BCS-BEC Crossover in Asymmetric Nuclear Matter with nn,pp,np Pairings

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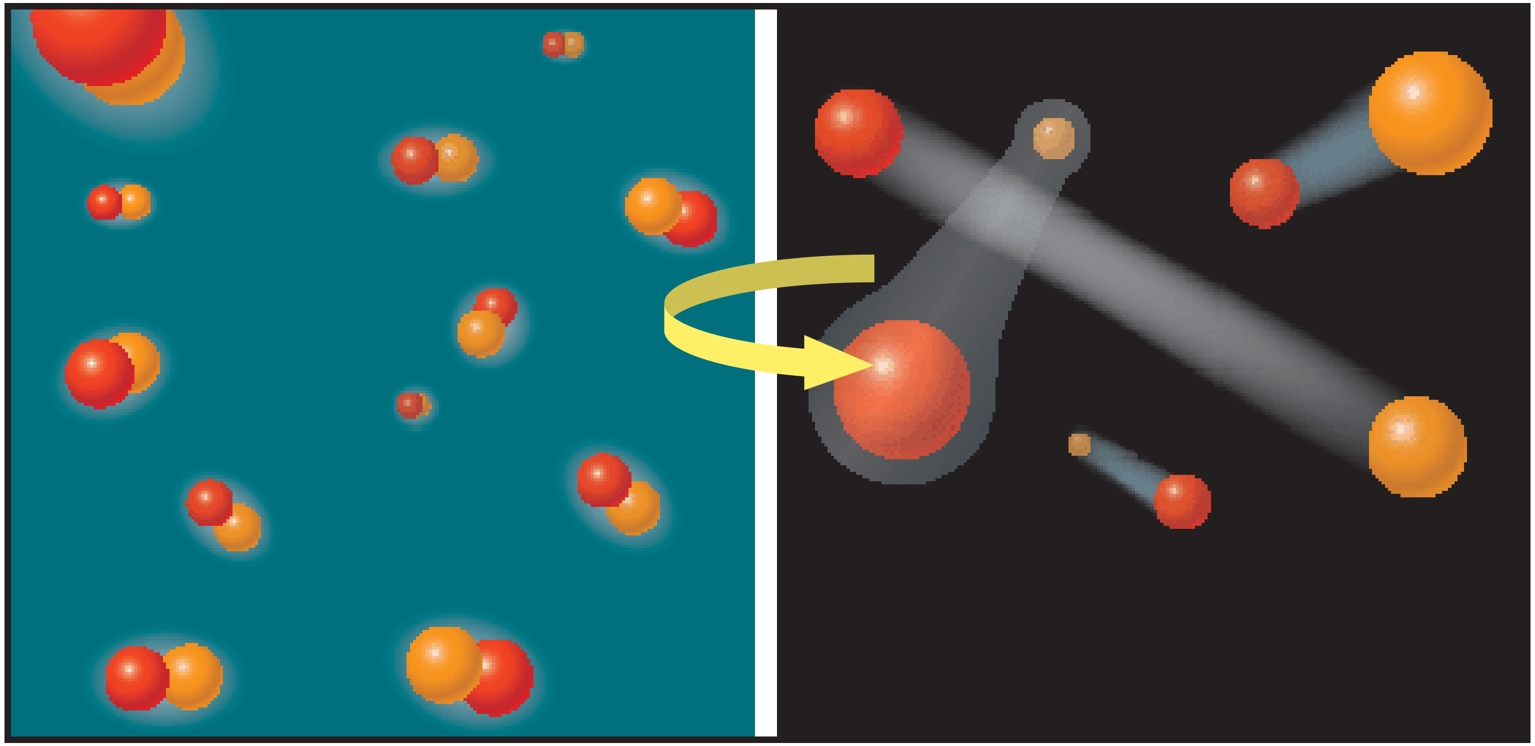
1. Introduction



BEC ← *BCS*

Work has been done…

★Transition from BCS pairing to BEc in low-density asymmetric nuclear matter

U. Lombardo, P. Nozières:PRC64, 064314 (2001)

★Spatial structure of neutron Cooper pair in low density uniform matter

Masayuki Matsuo:PRC73, 044309 (2006)

