Heavy QQbar potential

from a spectral decomposition of the thermal Wilson loop

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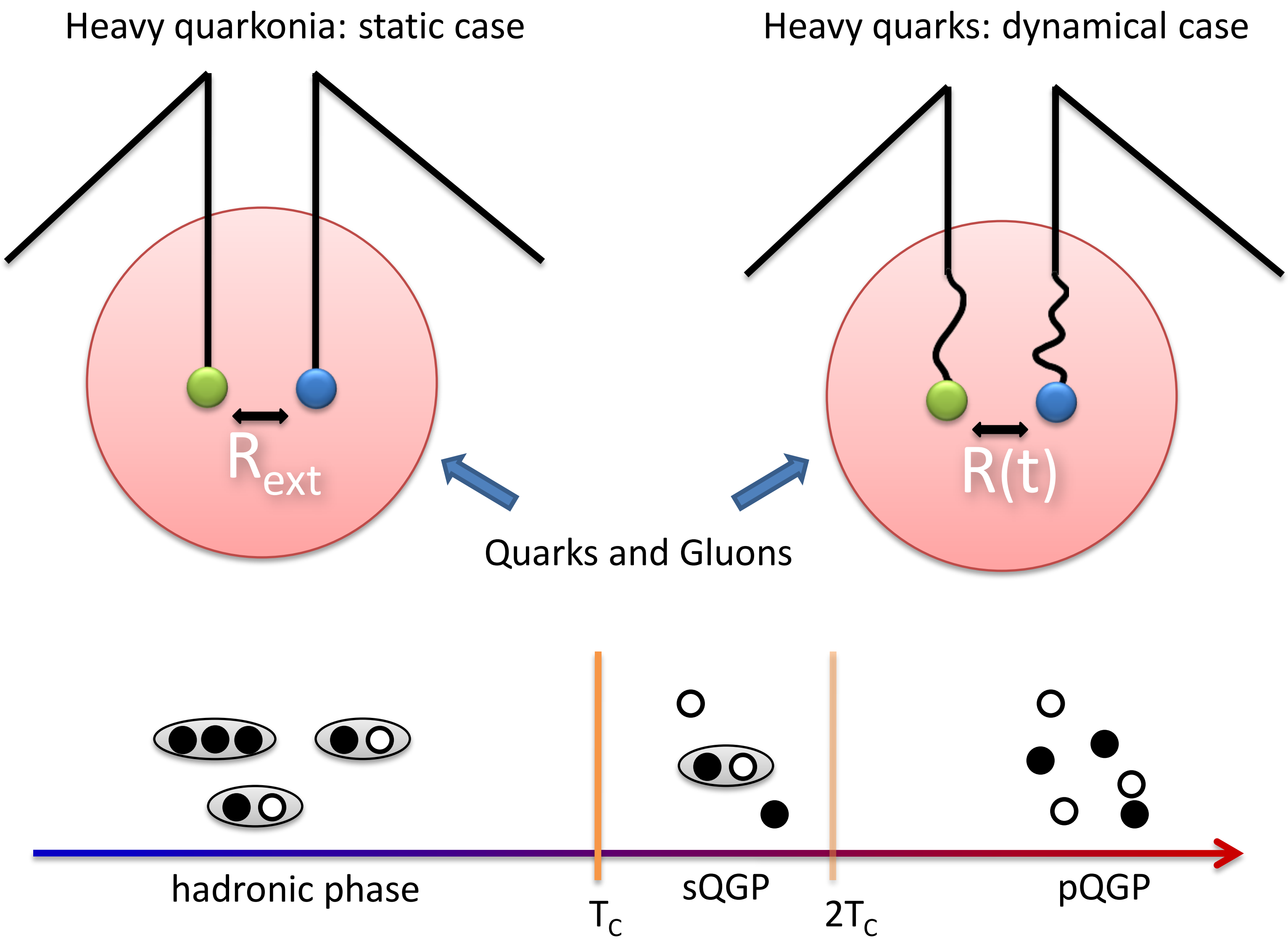
Motivation: Probing the QGP



