Particle production by Schwinger mechanism

Naoto Tanji
Univ. of Tokyo

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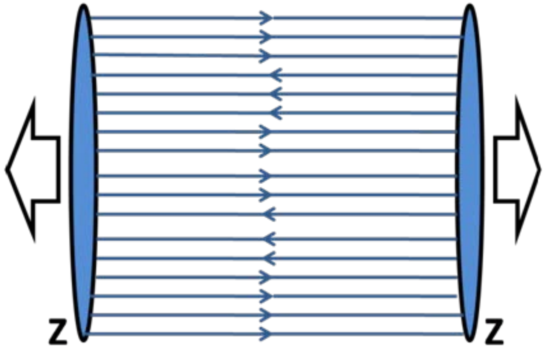
Non-perturbative particle production mechanism in heavy-ion collisions

color flux tube model



- Treat gluon fields as classical electric fields.
- The electric field decays into particles by Schwinger mechanism.

The momentum distribution of created particles and its time-evolution are not fully understood.

We investigate

- Dynamical view of pair creation

Non-perturbative particle production mechanism in heavy-ion collisions

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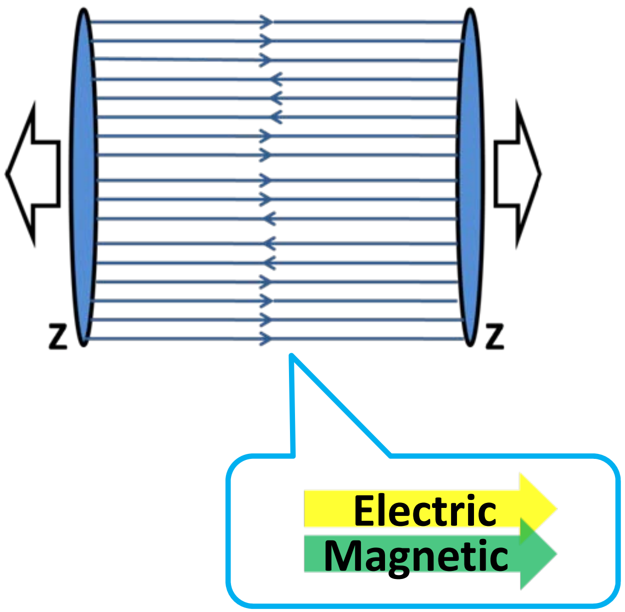

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- The electric field decays into particles by Schwinger mechanism.

The momentum distribution of created particles and its time-evolution are not fully understood.

Color glass condensate (Lappi,McLerran 2006)

Both **electric** and **magnetic** field exist

We investigate

- Dynamical view of pair creation
- Effects of a magnetic field

Schwinger mechanism

Schwinger 1951

Non-perturbative pair creation mechanism in an uniform and static classical electric field

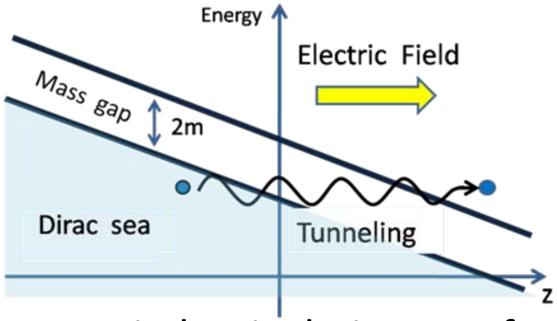
Vacuum persistence probability

$$\left|\langle 0,\mathrm{out}\,|\,0,\mathrm{in}\rangle\right|^2 = \exp(-2\,\mathrm{Im}\,L) = \exp\left(-\int w d^4x\right)$$

Pair creation probability per unit volume and unit time

$$w = \frac{2(eE)^2}{(2\pi)^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{n\pi m^2}{eE}\right) \quad \text{(fermion)}$$

no direct information about { particle number / time evolution }

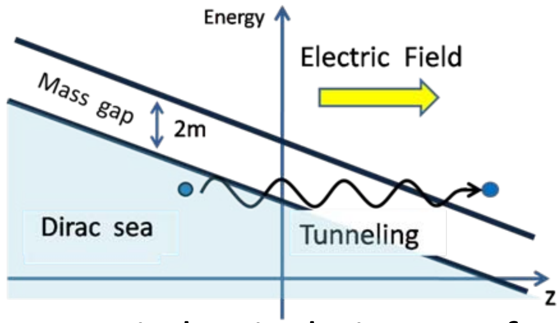


semi-classical picture of Schwinger mech.

Schwinger mechanism

Schwinger 1951

Non-perturbative pair creation mechanism in an uniform and static classical electric field



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direct information about { particle number / time evolution }

particle mean number

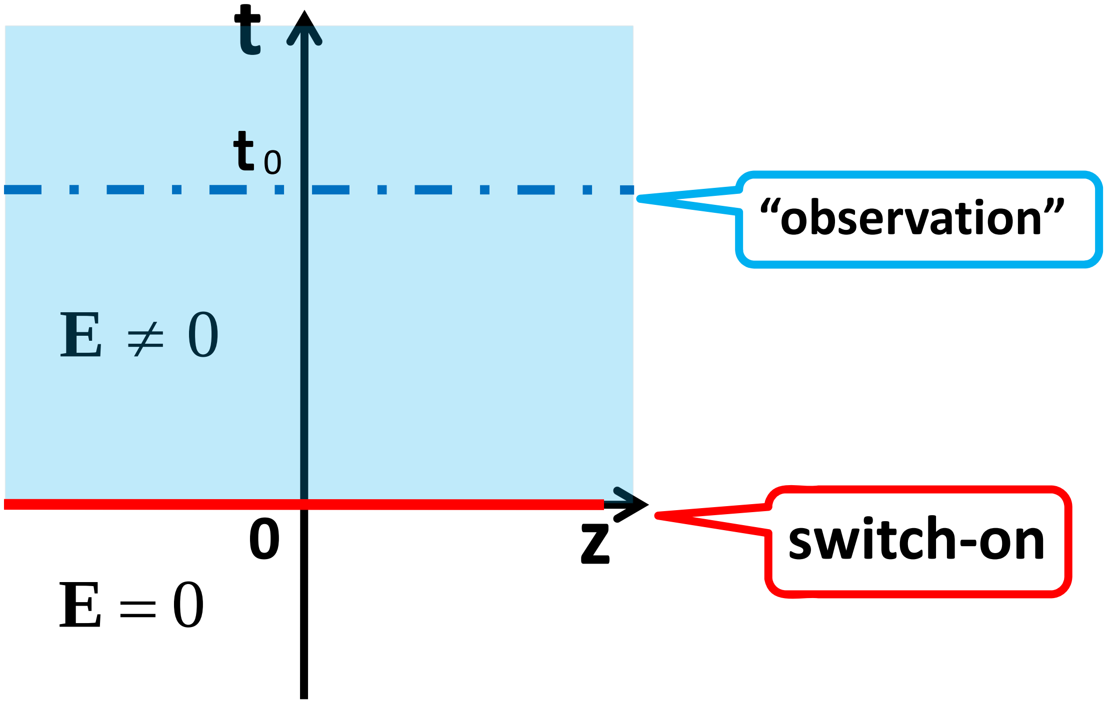
$$\langle 0,\mathrm{in}|a^\dagger_{\mathbf{p}}(t)a_{\mathbf{p}}(t)|0,\mathrm{in}\rangle$$