★BCS-BEC crossover of neutron pairs in symmetric and asymmetric nuclear matters

J. Margueron: arXiv:0710.4241vl [nucl-th] 23 Oct 2007

★ What we will do: considering both nn, pp, and np pairings

2.Basic Formalism

The Lagrangian:

$$L=$$

$$\sum_{\sigma=\uparrow,\downarrow}[p_\sigma^+(\vec{x})(-\frac{\partial}{\partial\tau}+\frac{\nabla^2}{2m}+\mu_p)p_\sigma(\vec{x})+n_\sigma^+(\vec{x})(-\frac{\partial}{\partial\tau}+\frac{\nabla^2}{2m}+\mu_n)n_\sigma(\vec{x})]$$

$$-\int d^3\vec{x}'V_{nn}(\vec{x}-\vec{x}')[n_\uparrow^+(\vec{x})n_\downarrow^+(\vec{x}')n_\downarrow(\vec{x}')n_\uparrow(\vec{x})]$$

$$-\int d^3\vec{x}'V_{pp}(\vec{x}-\vec{x}')[p_\uparrow^+(\vec{x})p_\downarrow^+(\vec{x}')p_\downarrow(\vec{x}')p_\uparrow(\vec{x})]$$

$$-\frac{1}{2}\int d^3\vec{x}'V_{np}(\vec{x}-\vec{x}')[n_\uparrow^+(\vec{x})p_\downarrow^+(\vec{x}')-p_\uparrow^+(\vec{x})n_\downarrow^+(\vec{x}')][p_\downarrow(\vec{x}')n_\uparrow(\vec{x})-n_\downarrow(\vec{x}')p_\uparrow(\vec{x})]$$

Paris Potential and effective mass

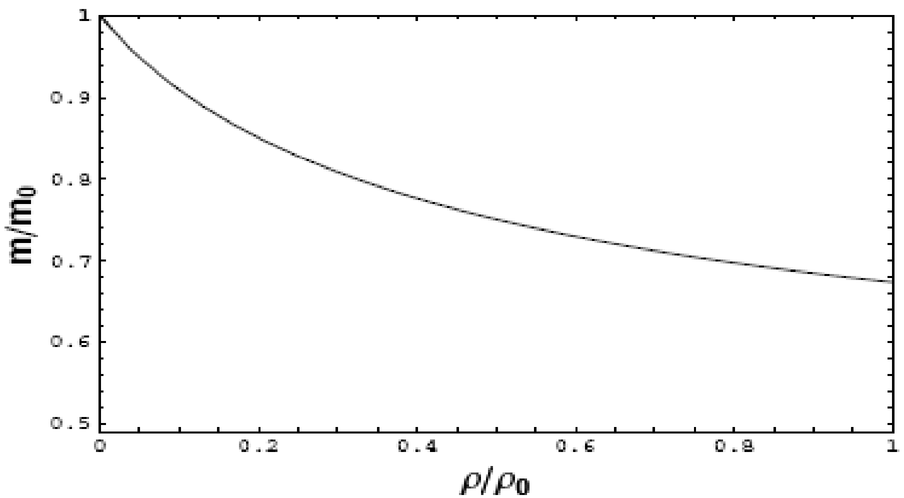
for uniform nuclear system:

$$V(\vec{x}-\vec{x}') = g_I \delta(\vec{x}-\vec{x}');$$

$$g_I = v_0[1-\eta_I(\frac{\rho}{\rho_0})^{\gamma_I}]$$

np Pairing (I=0): η= 0. v_0= - 530MeVfm^3;

nn, pp Pairing (I=1):η= 0.45.γ= 0.47, v_0= - 481Mevfm^3;



Cutoff energy:

$$\varepsilon_c = 60MeV$$

Gap Equation and Density Equation:

Partition function :

$$Z = \prod_{\sigma} \int [dn_{\sigma}][dp_{\sigma}][dn_{\sigma}^{+}][dp_{\sigma}^{+}] \exp\left(\int_0^{\beta} d\tau \int d^3\vec{x} L\right)$$

