ATHIC 2008
TSUKUBA

lattice QCD at finite temperature and density

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contents

- introduction to
  lattice QCD at $T > 0$ and/or $\mu \neq 0$
  - how we calculate
  - where we need care
- status: how far are we ?
  - wat's new at Lattice 2008 ?   (just keywords)
  - phase diagram

lattice QCD at $T > 0$ / $\mu \neq 0$

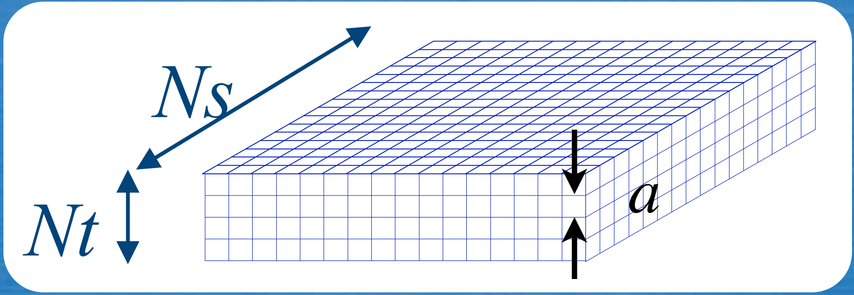
how we calculate • • •

- $T > 0$: Matsubara formalism

euclidian path integral of LQCD with finite euclidian-time extent.

$$Z = \mathrm{Tr}e^{-H/T} = \int DqD\bar{q}DU e^{-S}$$

$$T = \frac{1}{N_t a}$$



- vary T by varying a in terms of g at fixed N_t

$$a = \mathrm{const.} \times (b_0 g^2)^{-b_1/(2b_0^2)} e^{-1/(2b_0 g^2)}$$ + NP corrections

$$b_0 = \frac{1}{16\pi^2}\left(11 - \frac{2}{3}N_F\right),\ b_1 = \frac{1}{(16\pi^2)^2}\left(102 - \frac{38}{3}N_F\right)$$: asymptotic scaling

- *or* by varying N_t at fixed a

lattice QCD at $T > 0$ / $\mu \neq 0$
how we calculate • • • (2)

- thermodynamic quantities from derivatives of Z
  - trace anomaly (interaction measure)

$$\beta = 6/g^2$$

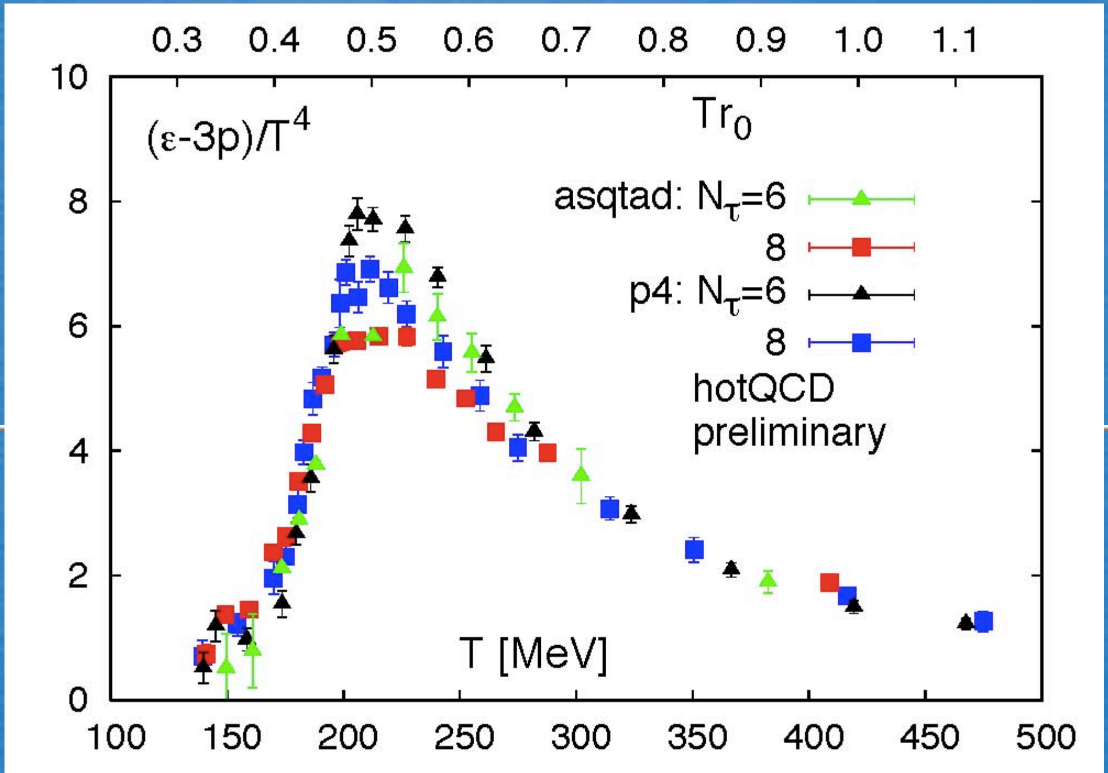
$$\epsilon - 3p = -\frac{T}{V}\frac{d\ln Z}{d\ln a} = -\frac{T}{V}\underbrace{\frac{\partial\beta}{\partial\ln a}}_{\text{NP beta fn.}}\underbrace{\frac{\partial\ln Z}{\partial\beta}}_{\langle\cdots\rangle}$$

For derivatives in V and T, e.g., $p = T\frac{\partial\ln Z}{\partial V}$,
we need separate derivatives in a_t and a_s on anisotropic lattice and Karsch coefficients, whose NP values not easy.

- integral method in conventional fixed N_t approach

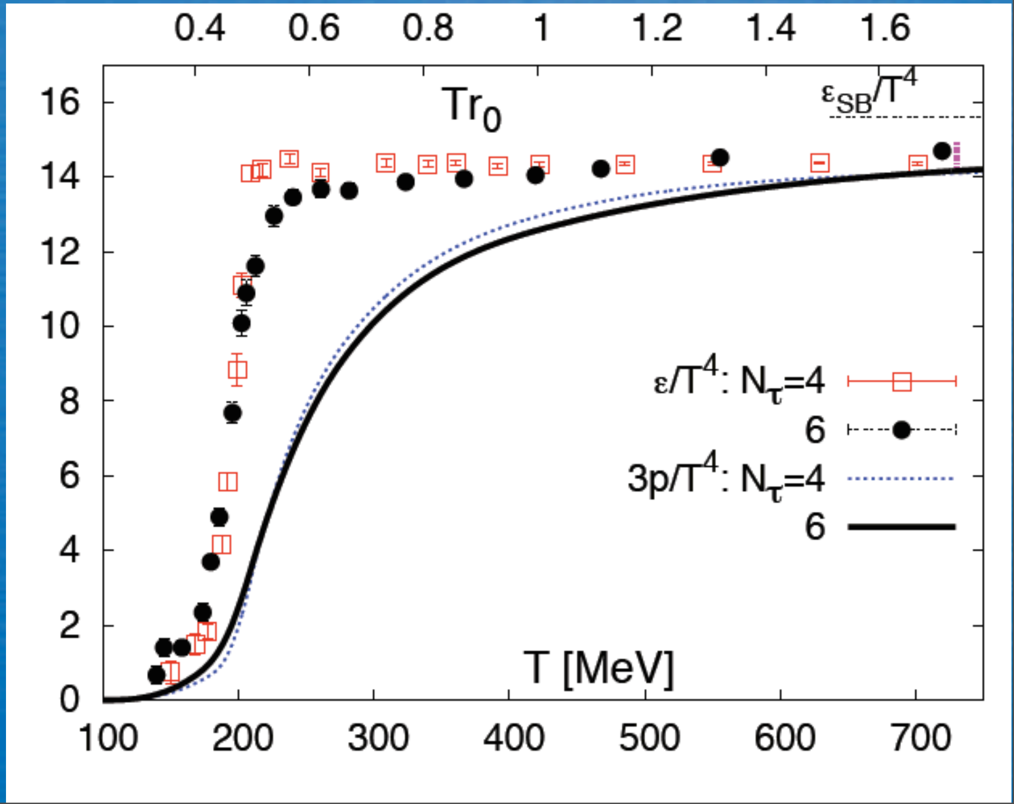
using the thermodyn. relation for large V,

$$p = \frac{T}{V}\ln Z = \frac{T}{V}\int_{\beta_0}^{\beta}\frac{\partial\ln Z}{\partial\beta}d\beta \qquad p(\beta_0)\approx 0$$



RBC-Bi
N_F = 2+1
improved staggered (p4)
$m_\pi \approx$ 220 MeV
M. Cheng et al.
PRD 77 (2008) 014511

HotQCD (≈ RBC-Bi + MILC)
N_F = 2+1
improved staggered (AsqTad, p4)
$m_\pi \approx$ 220 MeV
R. Gupta, Lattice 2008



lattice QCD at $T > 0$ / $\mu \neq 0$
how we calculate • • • (3)

This requires T=0 (large N_t) simulations at each β too.

• subtraction of T=0 UV divergences • determination of LCP • etc

=> 70-90% of the computer time!