Need to define particle picture specifically

sudden-switch-on electric field

To get dynamical view of pair creation,
we need to deal with non-steady electric fields.

In a static electric field,
a distribution is also static.



Canonical quantization in a constant electric field

Klein-Gordon field and Dirac field interacting with the classical electric field $\mathbf{E} = (0, 0, E)$

classical field

$$[(\partial_\mu + ieA_\mu)^2 + m^2]\phi(x) = 0 \; , \; [\gamma^\mu(i\partial_\mu - eA_\mu) - m]\,\psi(x) = 0$$

In the case of non-Abelian electric field $\mathbf{E}^a = (0, 0, E)n^a$, gluon field and quark field obey the equations of the same form as in Abelian case, up to first order.

Ambjorn,Nielsen,Olesen 1979
Gyulassy,Iwazaki 1985

But in the case of gluon, magnetic moment causes a difficulty

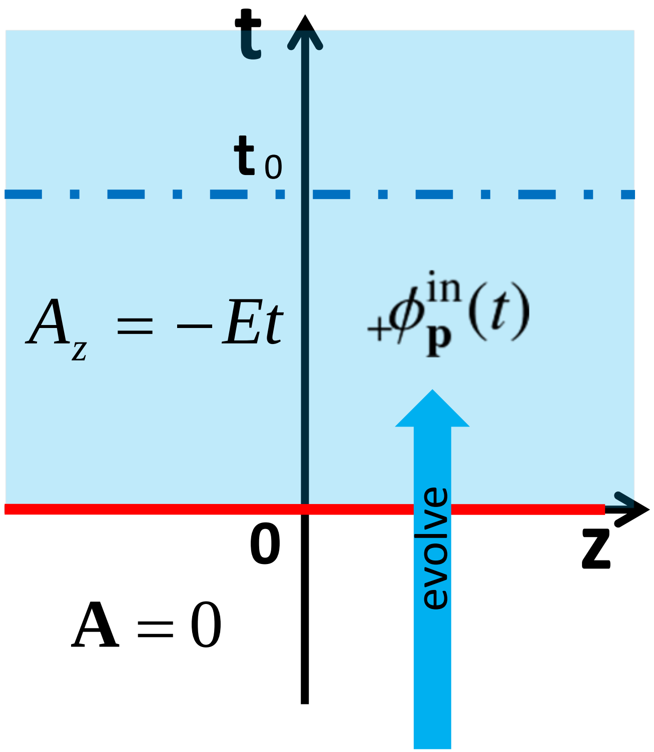
Quantization

$$[\phi(t, \mathbf{x}), \pi(t, \mathbf{x}')] = i\delta^3(\mathbf{x} - \mathbf{x}')$$

→ Particle picture is established.

↑ ambiguity in an external field

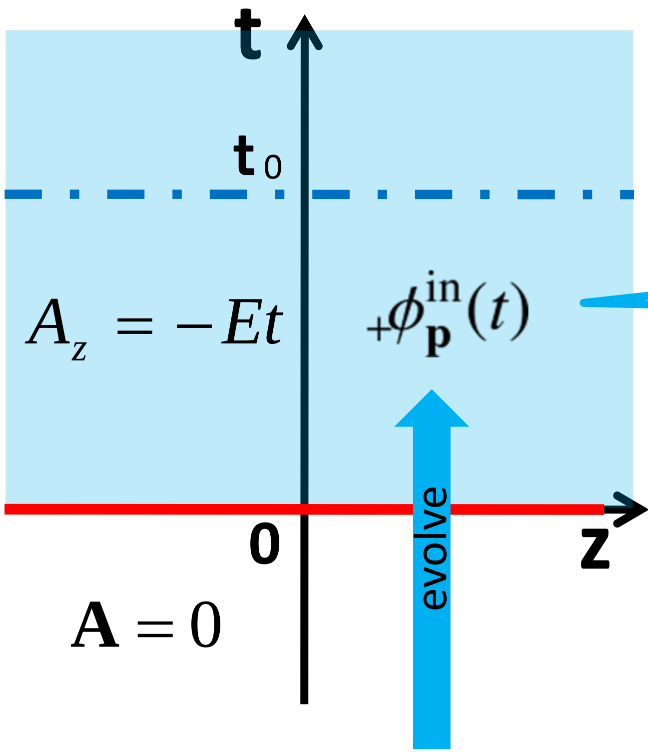
Instantaneous particle picture



$$ {}_{+}\phi_{\mathbf{p}}^{\text{in}}(t) = \frac{1}{\sqrt{2\omega_p}} e^{-i\omega t} $$

