QCD sum rules for ρ-meson at finite density or temperature

Youngshin Kwon

YONSEI UNIVERSITY

Heavy Ion Meeting at KNU

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contents based on :

YK., Procura & Weise [Phys. Rev. C78, 055203 (2008)]

YK, Sasaki & Weise [Phys. Rev. C81, 065203 (2010)]

Outline

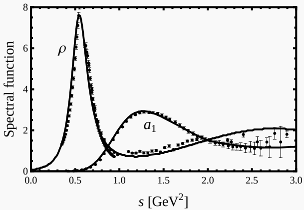
- Introduction & motivation
- QCD sum rules for ρ-meson, in vacuum and in medium
- Finite energy sum rules at finite density
- Finite energy sum rules at finite temperature
- Summary and outlook

Goal: reliable framework of in-medium QCD sum rules for vector mesons
$\Rightarrow$ constraints for the in-medium spectral properties

Motivation

◎ Spontaneous chiral symmetry breaking:

- quark condensate: $\langle \bar{q}q \rangle \neq 0$
- Goldstone bosons: π, K, etc.
  pion decay constant: $f_\pi \simeq 92.4$ MeV
- mass splitting of chiral partners
  (e.g. $\rho(770)$-$a_1(1250)$)



◎ Chiral symmetry restoration in nuclear medium:

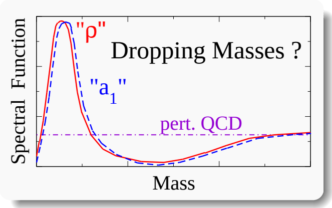
- degenerate chiral partners $\Rightarrow$ modifications of hadron spectrum

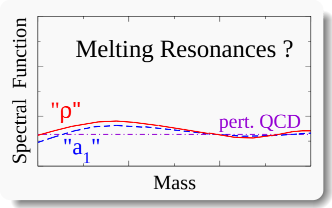
Restoration scenarios in medium

- Pole mass shift:
  - masses of parity partners degenerate in medium.
  - moving toward each other or going to zero (Brown-Rho).

Brown & Rho [PRL 66, 2720 (1991)]

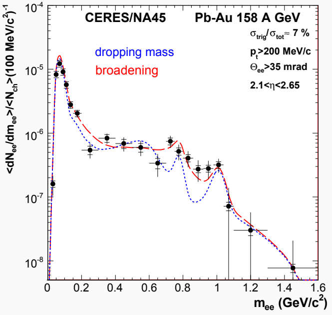
- Width broadening:
  - in-medium spectral functions are broadly distributed.
  - the continuum merges the broadened spectral distributions.

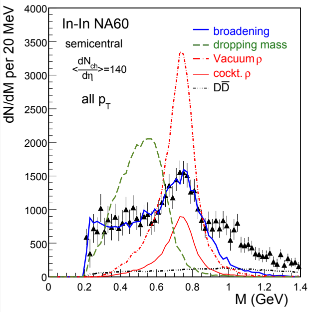




Dilepton spectroscopy

- Dilepton production in RHIC ($\gamma^* \rightarrow l^+ l^-$):
  - EM probe with pure information of the hot and/or dense region
  - dilepton emission ⇔ in-medium vector-meson spectroscopy





General review of QCD sum rules (in vacuum)

General review of QCD sum rules

- Current correlation function:

$$\Pi^{\mu\nu}(q) = i \int \mathrm{d}^4x\, e^{iq\cdot x} \langle \mathcal{T}\, j^\mu(x)\, j^\nu(0) \rangle$$

  - isovector vector- and axialvector-currents:

$$j^\mu_\rho = \tfrac{1}{2}\left(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d\right), \qquad j^\mu_A = \tfrac{1}{2}\left(\bar{u}\gamma^\mu\gamma_5 u - \bar{d}\gamma^\mu\gamma_5 d\right)$$

  - invariant correlator: $\Pi(q^2) = \frac{1}{3}\, g_{\mu\nu}\Pi^{\mu\nu}(q)$

- Operator product expansion (quark & gluon d.o.f.) at large $Q^2 = -q^2$:

$$\frac{12\pi^2}{Q^2}\Pi(Q^2) = -c_0 \ln\left(\frac{Q^2}{\mu^2}\right) + \frac{c_1}{Q^2} + \frac{c_2}{Q^4} + \cdots$$