- *T*-integral method in the fixed a approach (Talk by Umeda)

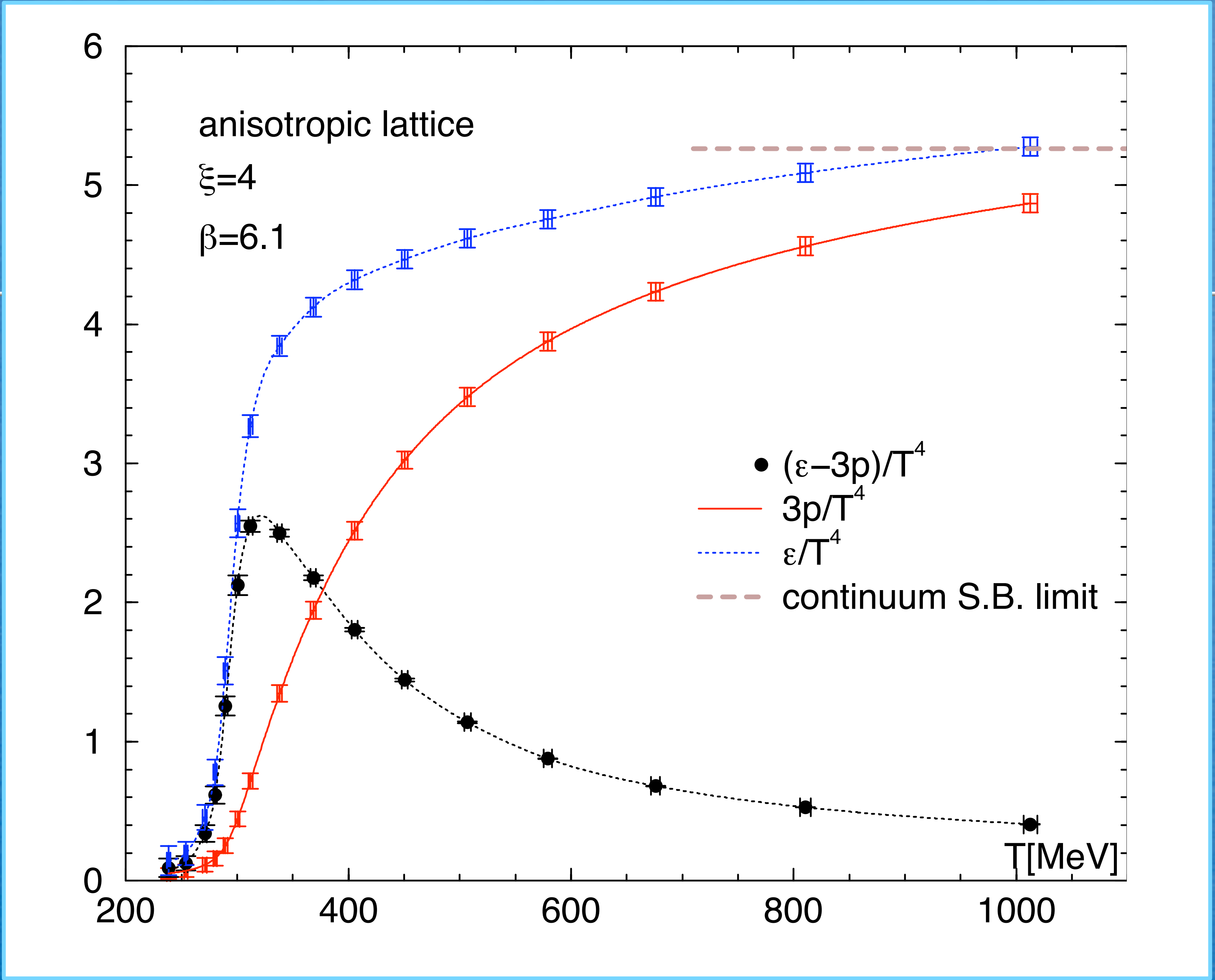
using the thermodyn. relation of grand canonical system at μ=0,

$$T\frac{\partial}{\partial T}\left(\frac{p}{T^4}\right) = \frac{\epsilon - 3p}{T^4} \quad ==> \quad \frac{p}{T^4} = \int_{T_0}^{T} dT\, \frac{\epsilon - 3p}{T^5}$$

Merits:

• subtraction by a common T=0 simulation • obviously on a LCP •

=> large reduction of the computer time.



T-integral method

(WHOT-QCD: Umeda et al. @ Lattice 2008)

lattice QCD at $T > 0$ / $\mu \neq 0$

how we calculate • • • (4)

- $\mu \neq 0$: sign (complex phase) problem

$$U_4 = e^{iaA_4} \quad \Longrightarrow \quad U_4 e^{ia\mu_q} \text{ (positive direction);}$$
$$U_4 e^{-ia\mu_q} \text{ (negative direction)}$$

$$Z = \int DqD\bar{q}DU e^{-S}; \quad S = S_g + \sum \bar{q}M[U]\,q$$

Quark kernel not γ_5 -hermete at $\mu \neq 0$

$$M(\mu)^\dagger = \gamma_5 M(-\mu)\gamma_5$$

=> complex Boltzmann weight $\int Dq\,D\bar{q}e^{-S_q(\mu)} = \det M(\mu)$

$$[\det M(\mu)]^* = \det M(-\mu) \neq \det M(\mu)$$

=> large cancellations due to phase fluctuatons

$\left\langle e^{i\theta} \right\rangle \sim e^{-V}$ while fluctuations ~ O(1), V = lattice vol.

=> MC simulation O(e^{+V}) expensive.

lattice QCD at $T > 0$ / $\mu \neq 0$
how we calculate • • • (4)

- $\mu \neq 0$

Methods for small μ

★ Taylor expansion in μ around $\mu = 0$ <= major studies
★ reweighting from $\mu = 0$
★ analytic continuation from imaginary μ
★ canonical ensemble

=> RHIC/LHC region OK
crit. point ??

still challenging

Intermediate-large μ ????

Bielefeld-Swansea

$N_F = 2$, improved KS (p4)

$m_q^{bare} / T = 0.4, \quad N_t = 4$

Allton et al., PRD 71 (2005) 054508

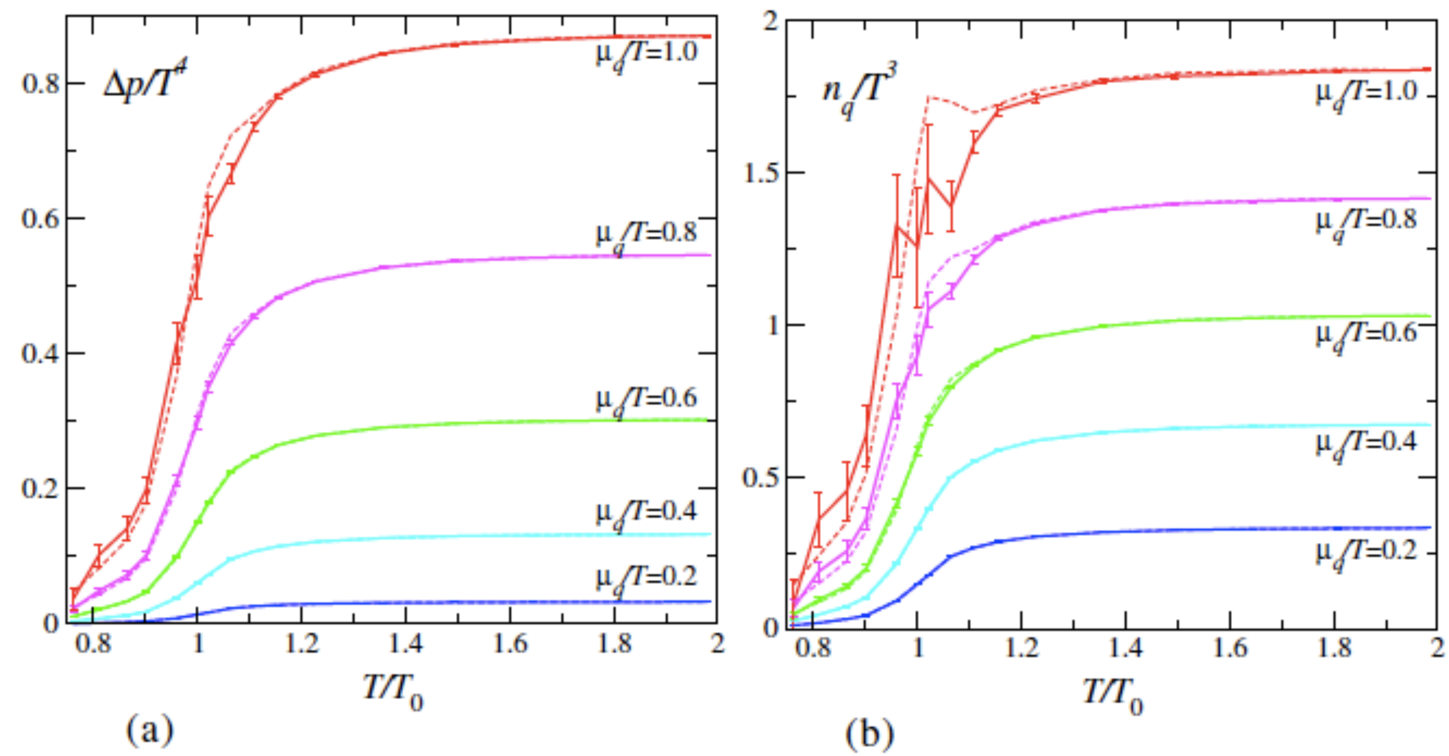


FIG. 2 (color online). The μ_q-dependent contribution to the pressure (left) and the quark number density (right) as functions of T/T_0 for various values of the quark chemical potential calculated from a Taylor series in 6th order. Also shown as dashed lines are results from a 4th-order expansion in μ_q/T.

lattice QCD at $T > 0$ / $\mu > 0$

where we need care • • •

- we are not very close to the cont. limit yet.
  - fixed N_t studies mostly at $N_t = 4$-8

    lattice artifacts due to large a and small N_t

    At $Tc \approx 180$ MeV, we have:

    => $N_t \geq 8$ hopefully

N_τ	4	6	8	10	12
a (fm)	0.27	0.18	0.14	0.11	0.09

  - T-integral method in fixed a approach

    large N_t around Tc ($N_t > 10$ with usual a)

    => lattice artifacts smaller there

lattice QCD at $T > 0$ / $\mu > 0$

where we need care • • • (2)

- lattice quarks
  - naïve lattice fermions: doubler problem
  - (improved) staggered (Kogut-Susskind) quarks
    - relatively cheap => most extensively used
    - degeneracy of 4 quarks with O(a^2) mixing.

      original idea= 4 flavors, but not easy to dissolve

      => "fourth root trick" to drop additional 3 "tastes"

$$\det M \Longrightarrow [\det M]^{1/4} \quad \text{by hand}$$

  - still many additional valence "quarks" => many "hadrons"
  - still different flavor+taste symmetry
  - nonlocality

  => universality class??