Outline

1. Motivation: Heavy Quark potential
2. Potential from the Spectral function
3. Lattice Reconstruction
4. Conclusion and Outlook

Heavy Quarks

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Apriori: At the phase transition tightly bound states do not have to vanish
See e.g. Hatsuda, Kunihiro (PRL 55, 1985)

However: Matsui & Satz predicted J/Psi melting at crtitcal T
comparison $V_{coulomb+lin.}(r,T)$ vs. $V_{Debye}(r\ ,T)$
"J/Psi Suppression by Quark-Gluon plasma formation" (Phys.Lett. 178, 1986)

Heavy Quarks allow separation of scales: Non-relativistic ($1/m_Q$)

$$\left[i\partial_t - \left(\quad 2m_Q - \frac{\nabla^2}{m_Q} + \mathcal{O}\left(\frac{1}{m_Q^3}\right)\right)\right]G^>(t,x) = 0$$

$$G^>(t,x) \equiv \left\langle J^\mu(t,x) J_\mu(0,0) \right\rangle$$

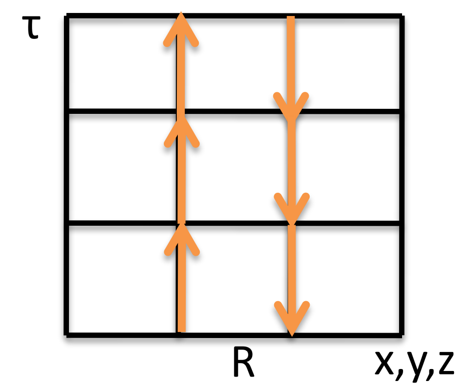
see e.g. Laine et al. JHEP03, 2007

How do we find the potential to use in that Schrödinger Equation?

Potential Searches

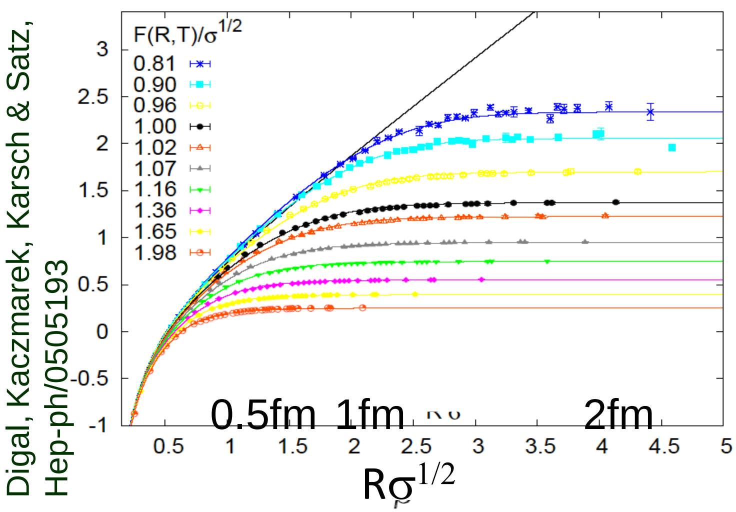
Use Lattice QCD since non-perturbative: $U_\mu(x) \sim e^{iagA_\mu}$

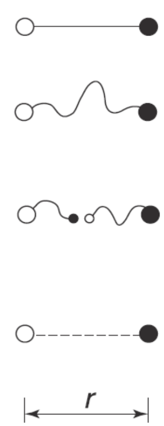
First approach: Free Energy as static potential



Free Energy from polyakov correlator

McLerran, Svetitsky
"Quark liberation at high temperature" (Phys. Rev. D 24, 1981)





$$e^{-\beta \Delta F_{Q\bar{Q}}} = \langle L(x_Q) L^\dagger(x_{\bar{Q}}) \rangle$$

$$L(x) = \frac{1}{N} \mathrm{Tr}\Big[\prod_{\tau=0}^{\tau=\beta} U_4(x,\tau) \Big]$$

Different approaches: Internal Energy, Linear combination Internal/Free energy
(Shuryak, Petreczky, Wong)

However: No direct relation to potential available!