positive frequency

Instantaneous particle picture



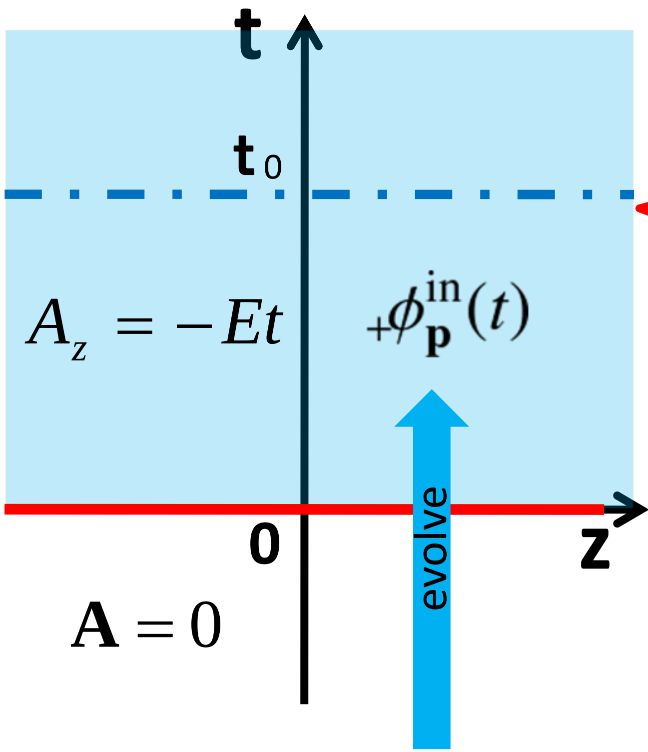
mixture of positive and negative frequency

consequence of pair creation

$${}_{+}\phi^{\text{in}}_{\mathbf{p}}(t) = \frac{1}{\sqrt{2\omega_p}} e^{-i\omega t}$$

positive frequency

Instantaneous particle picture



instantaneous positive and negative frequency

$${}_{+}\phi_{\mathbf{p}}^{(t_0)}(t) \;,\quad {}_{-}\phi_{\mathbf{p}}^{(t_0)}(t)$$

a solution under the gauge $A_z = -Et_0$

$${}_{+}\phi_{\mathbf{p}}^{\text{in}}(t) = \frac{1}{\sqrt{2\omega_p}} e^{-i\omega t}$$

positive frequency

Instantaneous particle picture

decompose ${}_{+}\phi^{\rm in}_{\mathbf{p}}(t)$ into positive and negative frequency instantaneously

$$ {}_{+}\phi^{\rm in}_{\mathbf{p}}(t) = \alpha_{\mathbf{p}}(t_0)\,{}_{+}\phi^{(t_0)}_{\mathbf{p}}(t) + \beta^{*}_{\mathbf{p}}(t_0)\,{}_{-}\phi^{(t_0)}_{\mathbf{p}}(t) $$

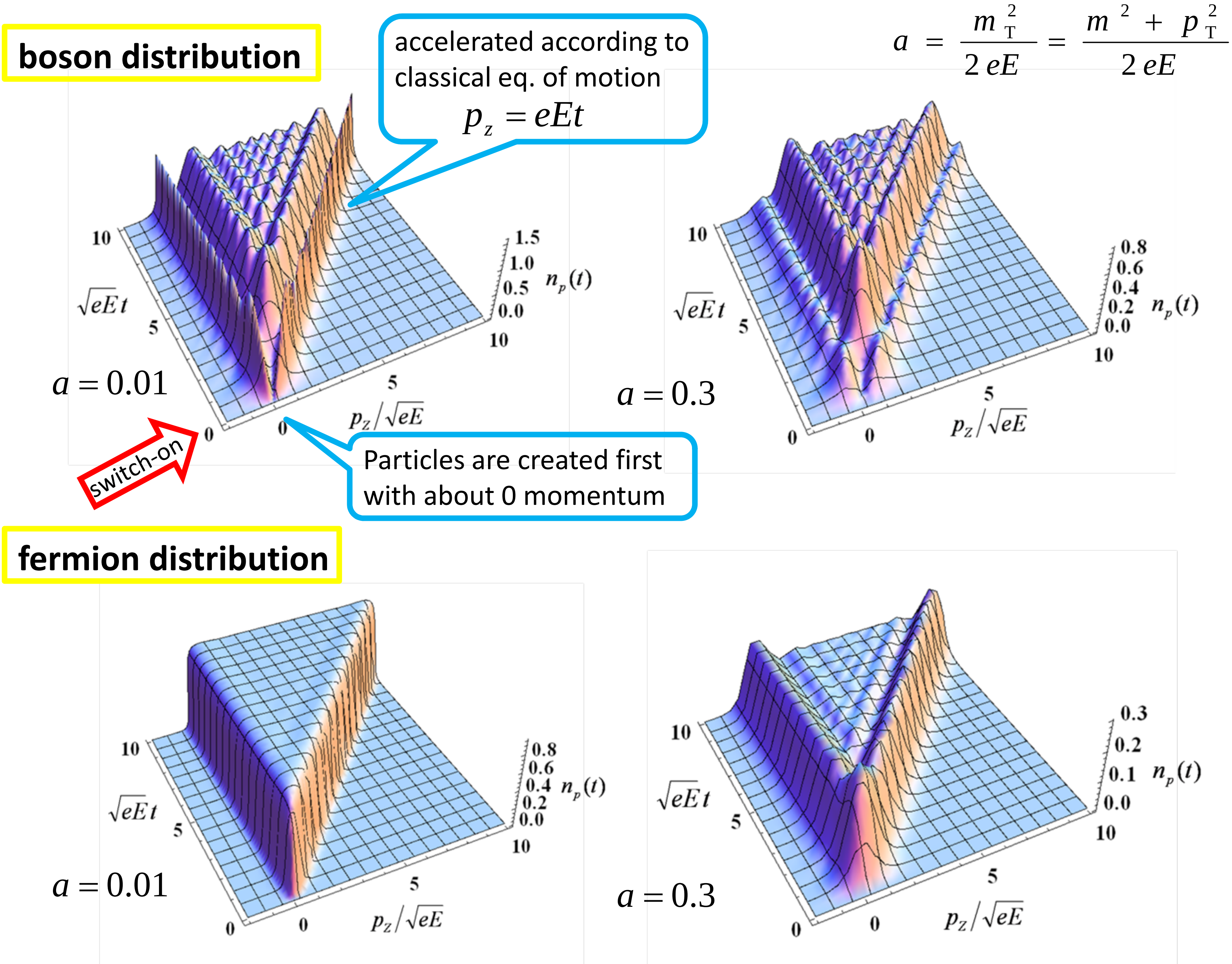
Field expansion

$$ \phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \left[{}_{+}\phi^{\rm in}(t) a^{\rm in}_{\mathbf{p}} + {}_{-}\phi^{\rm in}(t) b^{\rm in\dagger}_{-\mathbf{p}} \right] e^{i\mathbf{p}\cdot\mathbf{x}} $$
$$ = \int \frac{d^3p}{(2\pi)^{3/2}} \left[{}_{+}\phi^{(t_0)}(t) a_{\mathbf{p}}(t_0) + {}_{-}\phi^{(t_0)}(t) b^{\dagger}_{-\mathbf{p}}(t_0) \right] e^{i\mathbf{p}\cdot\mathbf{x}} $$

