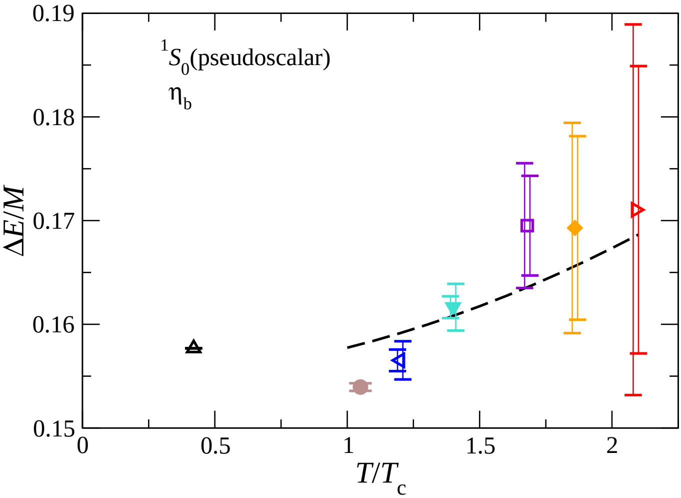
S-wave bottomonium spectral function



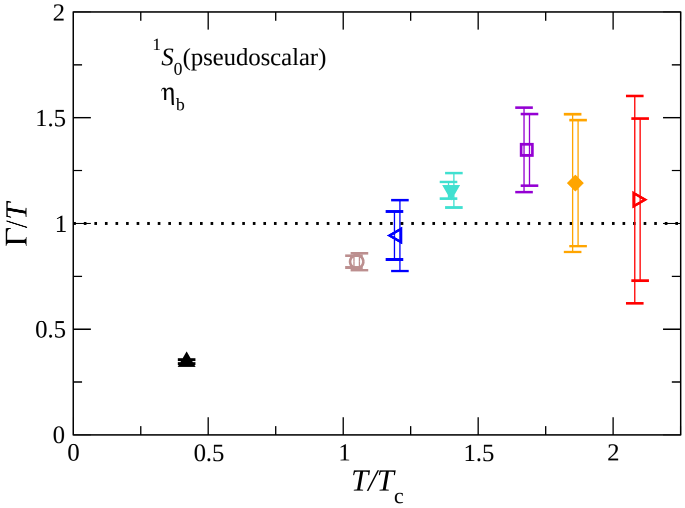
S-wave bottomonium spectral function



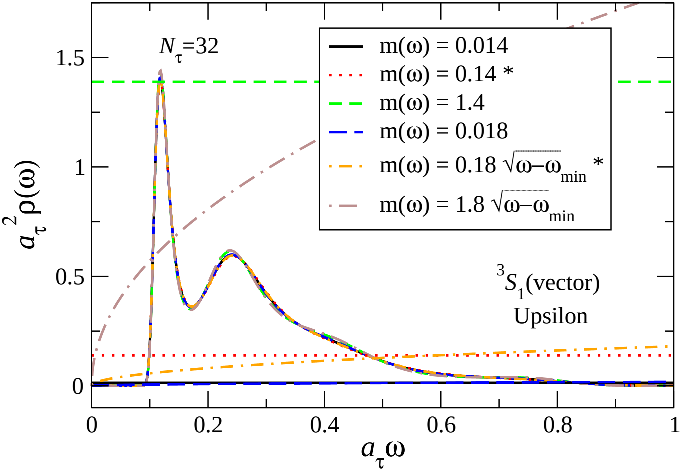
S-wave bottomonium spectral function



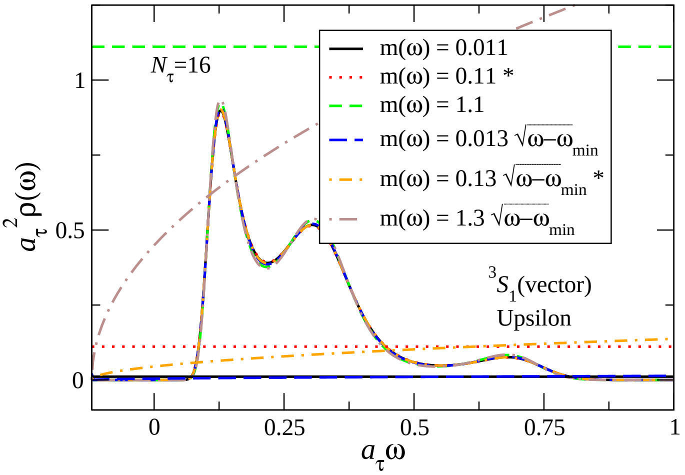
S-wave bottomonium spectral function



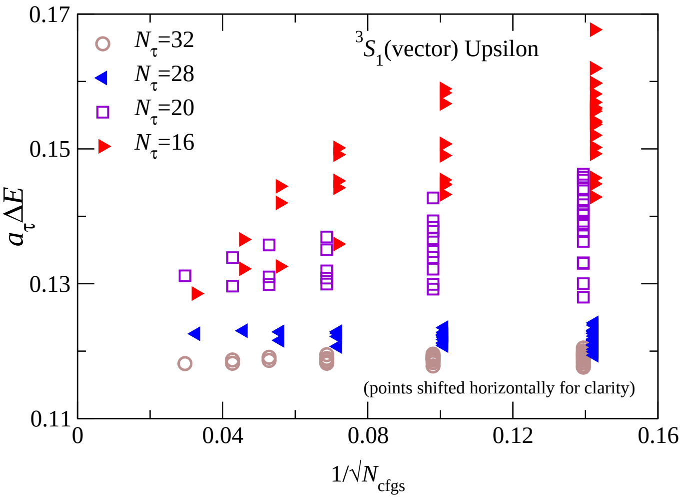
S-wave bottomonium spectral function



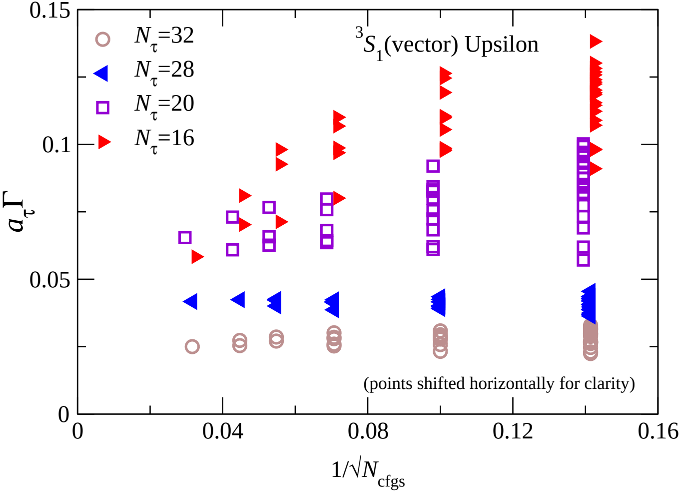
S-wave bottomonium spectral function