  - O(4) scaling for $N_F = 2$ QCD not seen yet.

lattice QCD at $T > 0$ / $\mu > 0$

where we need care • • • (3)

- lattice quarks (2)
  - (improved) Wilson fermions
    - more expensive => need various improvements
    - flavor symmetry & locality naturally realized

QCDPAX/CP-PACS $N_F = 2$, $\mu = 0$ ('96-'03):

O(4) scaling confirmed, phase structure

quark masses are still heavy

WHOT-QCD ('06-): screening masses, $\mu \neq 0$

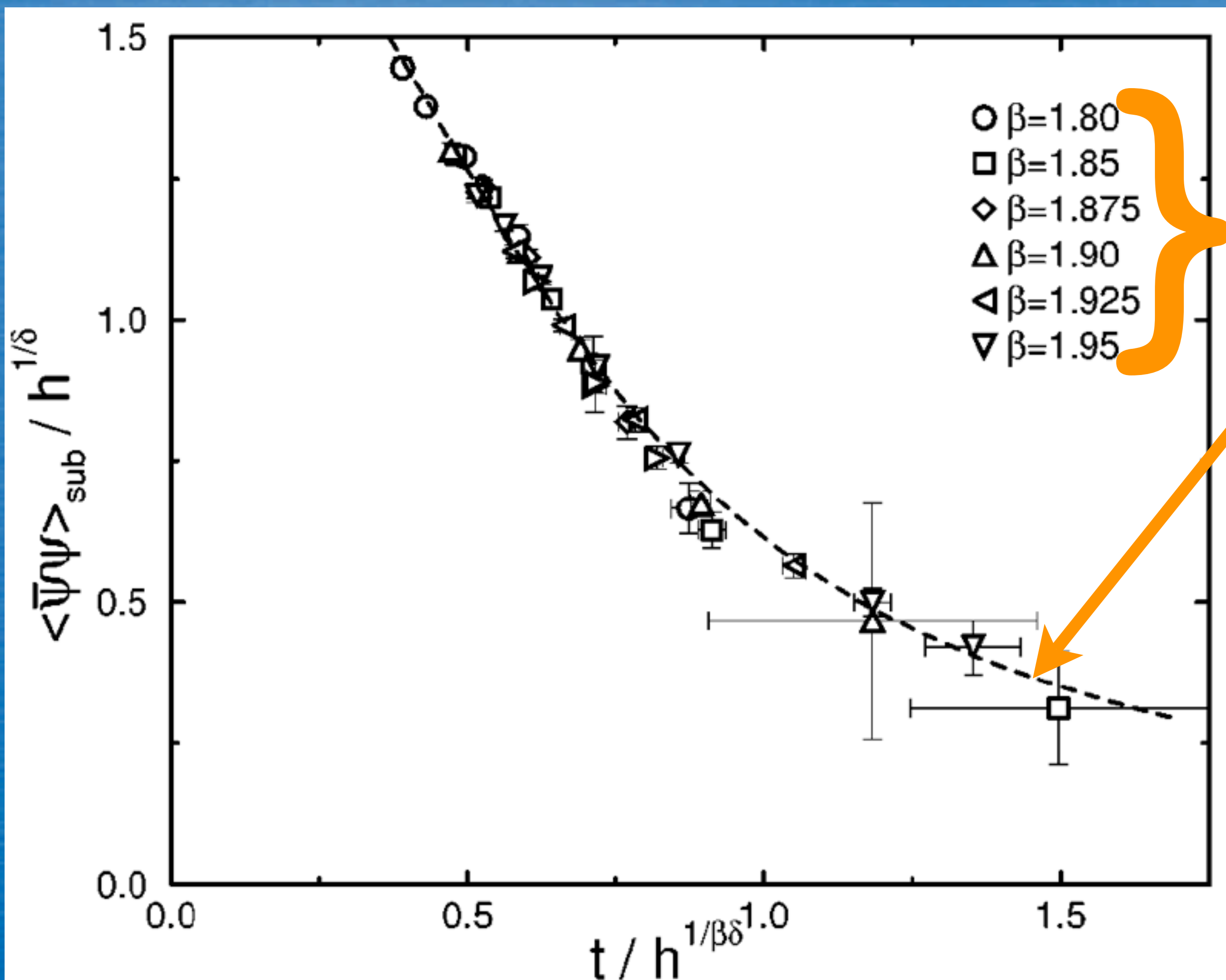
- Taylor expansion method with various improvements
- T-integral method for lighter quarks

CP-PACS

$N_F = 2$, improved Wilson

$m_{PS}/m_V = 0.65\text{-}0.95$, $N_t = 4$

AliKhan et al., PRD 63 (2000) 034502



QCD

O(4) Heisenberg model

$h = 2\, m_q$

$t = \beta - \beta_{\text{chiral trans.}}$

fit with

O(4) critical exponents

lattice QCD at $T > 0$ / $\mu > 0$
where we need care • • • (4)

- lattice quarks (3)
  - lattice chiral fermions (DW, overlap)

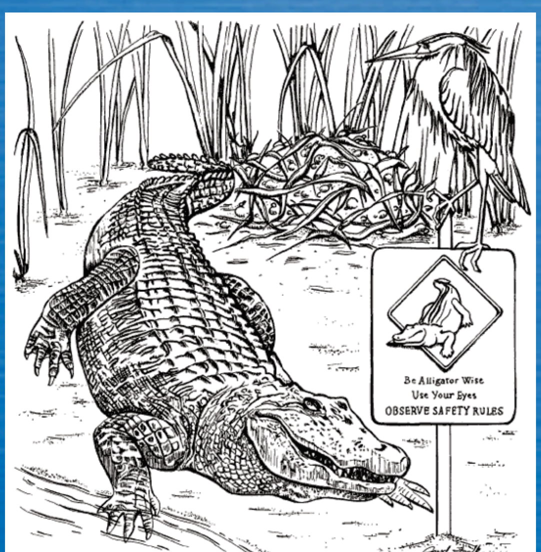
    still expensive (O(100) times more computer time)

    first results of DW simulations (RBC/HotQCD)

    "in infancy" (C. DeTar, plenary @ Lattice 2008)

lattice QCD at $T > 0$ / $\mu > 0$
where we need care • • • (5)

- and more
  - finite volume effects and FSS
  - violation of chiral symmetry
  - MEM
  - • • •



please enjoy

status

how far are we?

how was it at Lattice 2008 ?

Williamsburg, VA, USA, July 14-19, 2008

- 38 talks and 6 posters on $T > 0$ / $\mu > 0$

Started with plenaries by

- C. DeTar on “QCD Thermodynamics”
- S. Ejiri on “LQCD at finite density”

38 talks

$T > 0$

$\mu > 0$

Gert Aarts	Stochastic quantization at nonzero chemical potential
Masayuki Asakawa	Baryonic Spectral Functions above the Deconfinement Phase Transition
Bernd Berg	Minkowskian Dynamics of a Polyakov Loop Model under a Heating Quench
Michael Cheng	QCD Thermodynamics from Domain Wall Fermion
Guido Cossu	A test of first order scaling in Nf =2 QCD: a progress report
Gergely Endrodi	The curvature of the QCD phase transition line
Michael Fromm	Revisiting strong coupling QCD at finite baryon Density and temperature
Kenji Fukushima	Characteristics of the Dirac eigenvalue distribution in dense two-color QCD
Rajiv Gavai	Exact Chiral Fermions and Finite Density on Lattice
Steven Gottlieb	QCD equation of state at non-zero chemical potential
Rajan Gupta	The EOS from simulations on BlueGene L Supercomputer at LLNL and NYBlue
Sourendu Gupta	Finite chemical potential in Nt=6 QCD
Masatoshi Hamada	Quark Propagators at the confinement and deconfinement phases
Kay Huebner	Renormalized Polyakov loops in various Representations in finite Temperature SU(2) gauge theory
Ernst-Michael Ilgenfritz	The finite-temperature phase structure of lattice QCD with twisted-mass Wilson fermions
Frithjof Karsch	Fluctuation of Goldstone modes and the chiral transition in QCD
Masakiyo Kitazawa	Measurement of shear viscosity in lattice gauge theory without Kubo formula
Tamas Kovacs	Gapless Dirac spectrum at high temperature
Aleksi Kurkela	Center-symmetric dimensional reduction of hot Yang-Mills theory
Edwin Laermann	Recent results on screening masses
Anyi Li	Finite Density Simulation with the Canonical Ensemble
Yu Maezawa	Magnetic and electric screening masses from Polyakov-loop correlations
Xiangfei Meng	Winding number expansion in canonical approach to finite Density
Shin Muroya	Stochastic quantization of a finite temperature lattice field theory in the real time formula
Joyce Myers	Exotic phases of finite temperature SU(N) gauge theories with massive fermions: F, Adj, A/S
Michael Ogilvie	High Temperature Confinement in SU(N) Gauge Theories
Akira Ohnishi	Quarkyonic phase in the strong coupling region of lattice QCD
Hiroshi Ohno	Search for the Charmonium Dissociation Temperature with Variational Analysis in Lattice QCD
Marco Panero	Geometric effects in lattice QCD thermodynamics
Claudio Pica	Critical behavior of the energy and pressure correlation functions in SU(2) gauge theory
Yuji Sasai	Eigen-value Distributions of Quark Matrix at Finite Isospin Chemical Potential
Christian Schmidt	The QCD phase diagram and the equation of state at non-zero Density from a Taylor expansion of the pressure
Donald Sinclair	Con
Wolfgang Soeldner	Qua
Kalman Szabo	The
Takashi Umeda	Ther
Jacobus Verbaarscho	Phas
Philippe de Forcrand	The curvature of the critical surface (m_ud,m_s)^{crit}(mu), on finer and bigger lattices