Instantaneous particle picture (creation and annihilation operator) is introduced.

particle pair distribution function

$$ n_{\mathbf{p}}(t) = \langle 0, {\rm in} | a^{\dagger}_{\mathbf{p}}(t) a_{\mathbf{p}}(t) | 0, {\rm in} \rangle \frac{(2\pi)^3}{V} = \langle 0, {\rm in} | b^{\dagger}_{-\mathbf{p}}(t) b_{-\mathbf{p}}(t) | 0, {\rm in} \rangle \frac{(2\pi)^3}{V} $$
$$ = |\beta_{\mathbf{p}}(t)|^2 $$


boson distribution
accelerated according to classical eq. of motion
$p_z = eEt$
$a = \frac{m_T^2}{2eE} = \frac{m^2 + p_T^2}{2eE}$
$a = 0.01$
$a = 0.3$
switch-on
Particles are created first with about 0 momentum
$\sqrt{eE}t$
$p_z/\sqrt{eE}$
$n_p(t)$
fermion distribution
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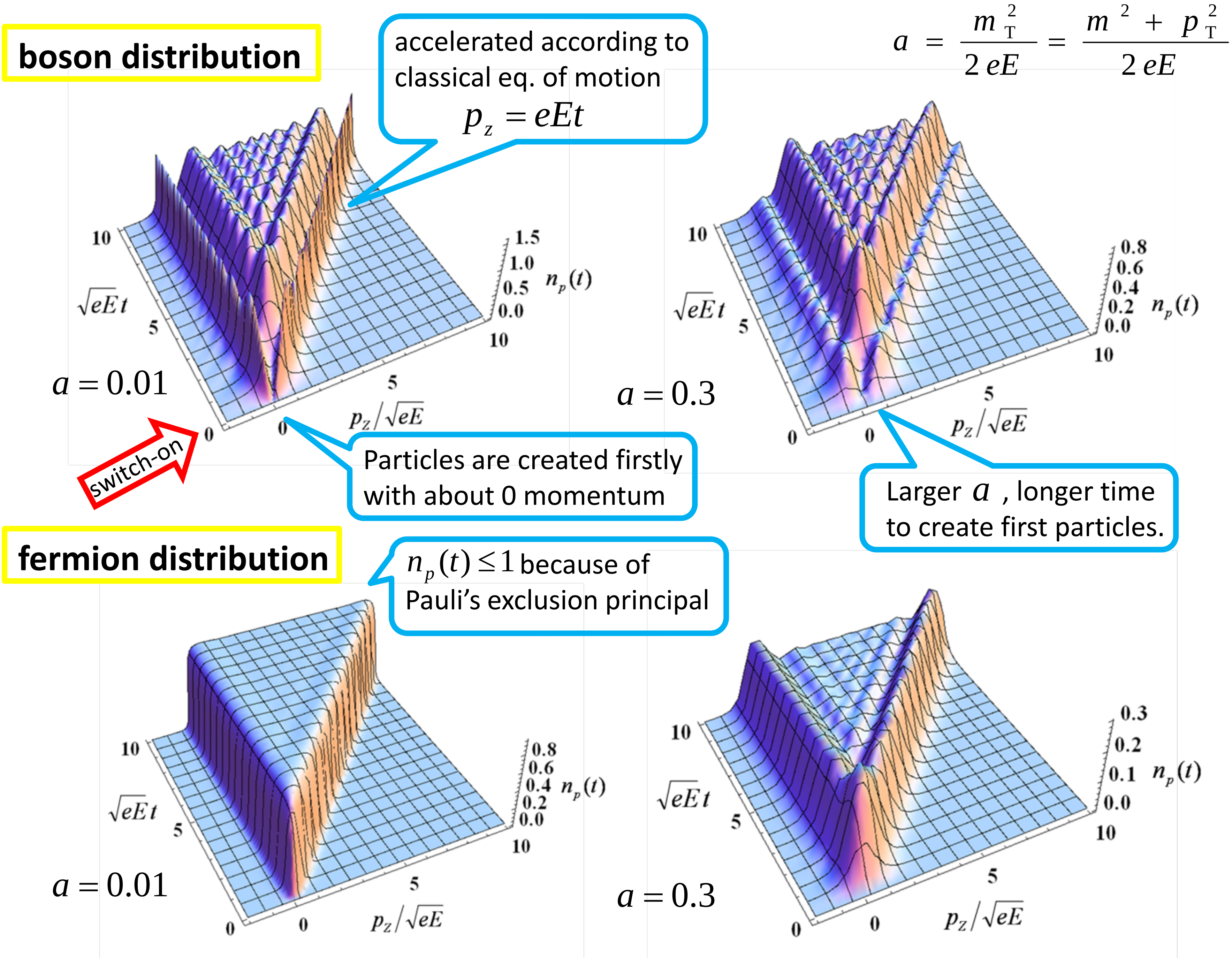

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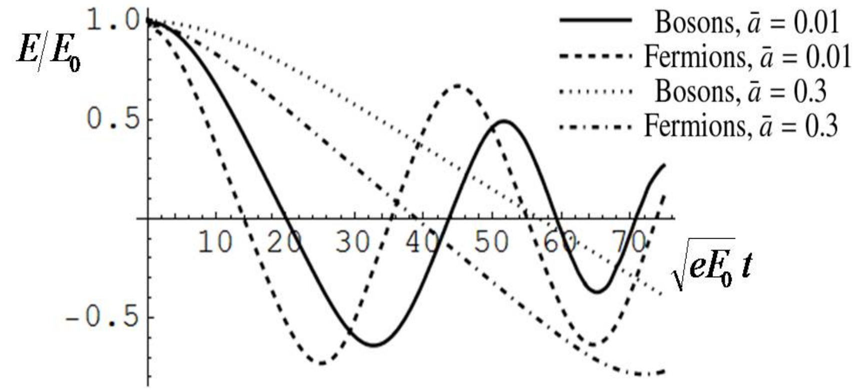
accelerated according to classical eq. of motion $p_z = eEt$
$a = \frac{m_T^2}{2eE} = \frac{m^2 + p_T^2}{2eE}$
$a = 0.01$
$a = 0.3$
switch-on
Particles are created firstly with about 0 momentum
Larger a , longer time to create first particles.
fermion distribution
$n_p(t) \leq 1$ because of Pauli's exclusion principal
$a = 0.01$
$a = 0.3$
$\sqrt{eE}t$
$p_z/\sqrt{eE}$
$n_p(t)$