- Spectral representation (hadronic d.o.f.) at resonance region:

$$\Pi(q^2) = \Pi(0) + q^2\Pi'(0) + \frac{q^4}{\pi}\int \mathrm{d}s\, \frac{\mathrm{Im}\,\Pi(s)}{s^2(s - q^2 - i\epsilon)}$$

General review of QCD sum rules

◎ Borel transformation:

$$12\pi^2\Pi(0) + \int_0^\infty \mathrm{d}s\, R(s)\, \mathrm{e}^{-s/M^2} = c_0 M^2 + c_1 + \frac{c_2}{M^2} + \frac{c_3}{2M^4} + \cdots$$

- dimensionless spectral function: $R(s) \equiv -\frac{12\pi}{s}\mathrm{Im}\,\Pi(s)$

◎ Coefficients c_n:

$$c_0 = \frac{3}{2}\left(1 + \frac{\alpha_s}{\pi}\right) + \cdots , \qquad c_1 \propto m_q^2 : \text{negligibly small}$$

$$c_2 = \frac{\pi^2}{2}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \pm 6\pi^2 \left(m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle \right)$$

$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle$: $0.005 \pm 0.004\,\mathrm{GeV}^4$

Ioffe [PPNP 56, 232 (2006)]

$m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle$: $\simeq -m_\pi^2 f_\pi^2 = -(0.11\,\mathrm{GeV})^4$

[Gellman-Oaks-Renner]

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$$c_3 \propto \mp\langle(\bar{q}q)^2\rangle \text{ uncertain value}$$

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- dimensionless spectral function: $R(s) \equiv -\frac{12\pi}{s}\mathrm{Im}\,\Pi(s)$
- expand for $s_0 \ll M^2$ and compare term by term

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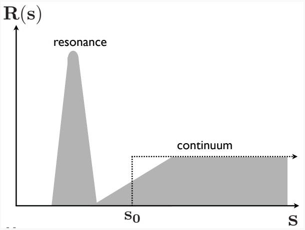
Finite energy sum rules

- Hierarchy of finite energy sum rules for moments of $R(s)$:

$$0^{th}\text{ moment}: \quad \int_0^{s_0} ds\, R(s) = s_0 c_0 + c_1 - 12\pi^2 \Pi(0)$$

$$1^{st}\text{ moment}: \quad \int_0^{s_0} ds\, s\, R(s) = \frac{s_0^2}{2} c_0 - c_2$$

- Spectral distribution (resonance + continuum):



$$R(s) = R_\rho(s)\,\theta(s_0 - s) + R_c(s)\,\theta(s - s_0)$$

- Assumption for vector channel;

$$\sqrt{s_0} \simeq 4\pi f_\pi$$

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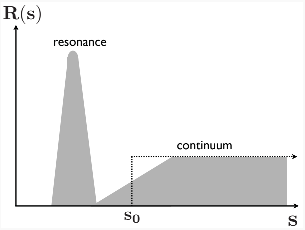

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► Assumption for vector channel;

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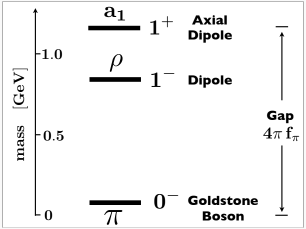
Consistency with current algebra

◎ Identification of $\sqrt{s_0}$ with $\Lambda_{\rm CSB} \simeq 4\pi f_\pi$:



- KSRF relation

  Kawarabayashi & Suzuki [PRL 16, 255 (1966)]
  Riazuddin & Fayyazuddin [PR 147, 1071 (1966)]

- Weinberg sum rules

  Weinberg [PRL 18, 507 (1967)]

$$\Rightarrow m_{a_1} = \sqrt{2}\, m_\rho = 4\pi f_\pi$$

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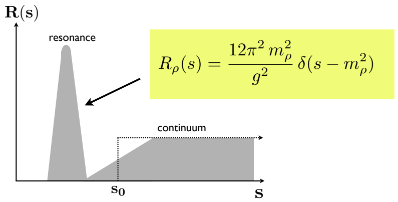