Thermodynamic potential in mean field approximation:

$$\Omega = -\frac{2\Delta^2}{g_0} - \frac{\Delta_n^2 + \Delta_p^2}{g_1} + \int \frac{d^3\vec{k}}{(2\pi)^3}\left[\varepsilon_n^- + \varepsilon_p^- - \sum_{i,j=\pm}\left(\frac{E_j^i}{2} + T\ln\left(1 + e^{-E_j^i/T}\right)\right)\right]$$

condensate

quasi-particle

gap equation:

$$\frac{\partial \Omega}{\partial \Delta_n} = 0, \quad \frac{\partial \Omega}{\partial \Delta_p} = 0, \quad \frac{\partial \Omega}{\partial \Delta} = 0,$$

density equation:

$$\mu_n, \mu_p \rightarrow \ \mu = (\mu_n + \mu_p)/2, \quad \delta\mu = (\mu_n - \mu_p)/2$$

$$\rho_n, \rho_p \rightarrow \ \rho = \rho_n + \rho_p = -\frac{\partial \Omega}{\partial \mu}, \ \delta\rho = \rho_n - \rho_p = -\frac{\partial \Omega}{\partial \delta\mu},$$

and define relative density asymmetry $\alpha = \dfrac{\delta\rho}{\rho}.$

3.Numerical Results of BCS-BEC Crossover at Zero Temperature

Ⅰ. qualitative description

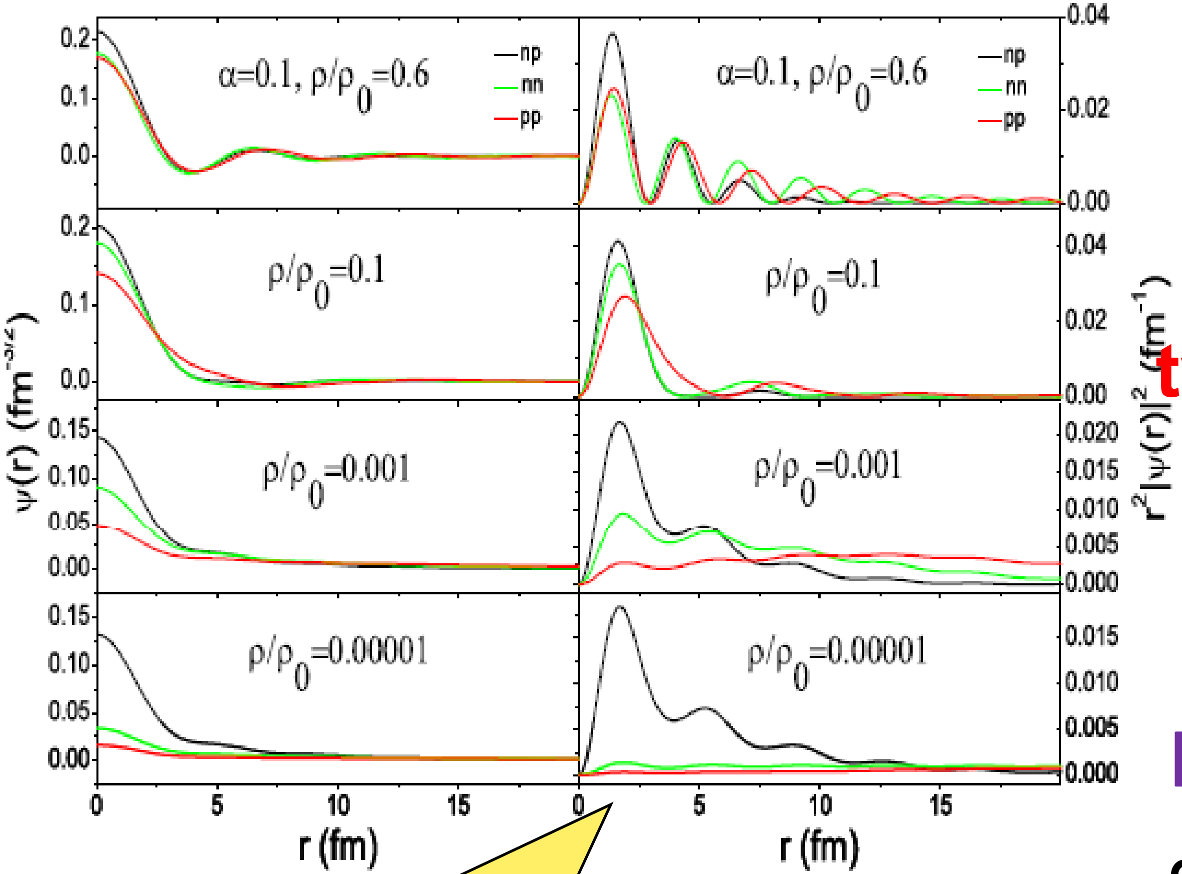
☆ wave function:

$$\psi_{ij}(r) = C\left\langle BCS \left| a^{+}_{i\uparrow}(\vec{x}) a^{+}_{j\downarrow}(\vec{x}+\vec{r}) \right| BCS \right\rangle$$

$$= C' \int \frac{d^3\vec{k}}{(2\pi)^3} \psi_{ij}(\vec{k}) e^{i\vec{k}\cdot\vec{r}},$$

☆ probability density: $r^2 \left|\psi_{ij}(r)\right|^2$

Friedel Oscillation



High density:

(n-n. p-p, n-p)
large spatial extension
and strong oscillation

typical BCS behavior

Crossover

Low density:

oscillation weakens;

wave function shrinks

possible BEC

np pair is stronger correlated than nn, pp pairs.