6 posters

Alexei Bazavov	Color singlet and adjoint free energy at finite temperature
Prasad Hegde	ark Number Susceptibilities with Domain-Wall Fermions
Kazuyuki Kanaya	uation of state at finite Density in two-flavor QCD with improved Wilson quarks
Kohtaroh Miura	ase diagram evolution by finite coupling effect in color SU(3) strong coupling lattice QCD at finite temperature and Density
Atsushi Nakamura	ite Density QCD with Wilson Fermions
C. Miao	Lattice Calculation of Hadronic ...

- C. DeTar on “QCD Thermodynamics”
  - large N_F=2+1 simulations near the physical point:

    HotQCD with impr.stag. at $m_\pi \approx$ 220 MeV.
  - new ideas:

    T-integral method for EOS (WHOT-QCD), etc.
  - Tc confusion diminished:

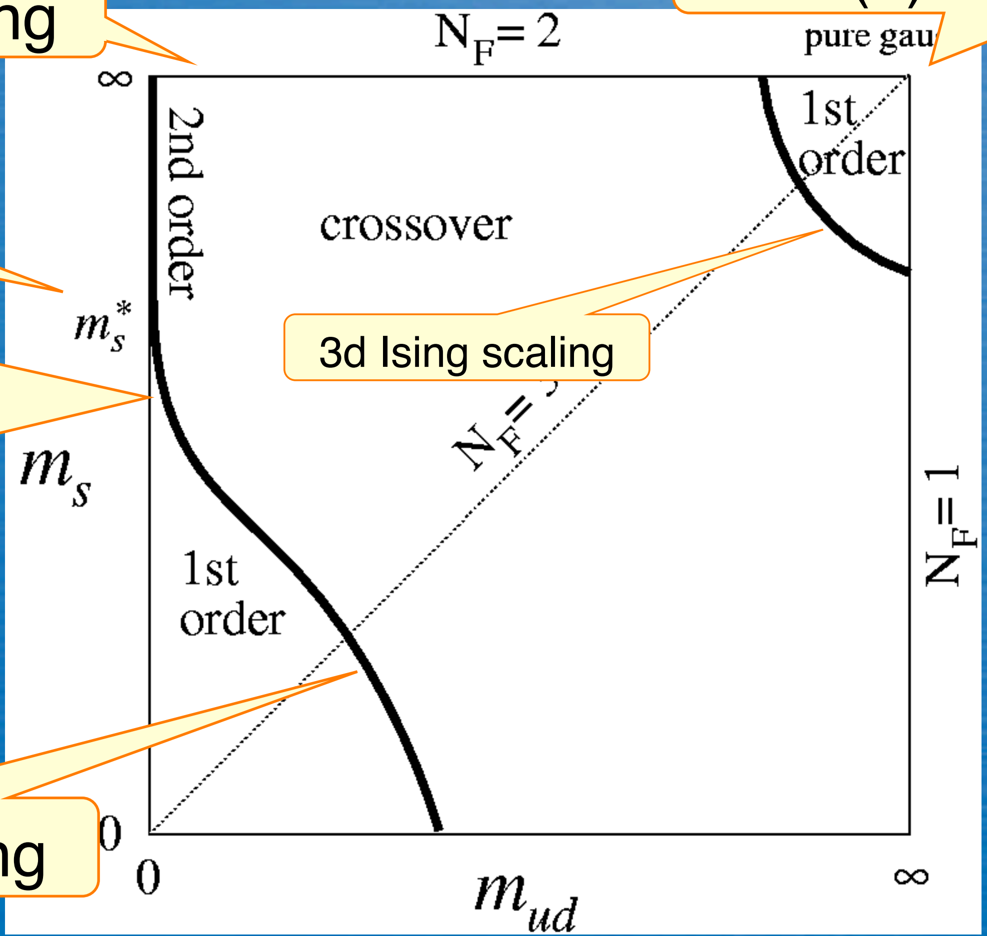
    chiral suscept. problematic for Tc

    $Tc \sim$ 170-190 MeV
  - . . .

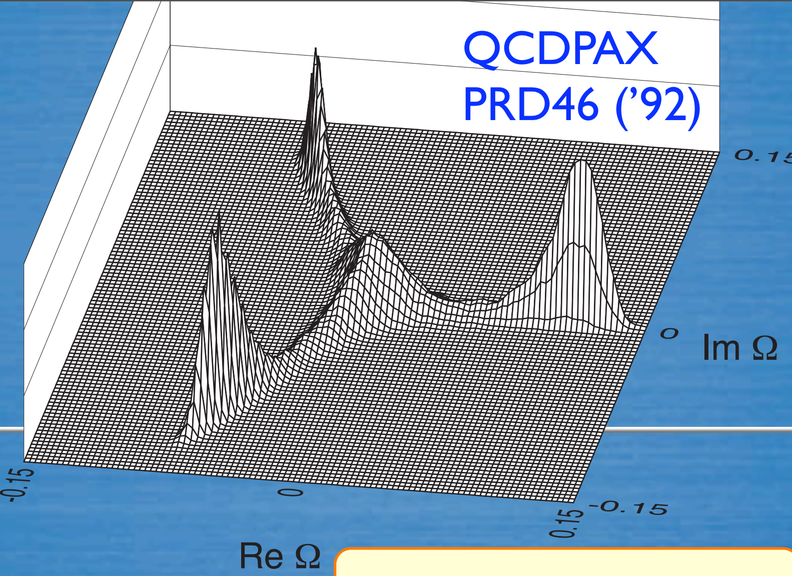
- S. Ejiri on “LQCD at finite density”
  - isentropic EOS (MILC, RBC-Bi, HotQCD)
  - results with Wilson-type quarks (WHOT-QCD):
    had. fluctuations enhanced toward crit. pt.
  - technical developments for $\mu > 0$ simulations
  - . . .

phase diagram at $\mu = 0$

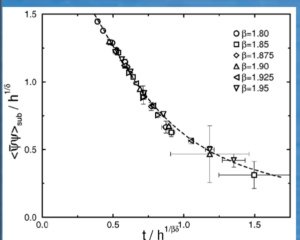
- theoretical expectations from effective models



phase diagram at $\mu = 0$ (2)



- LQCD simulations



3d O(4) scaling

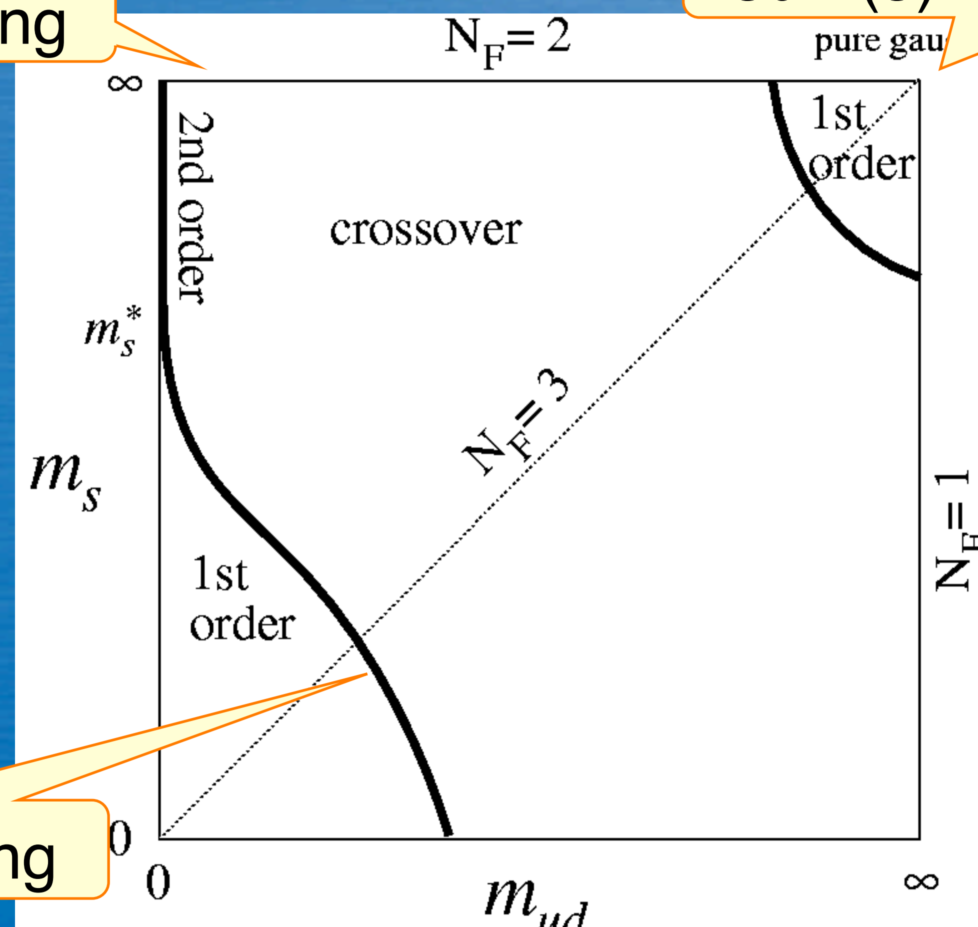
Wilson-type: OK
staggered-type: *not* seen
DW/overlap: not tested yet