J/Psi from first principles:

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2004: J/Psi survives in the deconfined phase up to $\lesssim 2T_C$

Asakawa, Hatsuda

"J/Psi and Eta$_C$ in the deconfined plasma from Lattice QCD" (PRL 92, 2004)

Fact: Spectral function contains all information of the system.

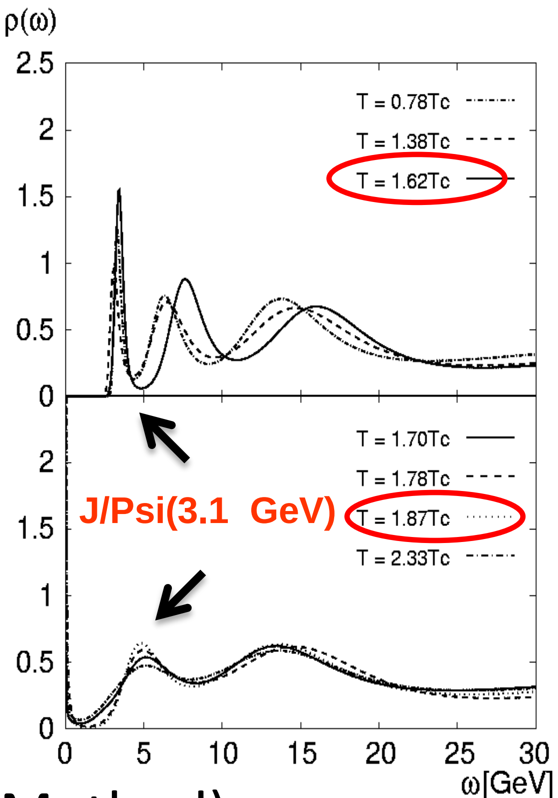
$$\rho(\omega,\vec{p}) = \int d^4x\, e^{-ip\cdot x}\mathrm{Im}(D^{ret}_{\mu\mu}(t,x))$$
$$= \int d^4x\, e^{-ip\cdot x}\mathrm{Im}(\langle J_\mu(t,x)J^\dagger_\mu(0,0)\rangle)$$

Fact: Vector channel SPF directly accessible by experiment:

$$\frac{d^8N_{l^+l^-}}{d^4xd^4p} = -\frac{\alpha^2}{3\pi^2 s}\frac{\rho(\omega,\vec{p};T)}{e^{\omega/T}-1}$$

Dilepton production rate — Feinberg (1976)

Lattice simulations in Euclidean time τ, no direct access to dynamical quantities. Reconstruction is ill-posed problem.



However possible with **MEM** (Maximum Entropy Method)

Asakawa, Nakahara, Hatsuda PRD 60 (1999) & Prog. Part. Nucl. Phys. 46 (2001)

What is the true potential?

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Challenge 1: Free energy potential has no direct theroetical basis (also J/Psi melts too quickly)

Challenge 2: Lattice potential does not include realtime dynamics, are we missing something?

Probably: Heavy Quark potential can be complex, i.e. J/Psi is transient
(using the Hard Thermal Loop approximation, i.e perturbative)

Laine et al.
"Realtime static potential in hot QCD" JHEP 0703 (2007)
Beraudo, Balizot, Ratti
"Real and imaginary-time QQbar correlators in a thermal medium" NPA 806 (2008)

HTL Potential: $$V_{\mathrm{HTL}}(r) = V_D(r) + V_{LD}(r)$$

Debye screening: (can be derived also from free energy)

$$V_D(r) \propto g^2 \left[m_D + \frac{e^{-m_D r}}{r} \right], \quad m_D \propto gT$$

Additional effect: Landau damping (scattering with light medium particles)

$$V_{LD}(r) \propto i g^2 T \int_0^\infty dz \frac{z}{(z^2+1)^2} \left[1 - \frac{sin(z m_D r)}{z m_D r} \right]$$