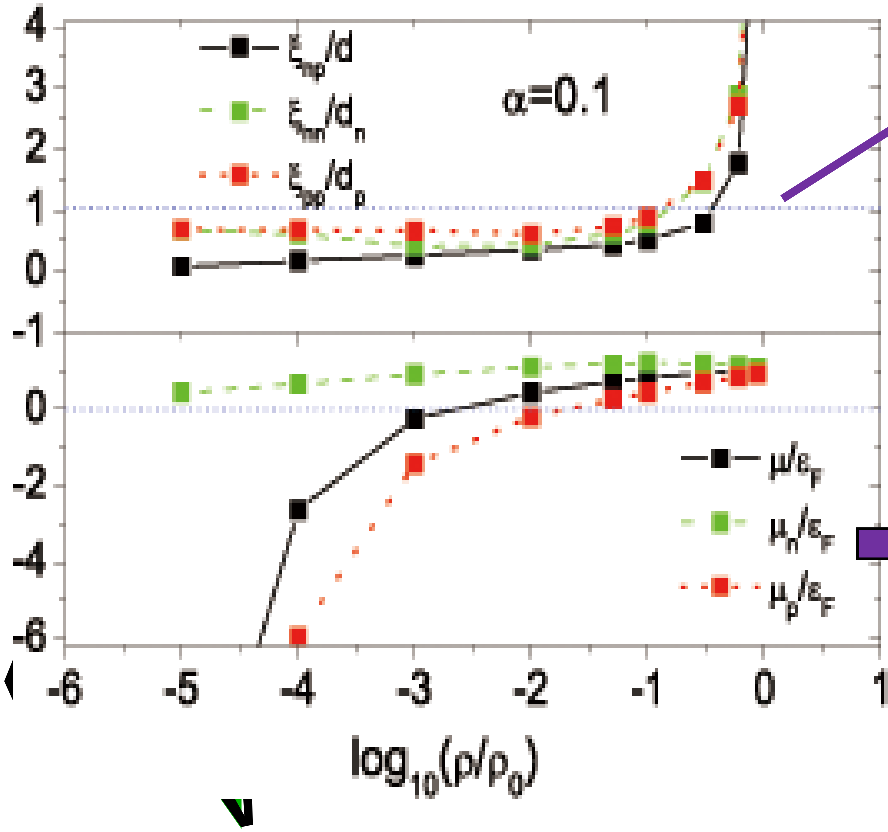
Ⅱ. more quantitative description

☆BCS boundary and BCS limit:

$\xi_{ij} / d_{ij} = 1,$

$(\xi_{ij} = \sqrt{\int_0^\infty r^4 \psi_{ij}^2 dr})$

$\mu_{ij} / \varepsilon_F \to 1$



I=0, $\rho > 0.5\rho_0 \to BCS$

I=1, $nn, \rho > 0.17\rho_0 \to BCS$

$pp, \rho > 0.14\rho_0 \to BCS$

I=0,1:

$\rho \approx 0.87\rho_0,$

BCS limit

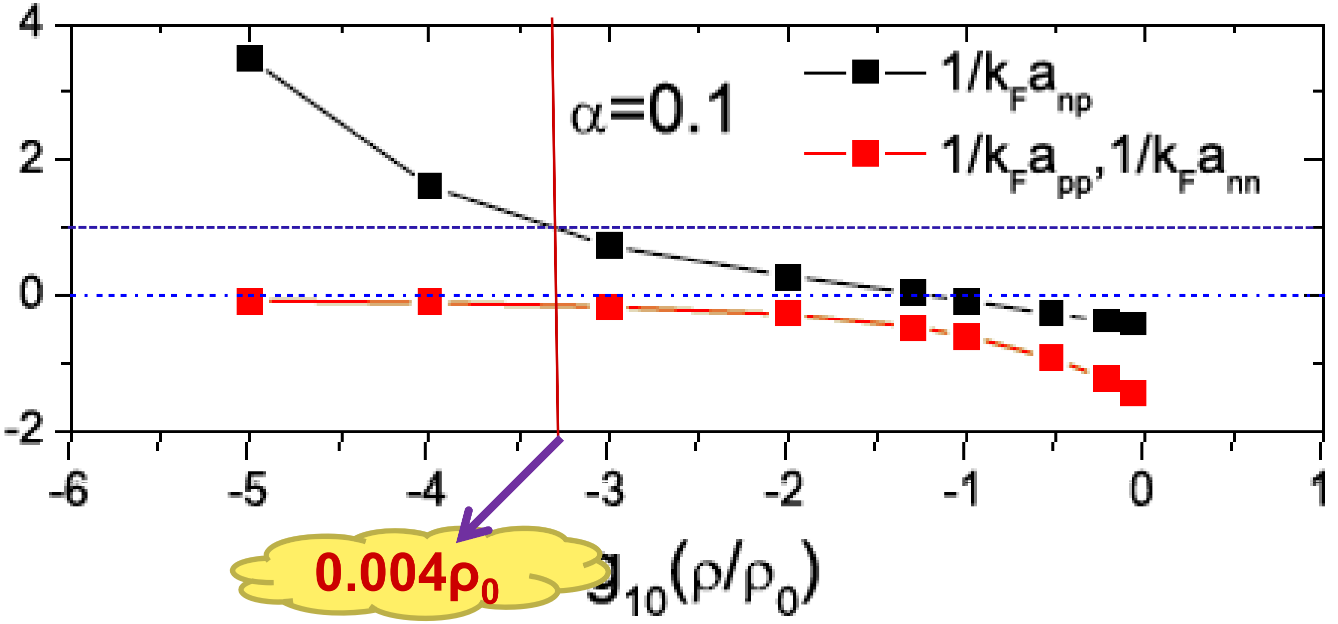
☆ BEC boundary and BEC limit

(i). BEC boundary:

s-wave scattering length: a_{ij}

$$\frac{m}{4\pi a_{ij}} = \frac{1}{g_{ij}} + \sum_{\vec{k}} \frac{1}{2\varepsilon_{\vec{k}}}$$

boundary condition: $\frac{1}{k_F a_{ii}} = 1$



(ii). BEC limit

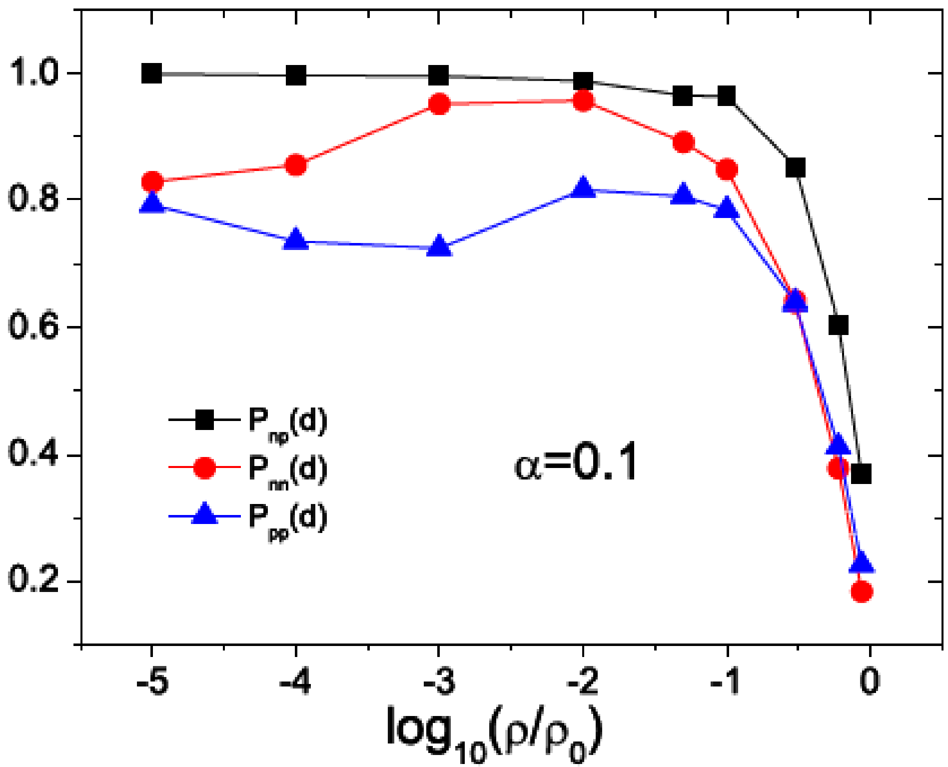
Probability P_{ij} (d) -----partners of the ij Cooper pair to come close to each other within the average distance d $(d = \rho^{-1/3})$

$$P_{ij}(d) = \frac{\int_0^d |\psi_{ij}(r)|^2 r^2 dr}{\int_0^\infty |\psi_{ij}(r)|^2 r^2 dr} \approx 1$$