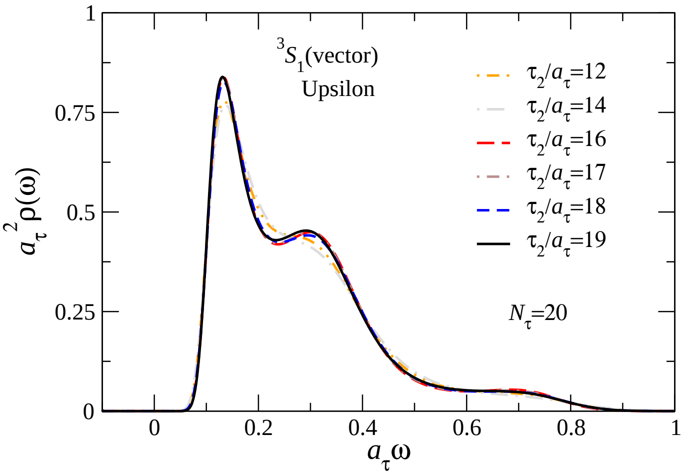
S-wave bottomonium spectral function



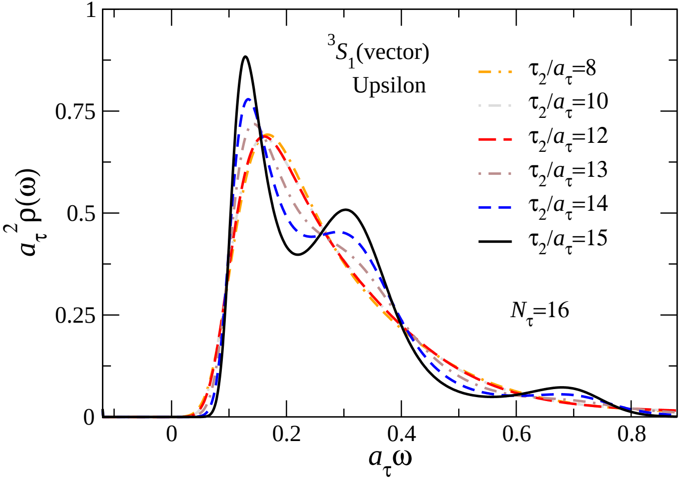
S-wave bottomonium spectral function



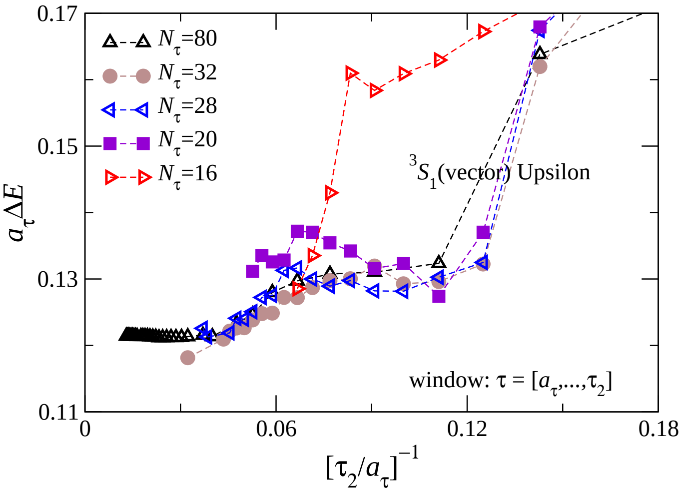
S-wave bottomonium spectral function



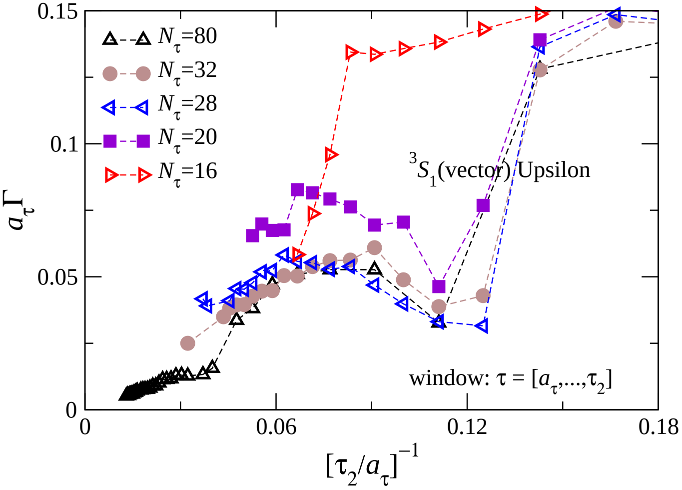
S-wave bottomonium spectral function



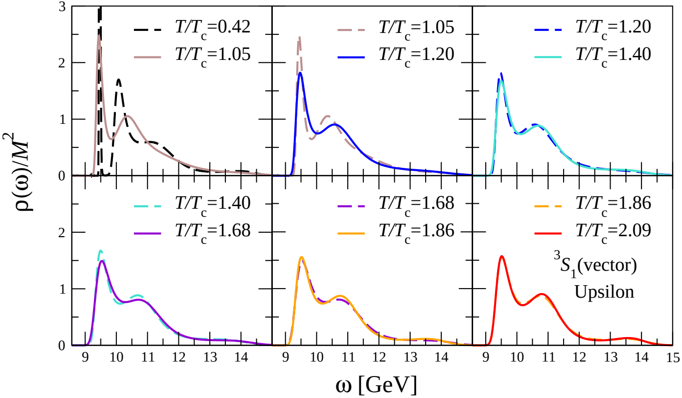
S-wave bottomonium spectral function



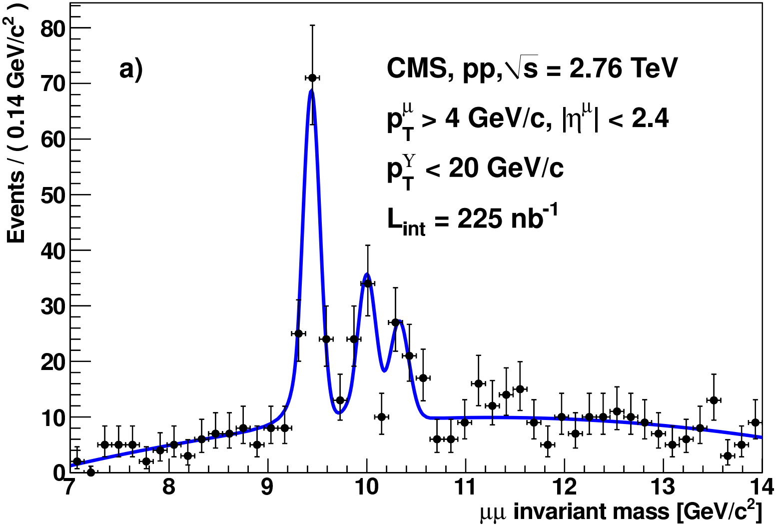
S-wave bottomonium spectral function