Back reaction (numerical , 1+3 dim)

Kluger et al. 1991

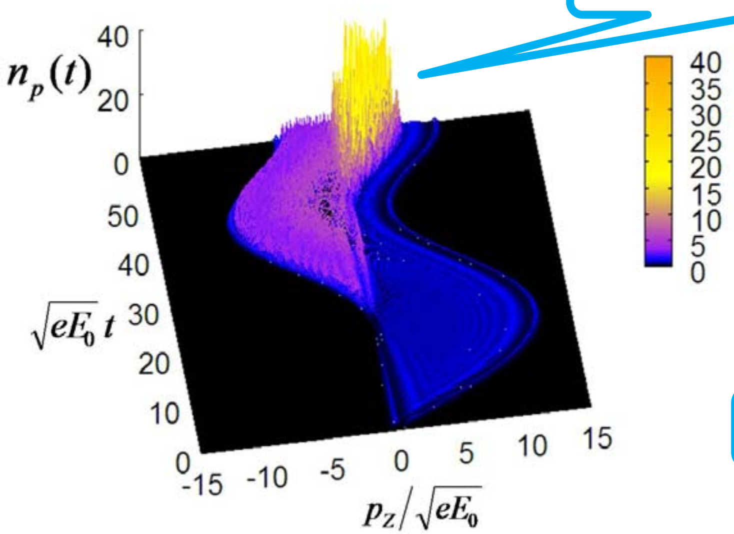
Couple the field eq. and Maxwell eq. $\frac{dE}{dt} = -j$

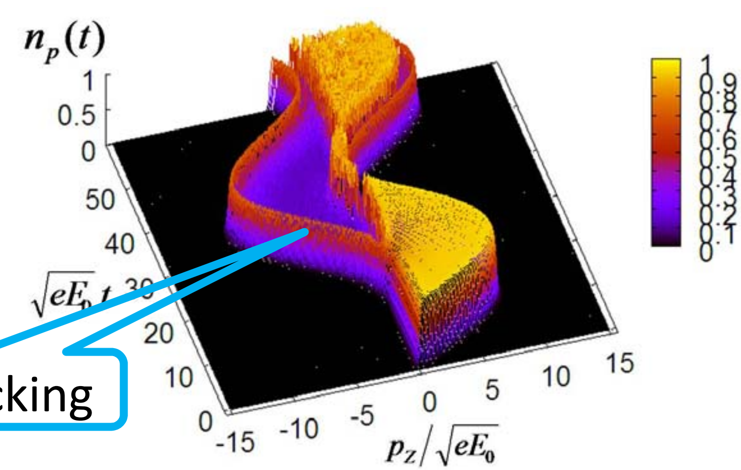


The time-evolution of the electric field

oscillating distribution in momentum space

Bose enhancement





Pauli blocking

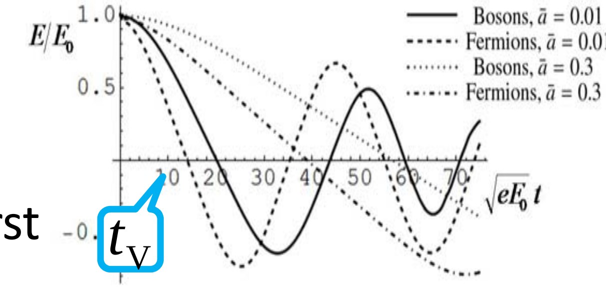
$a = 0.01$

boson

fermion

Time scale of back reaction

t_{V} : the time the electric field strength becomes 0 at first

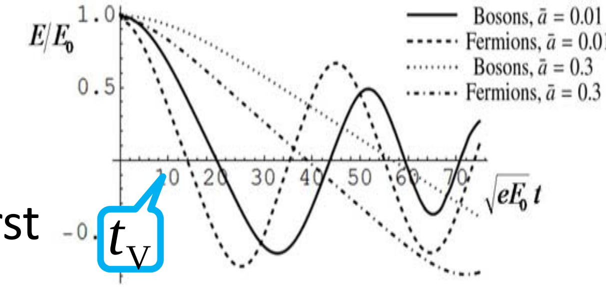


$$t_{\mathrm{V}} \gtrsim \sqrt{\frac{(2\pi)^3}{N_d g^3 E_0}} e^{\pi m^2/2gE_0} \approx \frac{3e^{\pi m^2/2gE_0}}{\sqrt{N_d g^2 \cdot gE_0\,[\mathrm{GeV}^2]}}\,[\mathrm{fm}]$$

roughly estimated from the current without back reaction

N_d : a number of inner degrees of freedom

Time scale of back reaction



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roughly estimated from the current without back reaction

$$m = 0$$
$$gE_0 = Q_{\mathrm{S}}^2 \approx 1\,\mathrm{GeV}^2$$

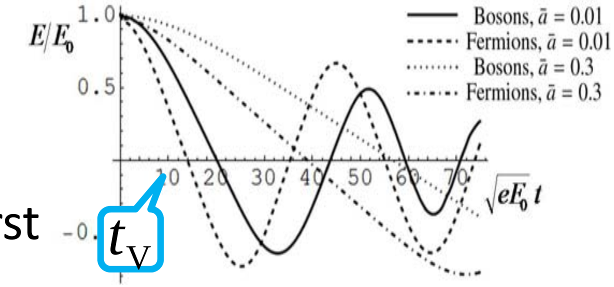
$$\Rightarrow \quad t_{\mathrm{V}} \approx \frac{3}{\sqrt{N_d g^2}} \,[\mathrm{fm}]$$

N_d : a number of inner degrees of freedom

$\Rightarrow$ If $g^2 \gtrsim O(1)$, t_{V} is the order or less than 1 fm.