A potential from Lattice QCD

Through MEM we can access dynamical information (SPF) directly from the lattice

What we need: Definition of the Heavy Quark Potential in terms of the spectral function

From **NRQCD**:

$$G^>(t,x) \overset{t\to\infty}{\propto} e^{i\mathrm{Re}[V(x)]t} e^{-\mathrm{Im}[V(x)]t} + \mathcal{O}\Big(\frac{1}{m_Q}\Big)$$

Heavy mass limit:

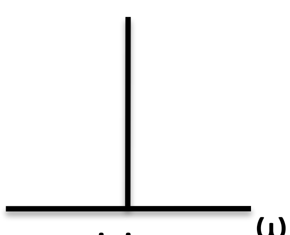
$$\rho(\omega) = \int_{-\infty}^{\infty} e^{i\omega t}\Big[G^>(t) - G^<(t)\Big] = \int_{-\infty}^{\infty} e^{i\omega t} G^>(t)$$

Lattice system is finite, therefore SPF **discrete**:

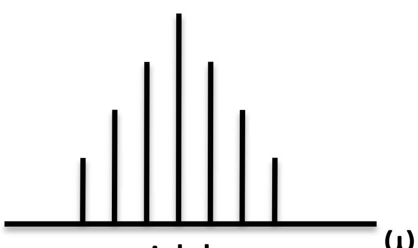
$$\rho(\omega) = \frac{1}{Z}\sum_{n,l} M_{(l,n)}\delta(\omega - (V_l(r) + 2m_Q - E_n))e^{-\beta E_n}$$

$$E_l = V_l(r) + 2m_Q$$

What we expect:



position:
Re[V]



width:
Im[V]

The heavy QQbar correlator

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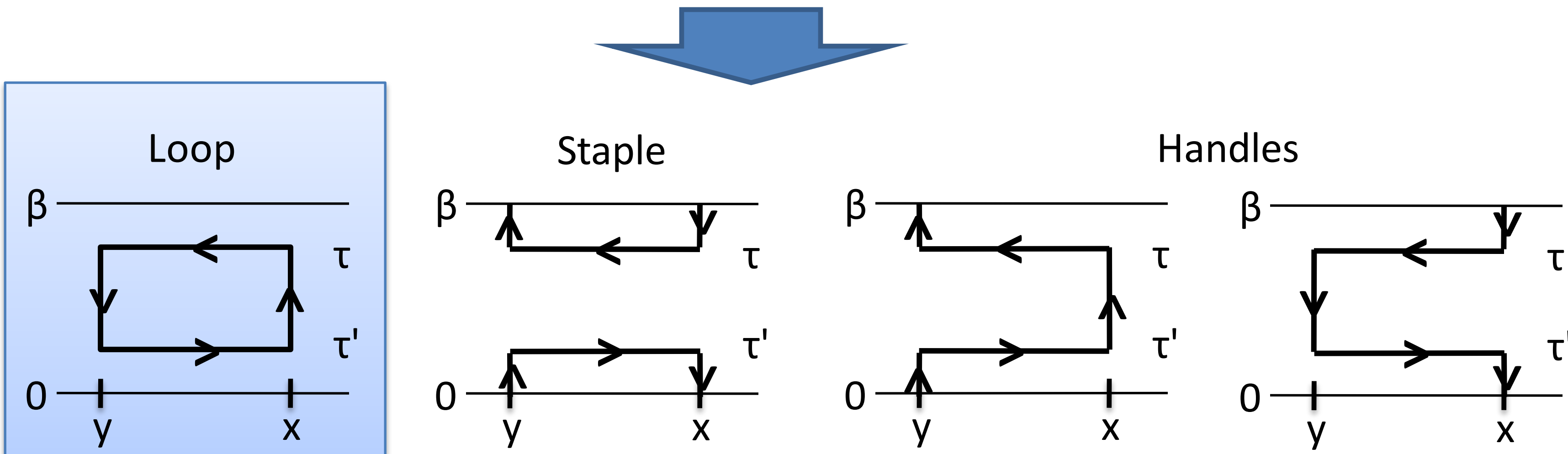
Can we simplify the QQbar correlator? In the heavy mass limit: YES!