Dissent argument by the Pisa group:
1st order at N_F=2
(Nt=4, unimproved staggered)

Wilson-type, staggered-type: look OK
DW/overlap: not tested yet

3d Ising scaling

3d Z(3) Potts

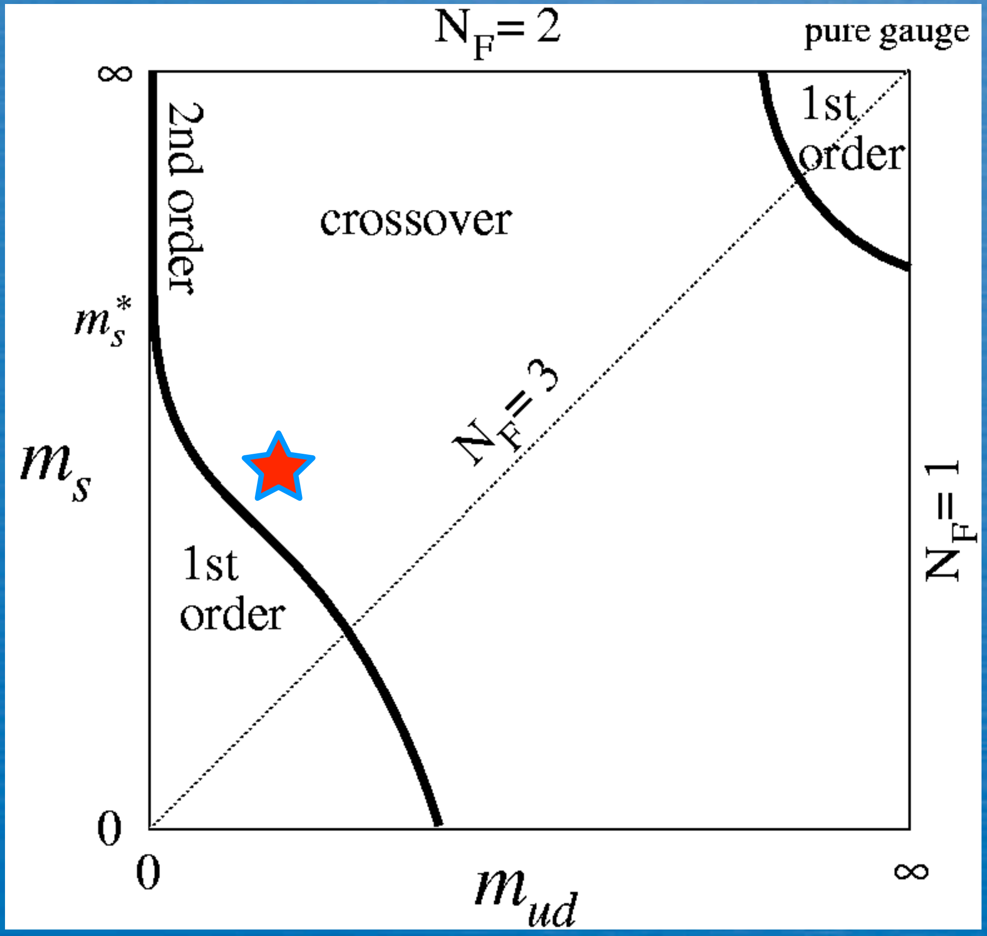


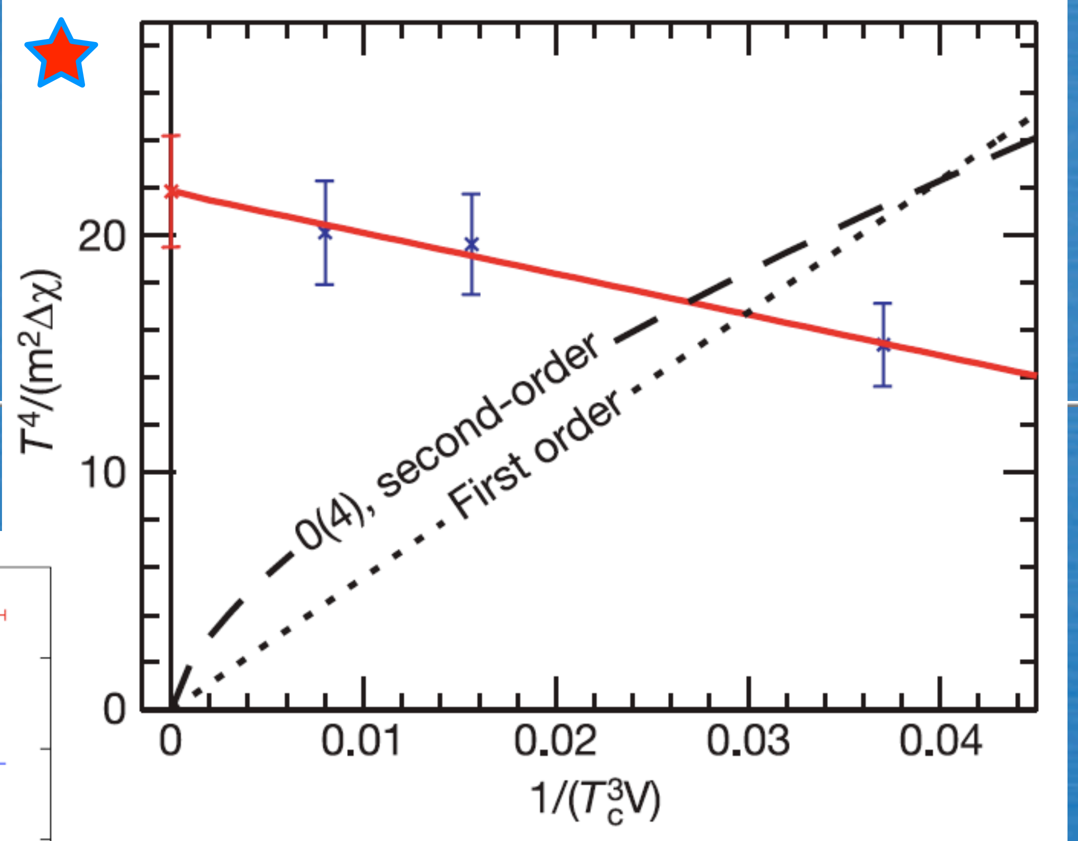
phase diagram at $\mu = 0$ (3)

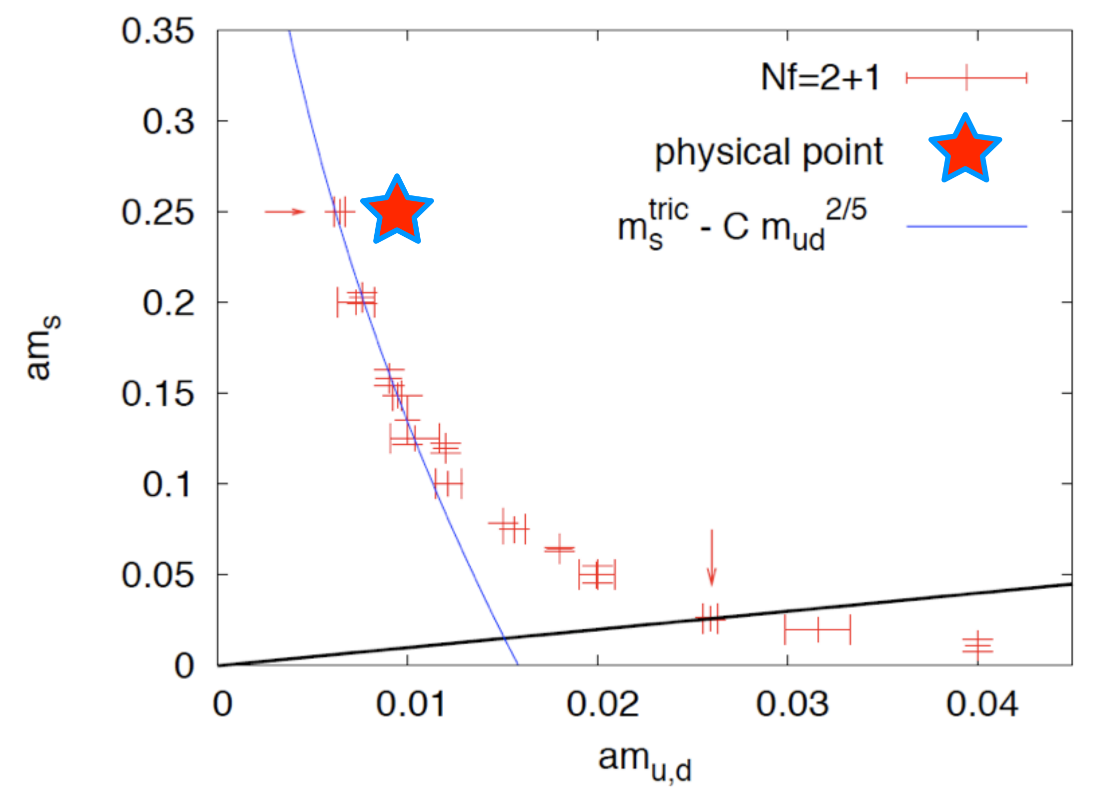
- location of the physical point

Intensively studied only with staggered quarks.
=> crossover

Caveats:
- Staggered quarks could not reproduce the O(4) scaling.
- How about Wilson-type ?? or DW/ovelap ??? (old unimpr.Wil.=> 1st order)







Y. Aoki et al., nature05120 ('06)
improved staggered (stout),
continuum extrap. with Nt=4-10
finite size scaling study
=> Crossover at the phys. pt.