Time scale of back reaction

t_{V} : the time the electric field strength becomes 0 at first



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$m = 0$
$gE_0 = Q_S^2 \approx 1\,\mathrm{GeV}^2$ ⇨ $t_{\mathrm{V}} \approx \frac{3}{\sqrt{N_d g^2}} [\mathrm{fm}]$

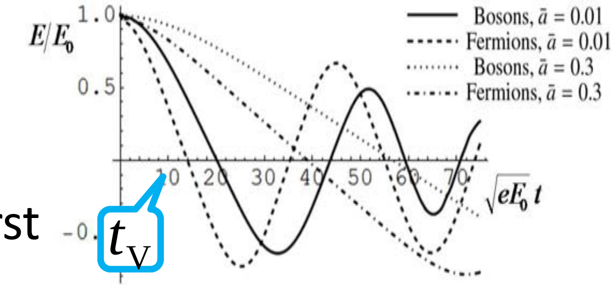
N_d : a number of inner degrees of freedom

⇨ If $g^2 \gtrsim O(1)$, t_{V} is the order or less than 1 fm.

Furthermore, a longitudinal magnetic field makes t_{V} smaller.

Time scale of back reaction

t_V : the time the electric field strength becomes 0 at first



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$$t_V \approx \frac{3}{\sqrt{N_d g^2}} [\mathrm{fm}]$$

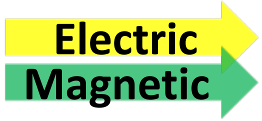
N_d : a number of inner degrees of freedom

If $g^2 \gtrsim O(1)$, t_V is the order or less than 1 fm.

Furthermore, a longitudinal magnetic field makes t_V smaller.

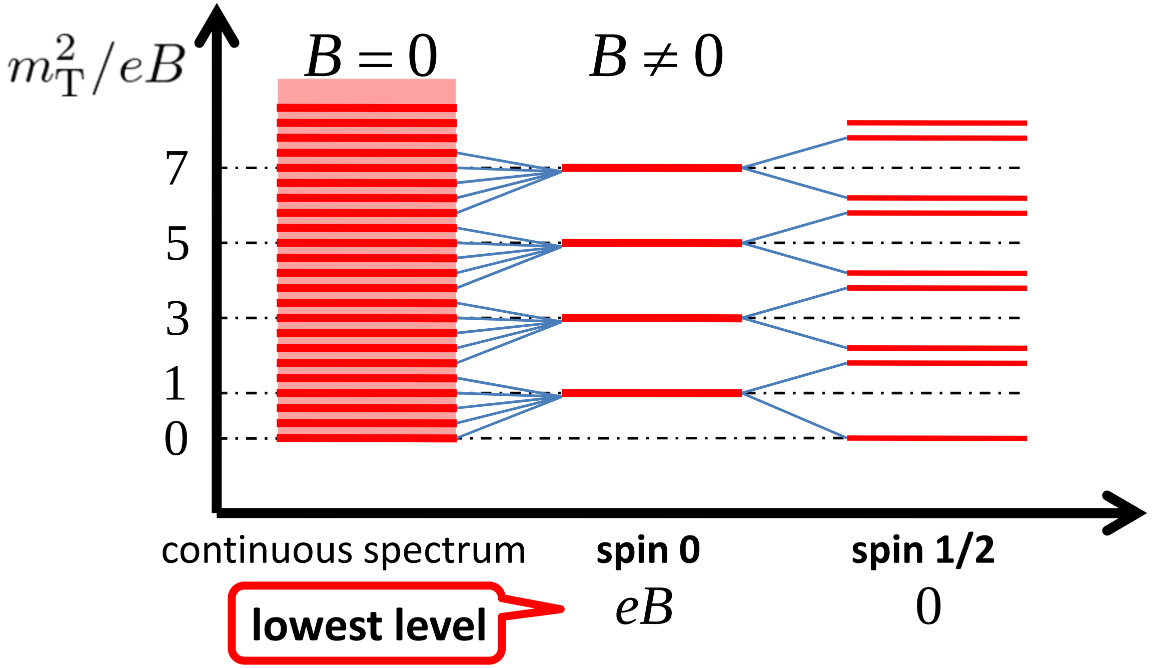
This decay mechanism of an electric field plays an important role in the initial stage of heavy-ion collisions.

The effect of a magnetic field



Longitudinal magnetic field $\mathrm{E} = (0,0,E)\,,\mathrm{B} = (0,0,B)$

- Landau level $p_\mathrm{T}^2 = p_x^2 + p_y^2 \rightarrow (2n+1)eB \quad (n = 0,1,2,\cdots)$
- Zeeman effect $\pm 2seB$



The effect of a magnetic field

Electric
Magnetic

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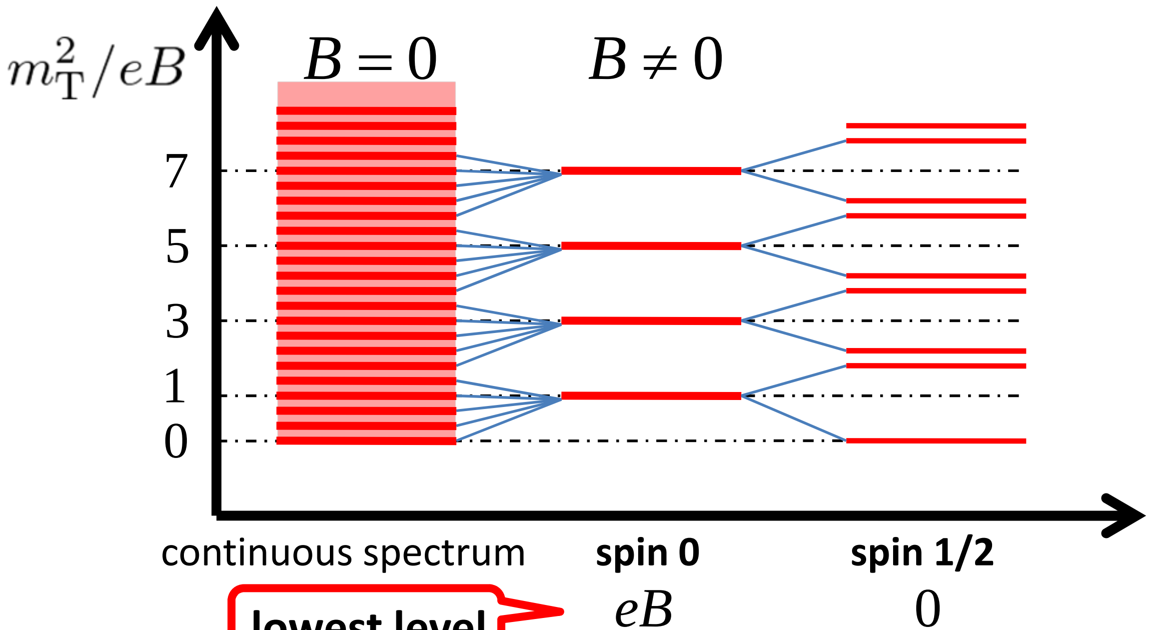
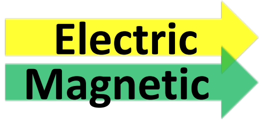


lowest level

The strong magnetic field makes particles "heavy" and suppresses pair creation.