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0th moment:

$$\int_0^{s_0} \mathrm{d}s\, R_\rho(s) = \frac{3}{2} s_0$$

$$\Rightarrow m_\rho^2 = 2 g^2 f_\pi^2$$

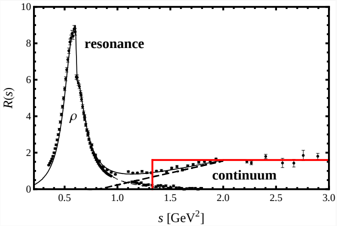
1st moment:

$$\int_0^{s_0} \mathrm{d}s\, s R_\rho(s) = \frac{3}{4} s_0^2$$

$$\Rightarrow g = 2\pi$$

Vacuum sum rule analysis

- Input: $R_\rho(s)$ from chiral effective field theory + vector mesons (VMD)



$$\int_0^{s_0} \mathrm{d}s\, R_\rho(s) = s_0 c_0 + c_1$$

$$\int_0^{s_0} \mathrm{d}s\, s R_\rho(s) = \frac{s_0^2}{2} c_0 - c_2$$

$$\sqrt{s_0} = 1.14 \pm 0.01\,\mathrm{GeV} \simeq 4\pi f_\pi$$

$$\bar{m}_\rho \equiv \sqrt{\frac{\int \mathrm{d}s\, sR(s)}{\int \mathrm{d}s\, R(s)}} = 0.78 \pm 0.01\,\mathrm{GeV}$$

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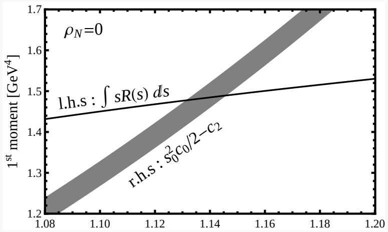

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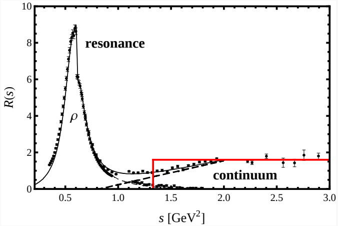
Sensitivity to threshold modeling

- replace the Heaviside step function with a ramp function:

$$R(s) = R_\rho(s)\,\theta(s_2 - s) + R_c(s)\,W(s)$$

with the weight function $W(s)$

$$W(x) = \begin{cases} 0 & \text{for } x \le s_1 \\ \dfrac{x - s_1}{s_2 - s_1} & \text{for } s_1 \le x \le s_2 \\ 1 & \text{for } x \ge s_2 \end{cases}$$



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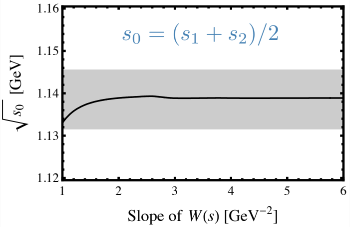

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Finite energy sum rules at finite density

Medium-modification of the sum rules

- The existence of nuclear matter causes breaking Lorentz invariance:
  - two invariant correlator: longitudinal and transverse parts.
  - choosing a preferred reference frame of the medium ($\mathbf{q} = 0$), longitudinal and transverse correlators coincide.

$$\Pi^{\mathrm{L}}(\omega, \mathrm{q} = 0) = \Pi^{\mathrm{T}}(\omega, \mathrm{q} = 0) \equiv \Pi(\omega, \mathrm{q} = 0)$$

- New operators with spin appear due to the broken Lorentz invariance:
  - the first moment of in-medium FESR involves twist-2 operator (e.g. $\langle \bar{q}\gamma_\nu D_\mu q \rangle$) to be considered.
- Medium-dependence in the OPE side contributes only to the condensates:
  - non-perturbative contributions in OPE appear to be clearly separated into the condensates.
  - medium effects are non-perturbative.

Density-dependence of OPE

Hatsuda & Lee [Phys. Rev. C 46, R34 (1992)]

- Expectation value: vacuum → ground state of nuclear matter

$$\langle 0|\mathcal{O}|0\rangle \equiv \langle \mathcal{O}\rangle_0 \rightarrow \langle \mathcal{O}\rangle_{\rho_N} = \langle N|\mathcal{O}|N\rangle$$