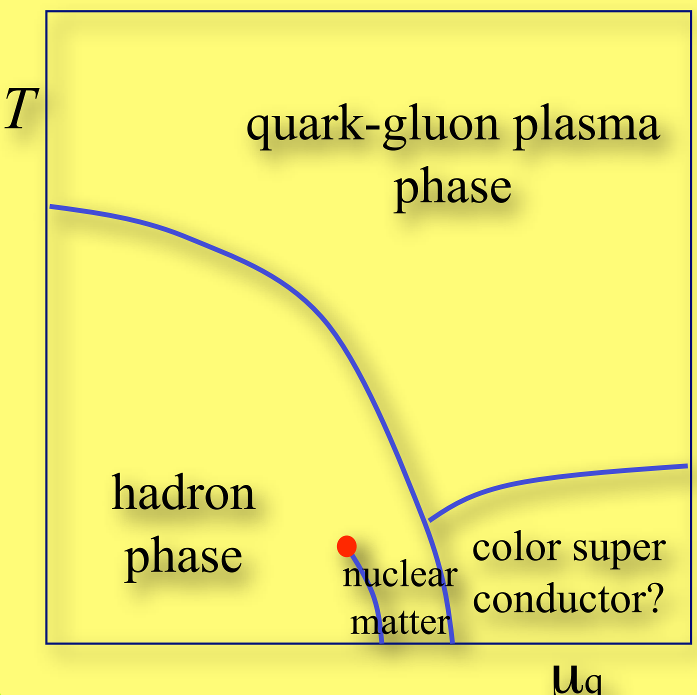
de Forcrand and Phillipsen, Lattice 2006
unimproved staggered, Nt=4, exact algorithm

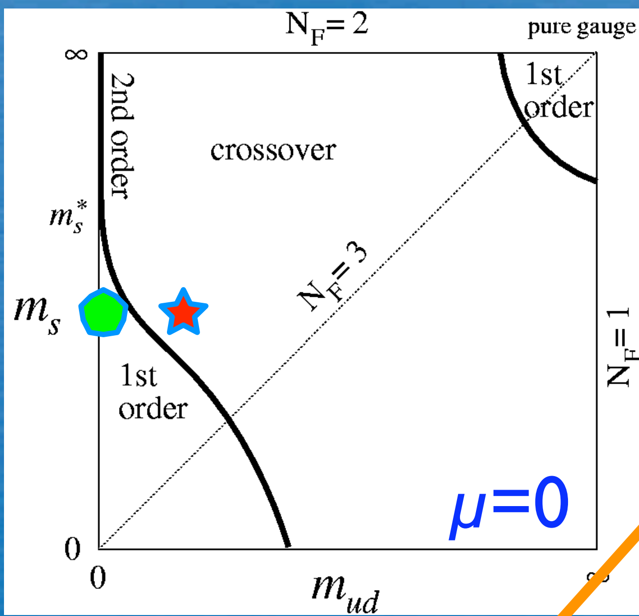
phase diagram at $\mu \neq 0$

- what usually assumed

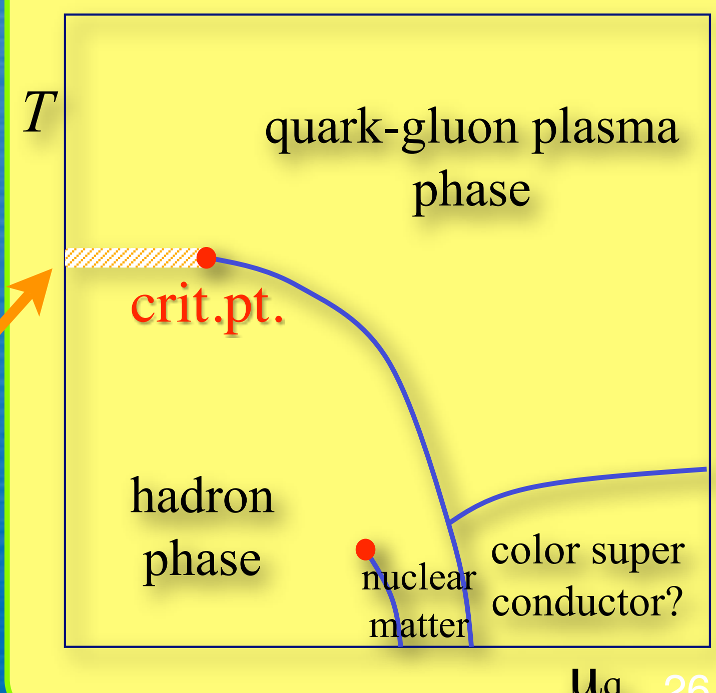
based on model studies + lattice staggered quark results