The effect of a magnetic field



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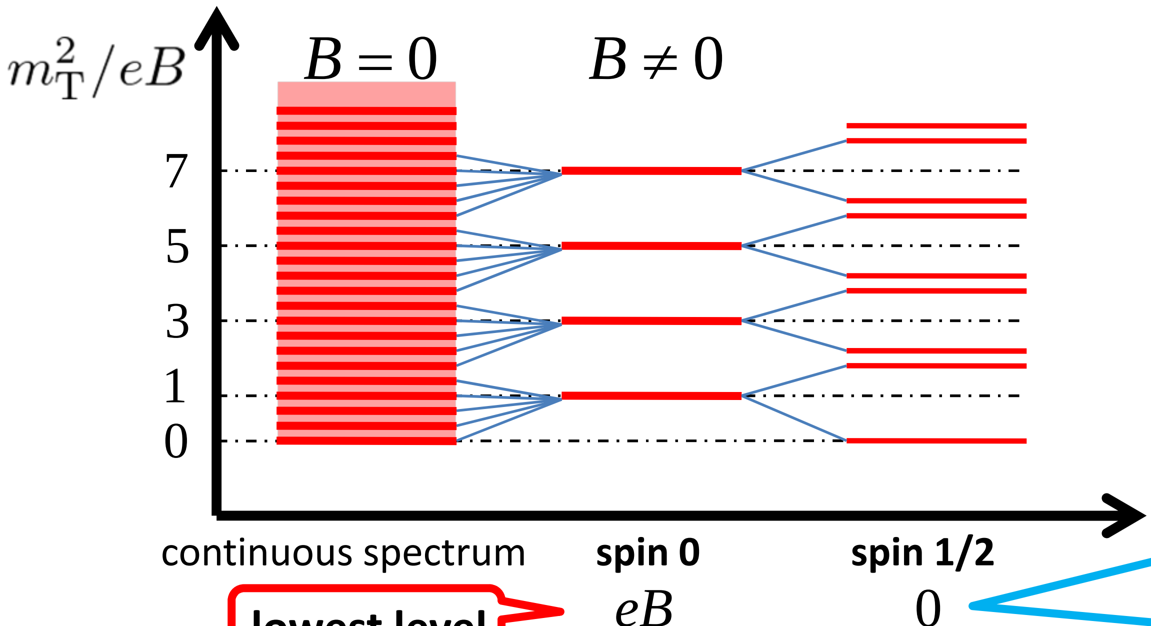


lowest level

independent of B

Pair creation is not suppressed.

The strong magnetic field makes particles "heavy" and suppresses pair creation.

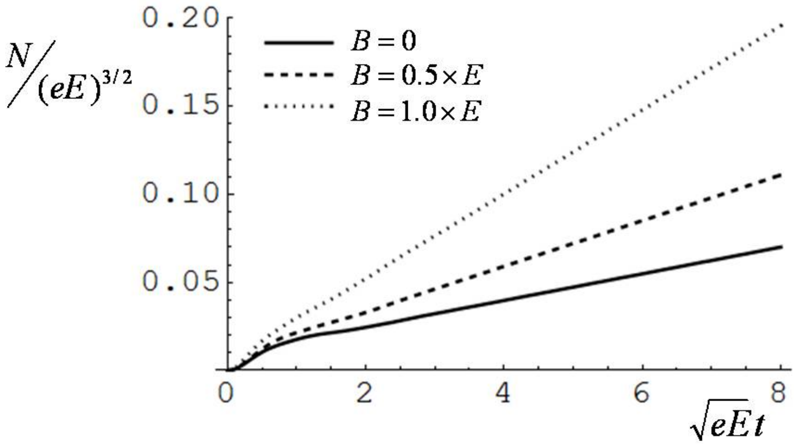
✚

The number of modes degenerating in one Landau level is proportional to B

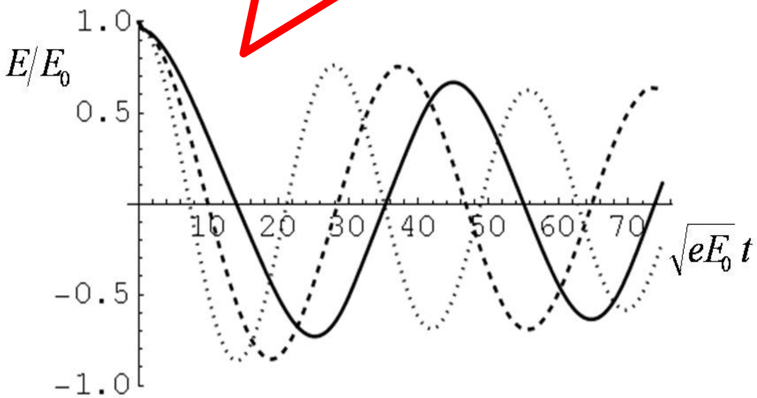
The effect of a magnetic field

Total particle number and current of fermions are enhanced by the magnetic field.

The time-evolution becomes faster due to the magnetic field.