1. Medium is vaccum state for Q and Qbar fields: No thermal excitation possible

2. QQbar correlator G(x,x',y,y') in terms of heavy quark correlators S = <Qbar Q>

$$G^{>}(x,\acute{x},y,\acute{y}) = \langle \bar{Q}(x)\Gamma U Q(y)\bar{Q(\acute{y})}\bar{\Gamma} U Q(\acute{x})\rangle$$
$$= -Tr[\Gamma U S(y,\acute{y})\bar{\Gamma} U S(\acute{x},x)]$$

3. Use m->∞ to simplify the form of S (periodic boundary condition in τ direction!)



Loop: $\propto e^{-2m_Q(\tau-\tau')}$

Staple: $\propto e^{-2m_Q(\beta-(\tau-\tau'))}$

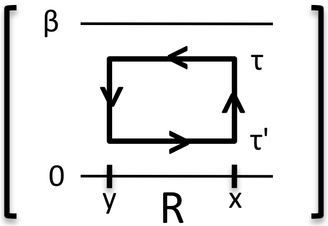
Handles: $\propto e^{-m_Q\beta}$

constant: ρ ∝ ω δ(ω) Umeda PRD75 2007

Exploring the QQbar potential

Combining both spectral function and the correlator gives:

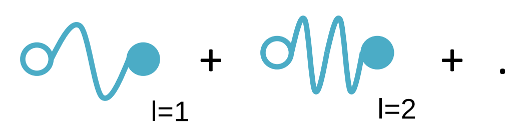
$$\frac{D^{>}(\tau,\tau',R)}{e^{-2m_Q(\tau-\tau')}} = \mathrm{Tr}\left[\right.$$



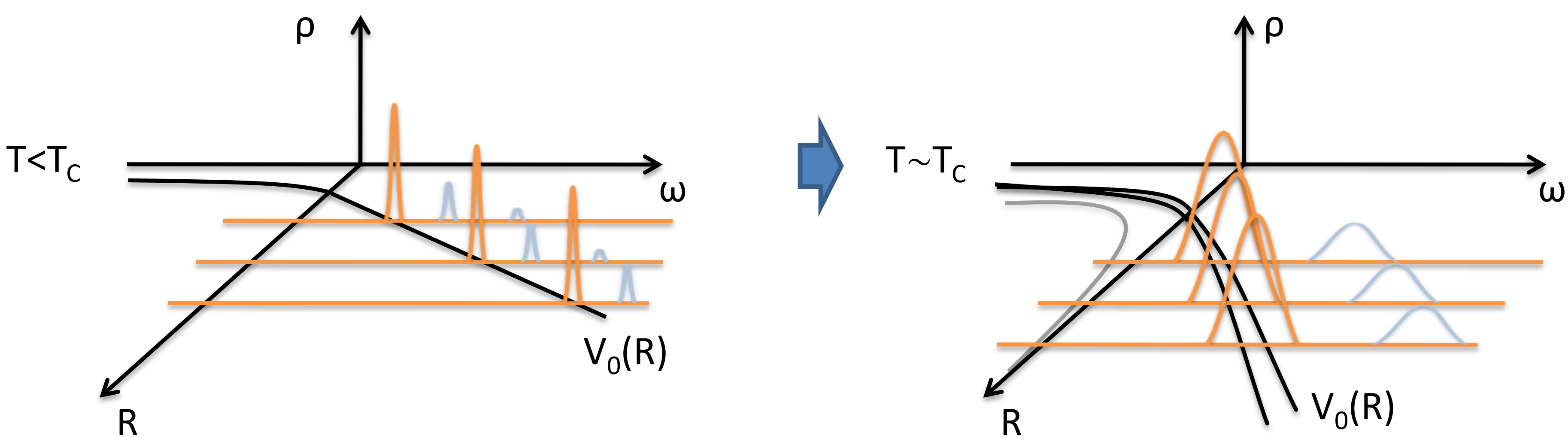
$$\left.\right] = \int_{-\infty}^{\infty} e^{-\bar{\omega}(\tau-\tau')}\rho(\bar{\omega},R)$$