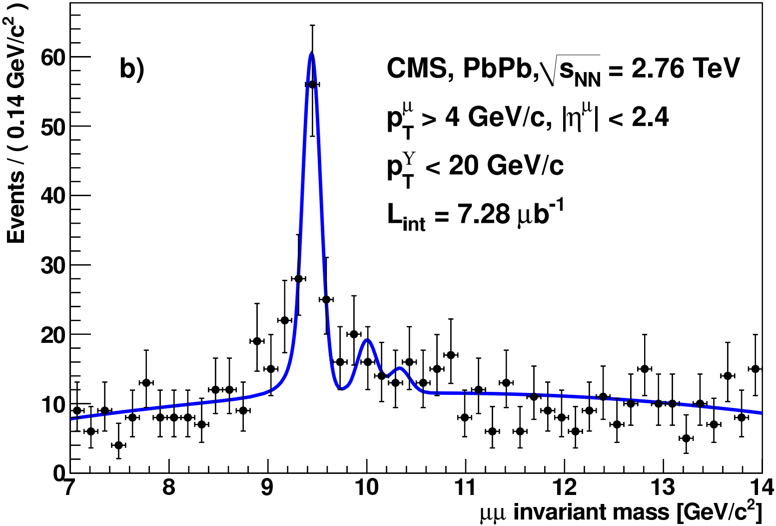
CMS collaboration, PRL107 (2011) 052302



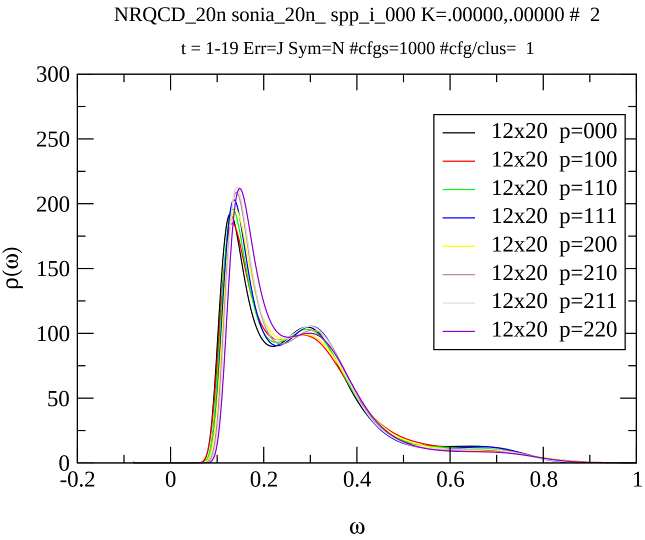
CMS collaboration, PRL107 (2011) 052302



CMS collaboration, PRL107 (2011) 052302



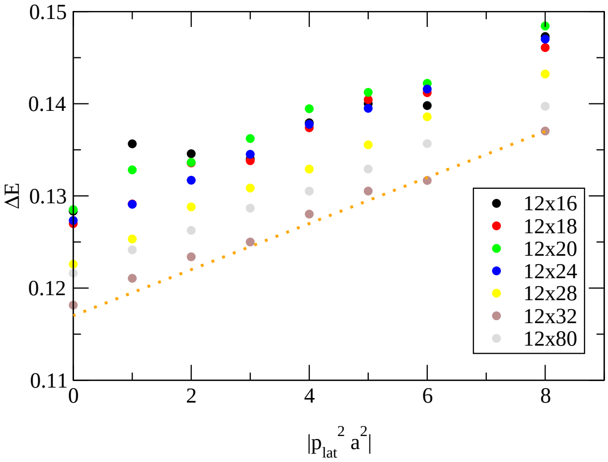
Upsilon moving in a thermal bath



Upsilon moving in a thermal bath

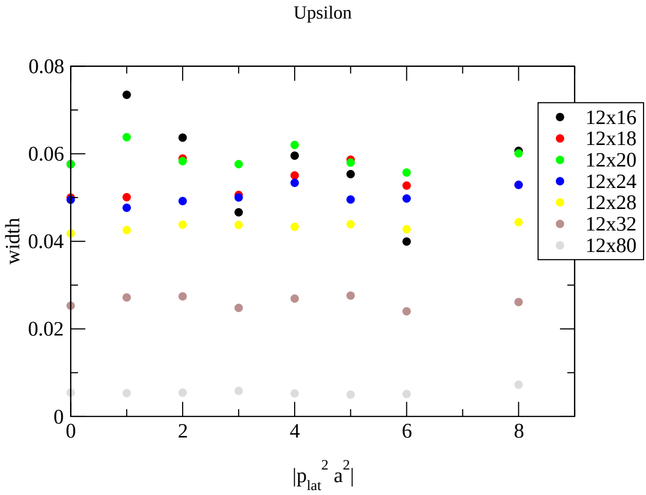
- observable heavy quarkonium velocity ($\frac{v^2_{\mathrm{Upsilon}}}{c^2} \sim 0.03$) effect on the S-wave state mass (NR dispersion $\sim \frac{\vec{p}^2}{2M_{\mathrm{Upsilon}}}$)

Upsilon



Upsilon moving in a thermal bath

- no observable v^2_{Upsilon} effect on the S-wave state "width" (Escobedo et al., PRD84 (2011) 016008, $\Gamma_v/\Gamma_0 \sim 1 - \frac{2}{3}v^2_{\text{Upsilon}}$)



Conclusion

• lattice NRQCD method for bottomonium on anisotropic lattice offers a method which is systematically improvable and is based on the first principel of quantum field theory (not a model)

• lattice NRQCD at zero temperature already produced accurate result and is producing more accurate result

• Υ and η_b (S-wave) show that the ground state survives but the excited states are suppressed as the temperature increases above T_c.

• χ_b (P-wave) follows a power-law decay at $T = 1.4T_c$ and is nearly consistent with free dynamics at $T = 2.09T_c$. This means that unlike S-wave, P-wave melts almost immediately above T_c

• further studies on bottomonium (including systematic error study) are under way.