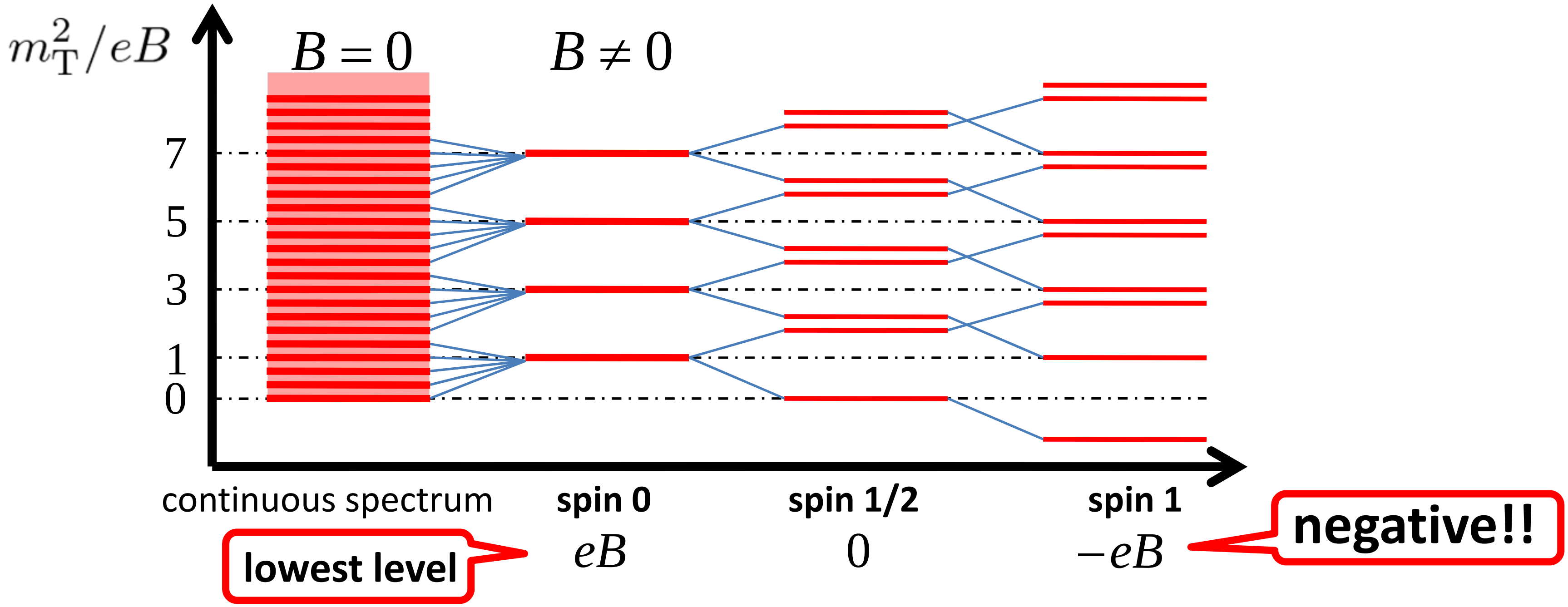
total particle number density
(without back reaction)



time-evolution of the electric field
(with back reaction)

Instability in spin-1 case

- Landau level $p_{\mathrm{T}}^2 = p_x^2 + p_y^2 \to (2n+1)eB \quad (n = 0,1,2,\cdots)$
- Zeeman effect $\pm 2seB$



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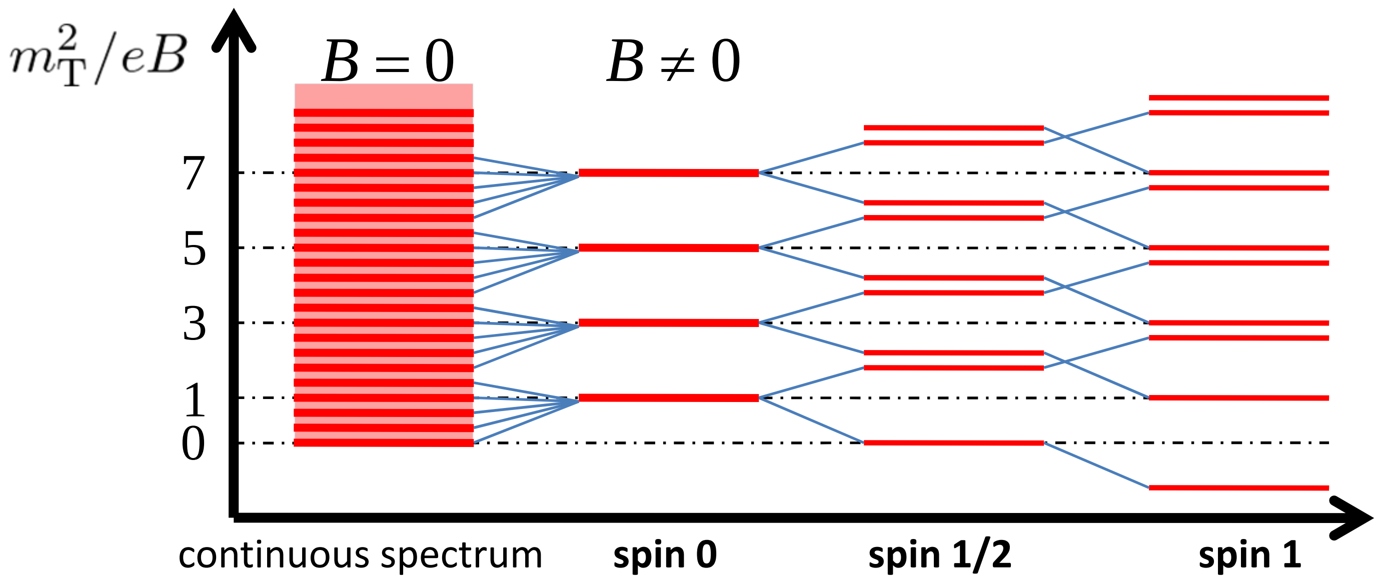

lowest level

eB $\quad 0 \quad -eB$

negative!!

classical instability
open problem

Nielsen-Olesen instability

$\omega_p = \sqrt{p_z^2 + m_T^2}$ can be pure imaginary.

$\exp(\mp i\omega_p t)$ become unstable.

Summary

- We have revealed the momentum distribution of created particles and its time-evolution in uniform electric fields by defining particle picture instantaneously.
- The time scale of the decay process of the initial electric field due to back reaction has been estimated.
- We have shown that a magnetic field speeds up the time-evolution of the fermion system.

Remaining problems

- More realistic configuration of an electromagnetic field, especially the case that a field exists only inside the light-cone
- Interaction through quantum gauge field (collision)
- Instability in gluon case