- In-medium coefficients: $c_n \rightarrow c_n + \delta c_n$

$$\delta c_2 = -3\pi^2 \left(\frac{4}{27} M_N^{(0)} - 2\sigma_N - A_1 M_N \right) \rho_N$$

density dependence of gluon condensate

$\left(M_N^{(0)} \simeq 0.88\,\mathrm{GeV} \right)$

density dependence of quark condensate

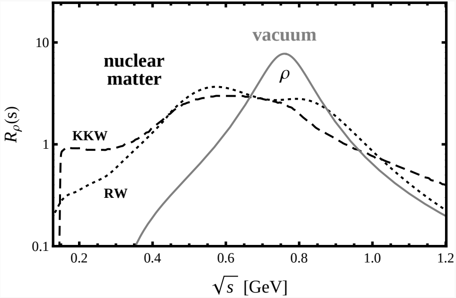
$(\sigma_N \simeq 45\,\mathrm{MeV})$

first moment of parton distribution from DIS

$(A_1 \approx 1.24)$

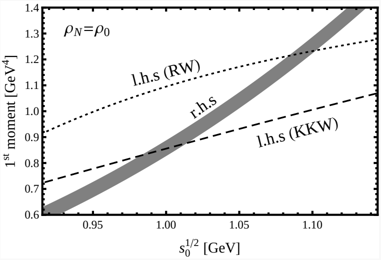
Spectral functions at finite density

◎ ρ-meson spectral functions in nuclear medium ($\rho_N = \rho_0 = 0.17\,\mathrm{fm}^{-3}$):



- KKW: SU(3) chiral dynamics with vector meson dominance
  Klingl, Kaiser & Weise [Nucl. Phys. A624, 527 (1997)]
- RW: particle-hole excitations ($\Delta(1232)$-h and $N^*(1520)$-h))
  Rapp & Wambach [Adv. Nucl. Phys. 25, 1 (2000)]

Results for ρ-meson at finite density



In vacuum: $\sqrt{s_0} \simeq 1.14\,\mathrm{GeV} \approx 4\pi f_\pi$

$$\bar{m}^2 \equiv \frac{\int_0^{s_0} ds\; s\, R(s)}{\int_0^{s_0} ds\; R(s)}$$

In-medium KKW spectrum:

$$\sqrt{s_0^*} \simeq 1.00 \pm 0.02\,\mathrm{GeV}$$

$$\sqrt{\frac{s_0^*}{s_0}} = \frac{f_\pi^*}{f_\pi} \approx 0.87 \simeq \frac{\bar{m}^*}{\bar{m}} \text{ : BR-scaling}$$

In-medium RW spectrum:

$$\sqrt{s_0^*} \simeq 1.09 \pm 0.01\,\mathrm{GeV}$$

$$\sqrt{\frac{s_0^*}{s_0}} \simeq \frac{\bar{m}^*}{\bar{m}} \approx 0.96$$

Kwon, Procura & Weise [PRC 78, 055203 (2008)]

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Finite energy sum rules at finite temperature

Temperature-dependence of OPE

Hatsuda, Koike & Lee [Nucl. Phys. B 394, 221 (1993)]

- Thermal expectation value:

$$\langle \mathcal{O} \rangle_0 \to \langle \mathcal{O} \rangle_T = \frac{\mathrm{Tr}\, \mathcal{O} \exp(-H/T)}{\mathrm{Tr}\, \exp(-H/T)}$$

- In-medium coefficients: $c_n \to c_n + \delta c_n$

$$\delta c_2 = -\frac{3}{2}\left(\frac{2}{9} \mp 3 + A_1\right) m_\pi^2 T^2 \int_{m_\pi/T}^{\infty} \mathrm{d}y \frac{\sqrt{y^2 - \left(\frac{m_\pi}{T}\right)^2}}{\mathrm{e}^y - 1}$$

$\frac{2}{9}$: T-dependence of gluon condensate

3: T-dependence of quark condensate

A_1: first moment of parton distribution from DIS

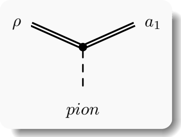
Vector & axialvector mixing with temperature

- Mixing of vector and axialvector:

$$R_V(s,T) = R_V(s,0)\,(1-\epsilon(T)) + R_A(s,0)\,\epsilon(T)$$
$$R_A(s,T) = R_A(s,0)\,(1-\epsilon(T)) + R_V(s,0)\,\epsilon(T)$$

Eletsky & Ioffe [PRD 47, 3083 (1993), PRD 51, 2371 (1995)]

  - the mixing parameter $\epsilon(T)$ is given by the thermal pion loop:



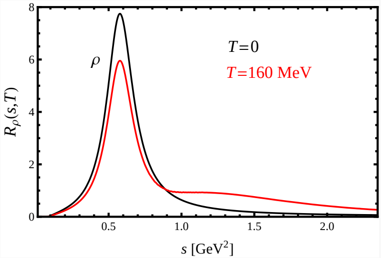
$$\epsilon(T) = \frac{2}{f_\pi^2}\int \frac{d^3k}{\omega\,(2k)^3}\,\frac{1}{e^{\omega/T}-1} \xrightarrow{m_\pi\to 0} \frac{T^2}{6\,f_\pi^2}$$

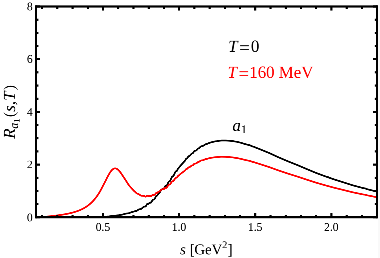
where $\omega^2 = k^2 + m_\pi^2$.