$m_s \approx m_s^{phys}$
$m_{ud} \approx 0$

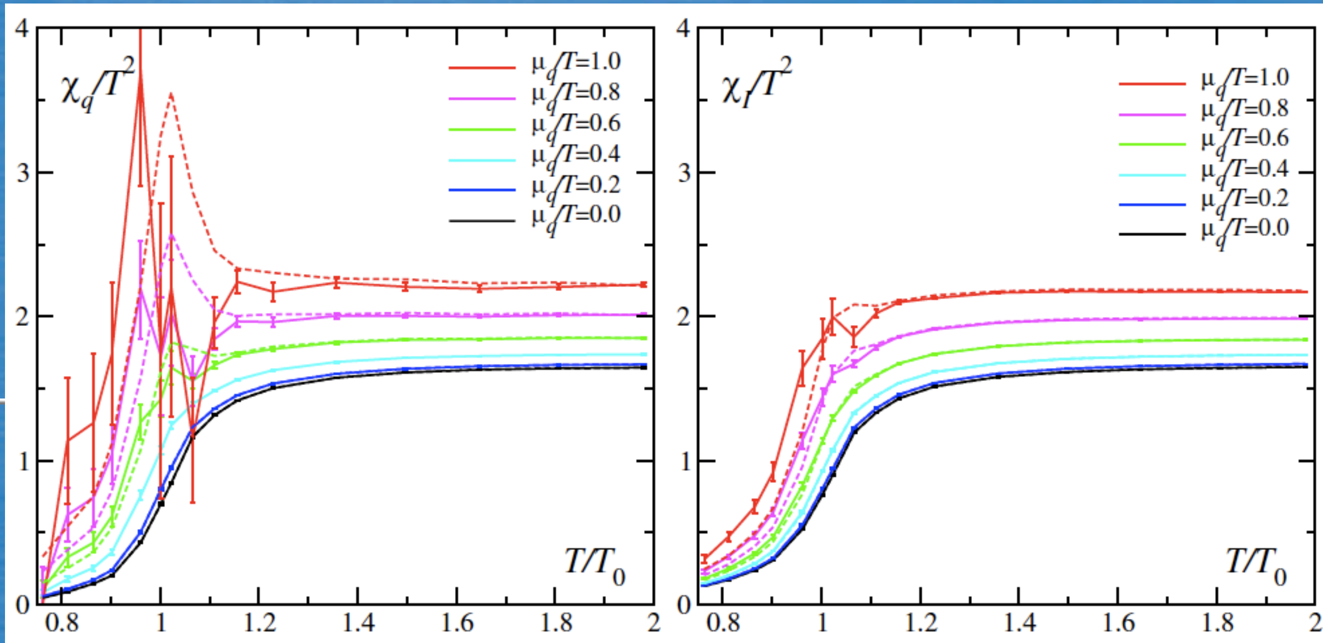




$m_s \approx m_s^{phys}$
$m_{ud} \approx m_{ud}^{phys}$

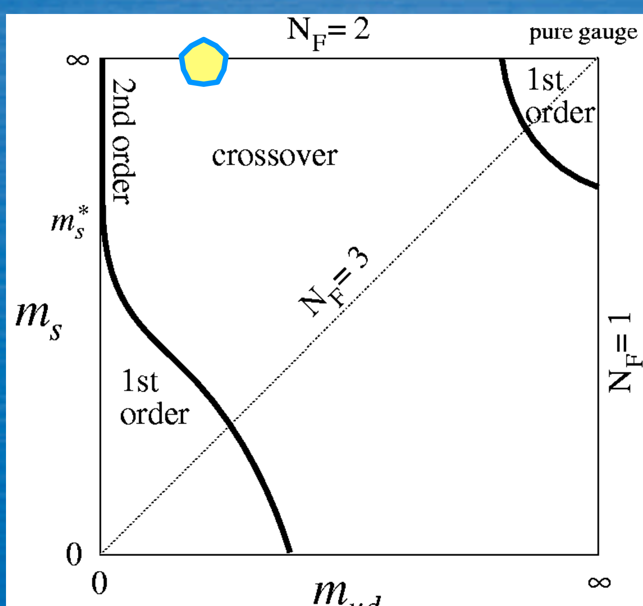


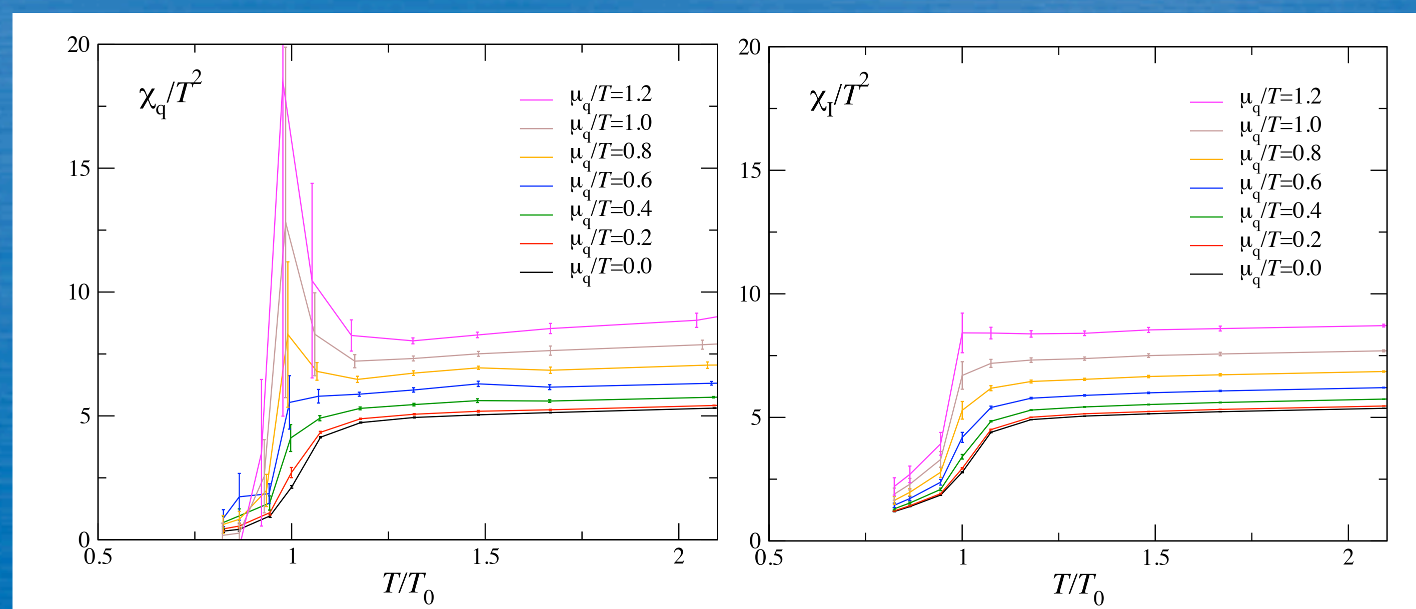
Assuming crossover at μ = 0.

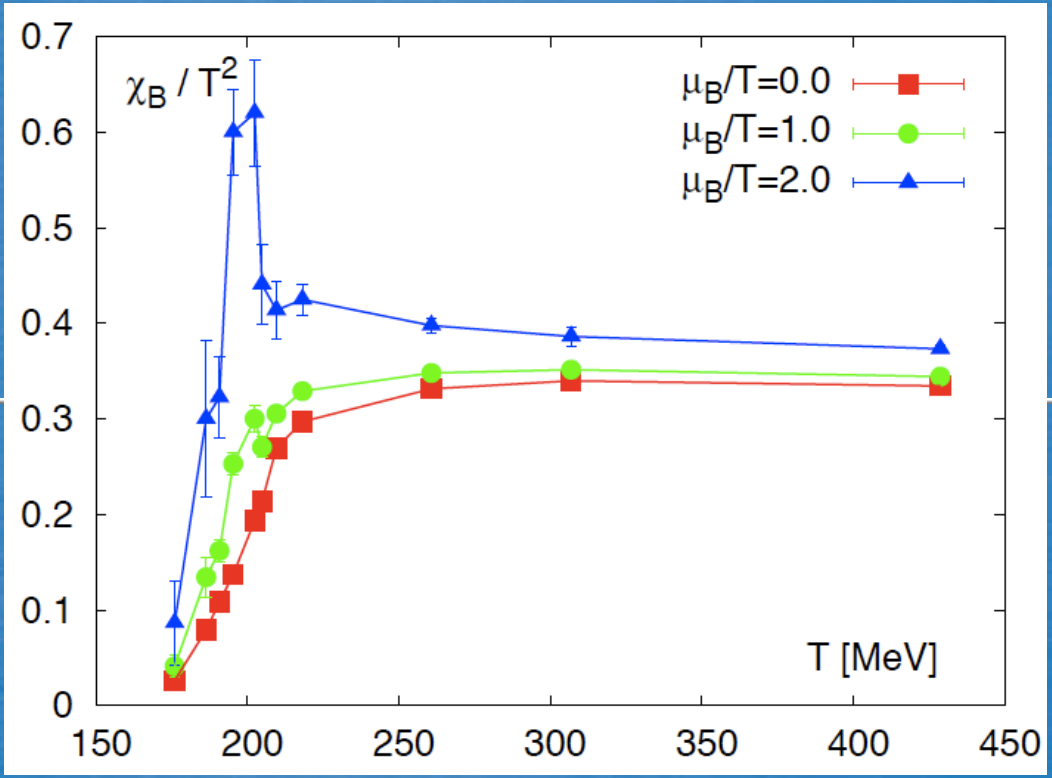


Bielefeld-Swansea
$N_F = 2$, $m_{ud}/T = 0.4$
improved stag. (p4), $N_t = 4$
Allton et al.,
PRD71, 054508 ('05)

WHOT-QCD
$N_F = 2$, $m_{PS}/m_V = 0.65$
improved Wilson, $N_t = 4$
KK, Lattice 2008





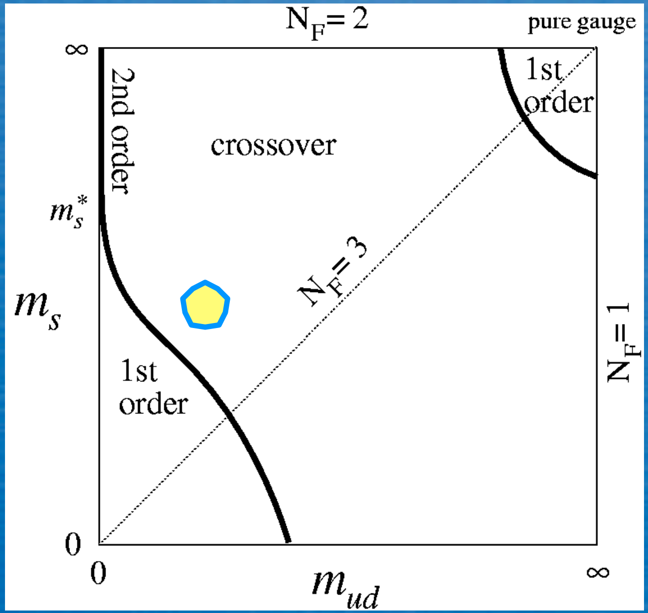


RBC-Bi

$N_F = 2{+}1$, $m_\pi \approx 220$ MeV

improved stag. (p4), $N_t = 4, 6$

C. Miao, Lattice 2008

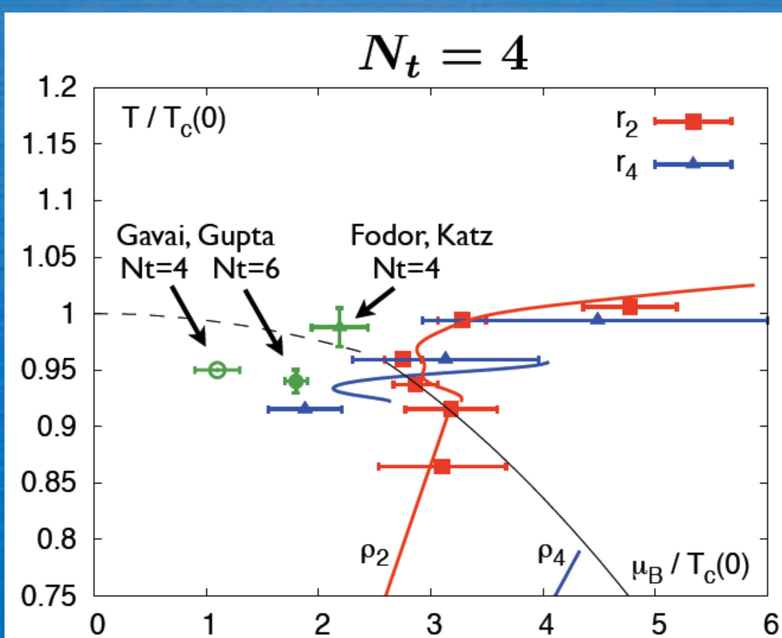


$\mu = 0$

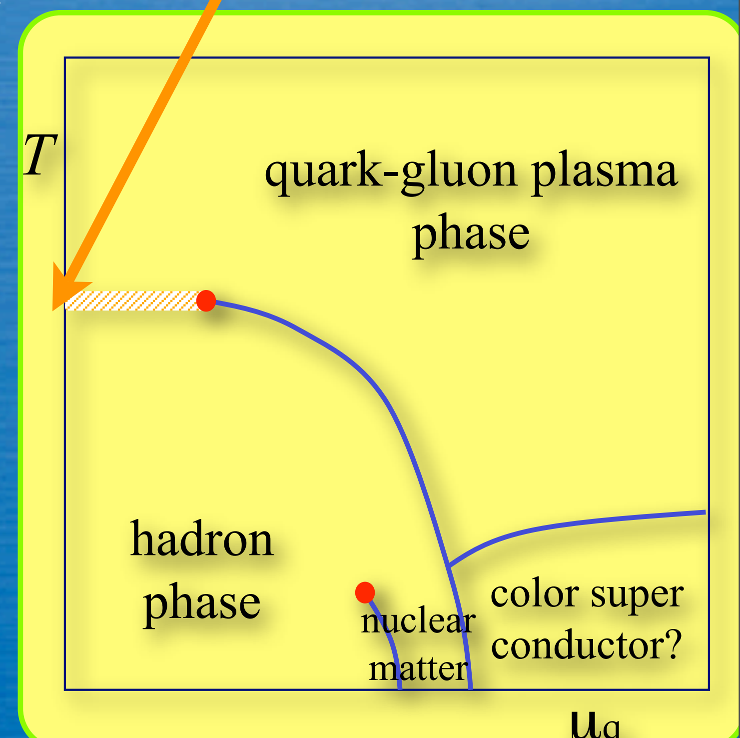
phase diagram at μ ≠ 0 (2)