Reconstruct spectral function for **different R**: map the shape of the potential

$$\rho(\bar{\omega},R) = \sum_{n,l} M_{n,0}\,\delta(\bar{\omega} - V_0(R) - E_n) + \Bigg[$$



$$\Bigg]$$



Conclusion & Outlook

1. Free energy as heavy quark potential not consistent with Lattice QCD and HTL
2. Nonpertrubative definition of the potential possible:
   Wilson contour (loop) $\xrightarrow{\text{Lattice}}$ Spectral Function $\xrightarrow{\text{MEM}}$ Potential
3. Obtaining Wilson Loop from Lattice Simulations (quenched)
   20^3x46 β=7.0 ξ=3.5 and 20^3x40 β=7.0 ξ=3.5
4. Reconstructing the spectral function from the lattice data using the Maximum Entropy Method (in progress)

↓

5. Extracting the QQbar potential both **Re[V]** and **Im[V]**

Thank you for your attention

ありがとうございました

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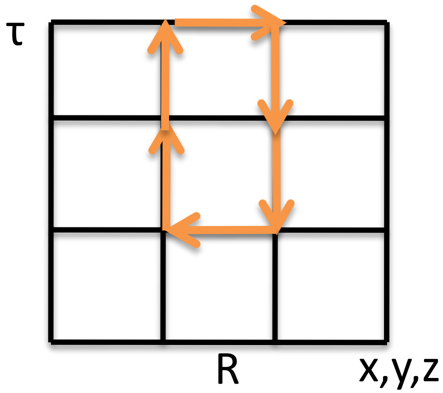
Additional Material

Extracting the SPF

Maximum Entropy Method:

Deconvolution prescription using Inference:
What is the most probable image for given data?

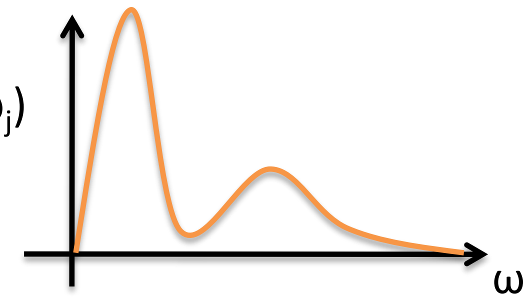


$D(R,\tau_i)$

ill - posed

cannot use χ^2 fitting

$\rho\ (R,\omega_j)$



$N_\tau \sim O(10)$

$N_\omega \sim O(1000)$

$$D(\tau,p) = \int_0^\infty K(\tau,\omega)\rho(\omega,p)$$

1. No need for low ω region parametrization (for $\omega \to \infty$: $\rho(\omega) \sim \omega^2$)
2. If a solution for ρ exists it is unique
3. Satistical errors for ρ can be estimates

MEM: Bayes Law

Probability of the Data given the solution SPF (fit ρ onto the data set)

$$\propto Exp\Big[-\frac{1}{2}\sum_{i,j}\Big(D(\tau_i)-D_\rho(\tau_i)\Big)C_{ij}^{-1}\Big(D(\tau_j)-D_\rho(\tau_j)\Big)\Big]$$

For many lattice measurements, central limit theorem gives gaussian distribution

Probability of the SPF given the prior knowledge. (fit ρ onto the prior model m)

Given by Shannon-Janes Entropy

$$\propto Exp\Big[\alpha\int_0^\infty \Big\{\rho(\omega)-m(\omega)-A(\omega)log\Big(\frac{A(\omega)}{m(\omega)}\Big)\Big\}d\omega\Big]$$

$$P[\rho|Dm]=\frac{P[D|\rho m]P[\rho|m]}{P[D|m]}$$

Probability of SPF given the Data D and prior knowledge m

Normalization, irrelevant

$$\frac{\delta}{\delta\rho}P[\rho|Dm]\overset{!}{=}0$$