  - At critical temperature where $\epsilon \simeq \frac{1}{2}$, R_V and R_A become identical.

Mixing of finite-width spectrum

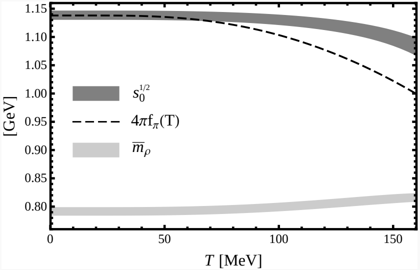
◎ Spectral functions with finite decay width:





Mixing of finite-width spectrum

◎ Sum rule result for vector channel:



- Average ρ-meson mass:

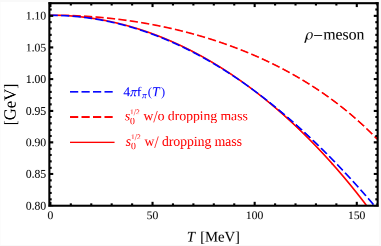
$$\bar{m}_\rho^2 = \frac{\int_0^{s_0} \mathrm{d}s \; s\, R_\rho(s)}{\int_0^{s_0} \mathrm{d}s \; R_\rho(s)}$$

- Comparison with ChPT:

$$f_\pi(T) = f_\pi\left(1 - \frac{1}{2}\epsilon(T)\right)$$

Simple test beyond V-A mixing

◎ Dropping pole mass in addition to the V-A mixing:



The simplest ansatz (zero width):

$$R_\rho(s,0) = F_\rho^2\,\delta\left(s - m_\rho^2\right)$$
$$R_a(s,0) = F_a^2\,\delta\left(s - m_a^2\right)$$
$$R_\rho(s,T) = R_\rho(s,0)\,(1-\epsilon) + R_a(s,0)\,\epsilon$$

Brown-Rho scaling hypothesis:

$$m_\rho^2 \to m_\rho^2\left(1 - \tfrac{1}{2}\epsilon(T)\right)^2$$

⇒ better agreement : $\sqrt{s_0} = 4\pi\, f_\pi(T)$

About four-quark condensates

- Sum rules for 0th and 1st moments: RHS quantities are accurately determined (pQCD and leading condensates)
- Sum rules for 2nd moment: involving four-quark condensates

$$\int_0^{s_0} \mathrm{d}s\, s^2 R(s) = \frac{s_0^3}{3} + c_3$$

$$c_3 = -6\pi^3 \alpha_s \left[\langle (\bar{u}\gamma_\mu\gamma_5\lambda^a u - \bar{d}\gamma_\mu\gamma_5\lambda^a d)^2 \rangle + \frac{2}{9} \langle (\bar{u}\gamma_\mu\lambda^a u + \bar{d}\gamma_\mu\lambda^a d) \sum_{q=u,d,s} \bar{q}\gamma^\mu\lambda^a q \rangle \right]$$

- Ground state saturation ($\kappa = 1$)

$$\langle (\bar{q}\gamma_\mu\gamma_5\lambda^a q)^2 \rangle = -\langle (\bar{q}\gamma_\mu\lambda^a q)^2 \rangle = \frac{16}{9}\kappa \langle \bar{q}q \rangle^2$$

valid approximation? $\Rightarrow$ Always $\kappa > 3$ and large uncertainties.

Summary

- The sum rules for the lowest two moments of the ρ-meson spectral function involve perturbative contributions and only leading condensates as small corrections: accuracy both in vacuum and in medium
- Chiral gap scale: $4\pi f_\pi$ meaningful both in vacuum and in-medium.
- For broad spectral distributions, “mass shift” vs. “broadening” discussion must be specified in terms of first moment.
- Brown-Rho scaling as a statement involving the lowest two moments in the window of low-mass enhancement.
- Further step: extension to nonvanishing three-momentum.

Thank you for your attention!