- where is the critical point?
  - Lee-Yang zero (Fodor and Katz)
    <= critique by Ejiri (PRD73,054502('06))
  - radius of convergence of Taylor expansion:
    N_F=2, 2+1, staggered-type quarks, Nt=4 mostly

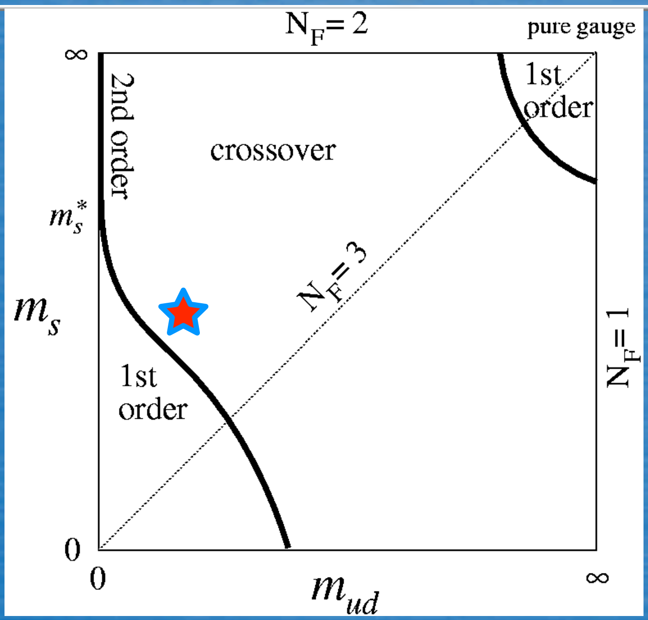
Assuming crossover at μ = 0.

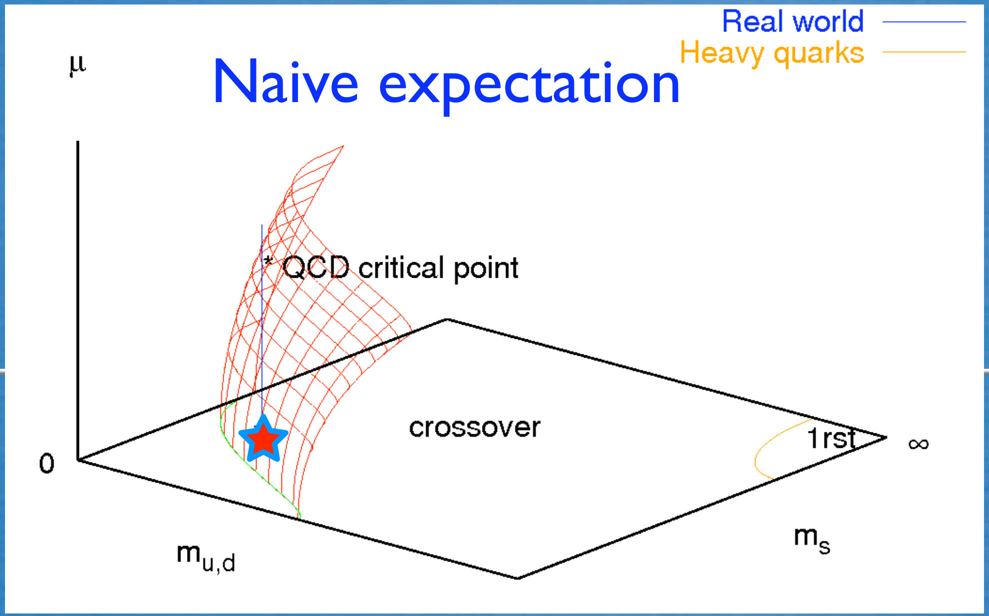


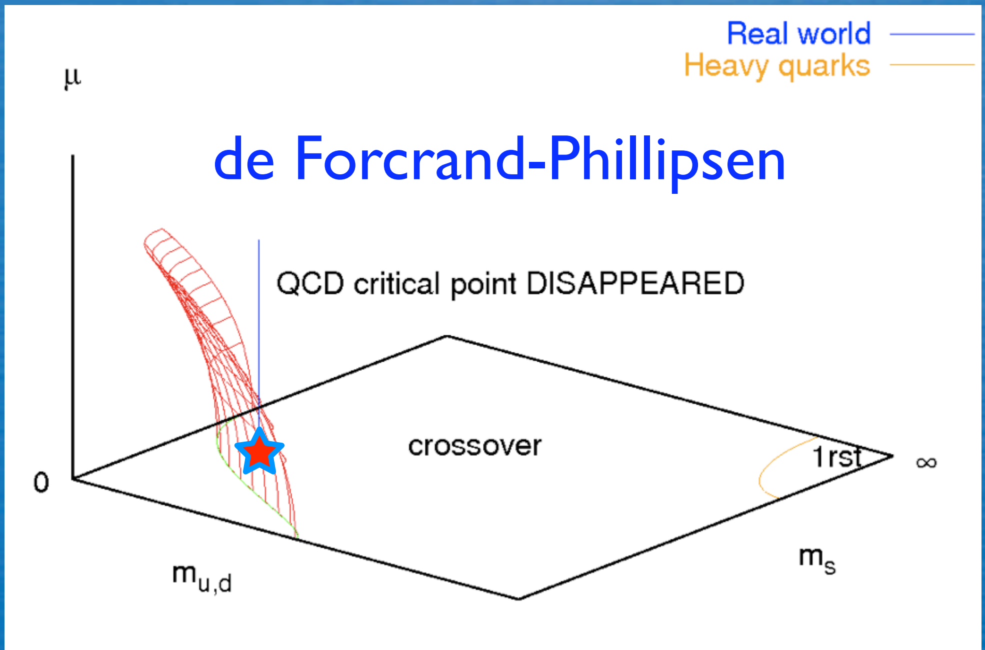
compiled by C. Schmidt, Lattice 2008



phase diagram at $\mu \neq 0$ (3)







imaginary μ method
unimproved stag., *Nt*=4
JHEP01 077, Lattice 2008
=> slightly negative curvature at μ=0.
The crit. surface should bend back!

summary

LQCD: direct bridge between

1st principles of QCD <=> hadron / QGP physics

Predictions availabele.

caveats: several systematic errors not well controlled yet.

Simulations becoming constantly realistic.

Direct studies just at the physical point started.

(plenary by Y. Kuramashi, Lattice 2008)

Feed back to finite temperature and density will be stareted soon.

thank you

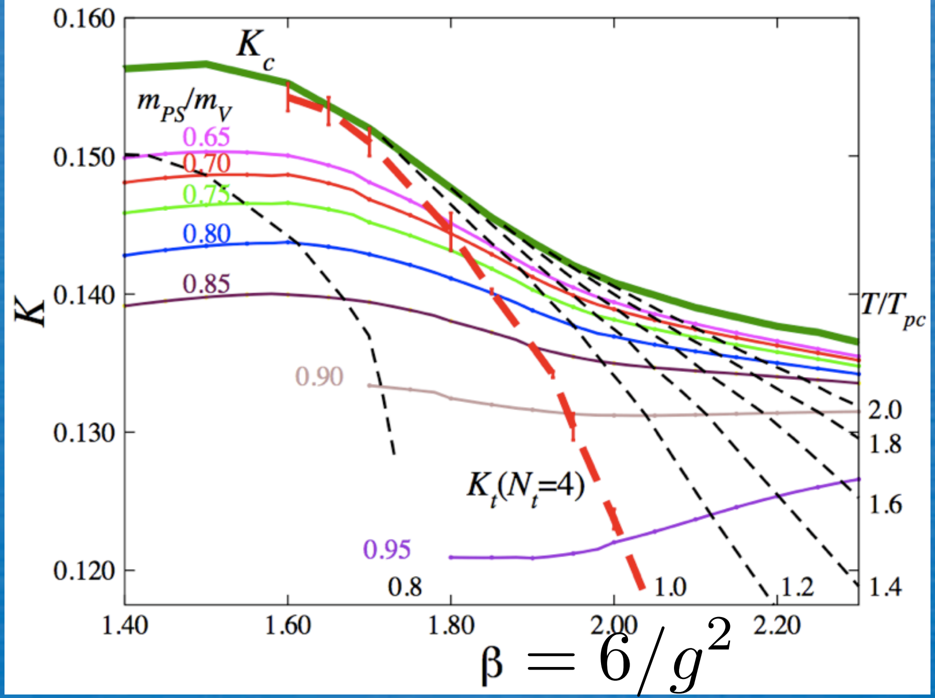
lattice QCD at T > 0

how we calculate • • •

- line of constant physics (LCP)

A physical system (with various a, i.e various g) is given by a line in the coupling parameter space:

Nf=2 QCD with improved Wilson quarks
(CP-PACS Collab., WHOT-QCD Collab.)
LCP by $m_{\rm PS}/m_{\rm V}$ at $T=0$.



Different line = different world
Our world is given by LCP for

$$m_{\rm PS}/m_{\rm V} = m_\pi/m_\rho = 135/770$$

To heat up a given physical system in fixed N_t approaches, we have to follow the LCP for this system.