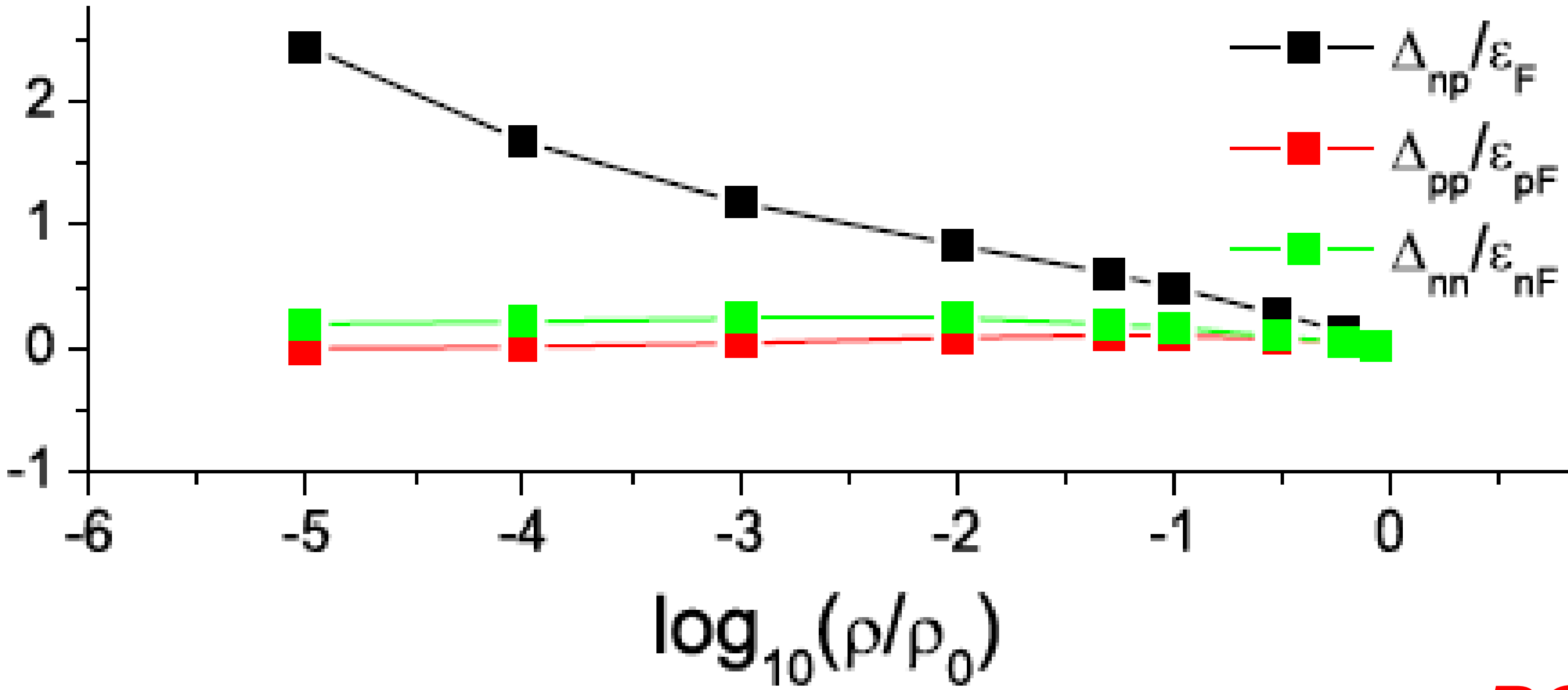
at low density:

- $P_{np}(d) \approx 1, \Rightarrow$ **deuteron BEC**
- $P_{nn}(d), P_{pp}(d) \approx 0.8$

⇒ **no true BEC**



☆ **BCS-BEC Crossover**



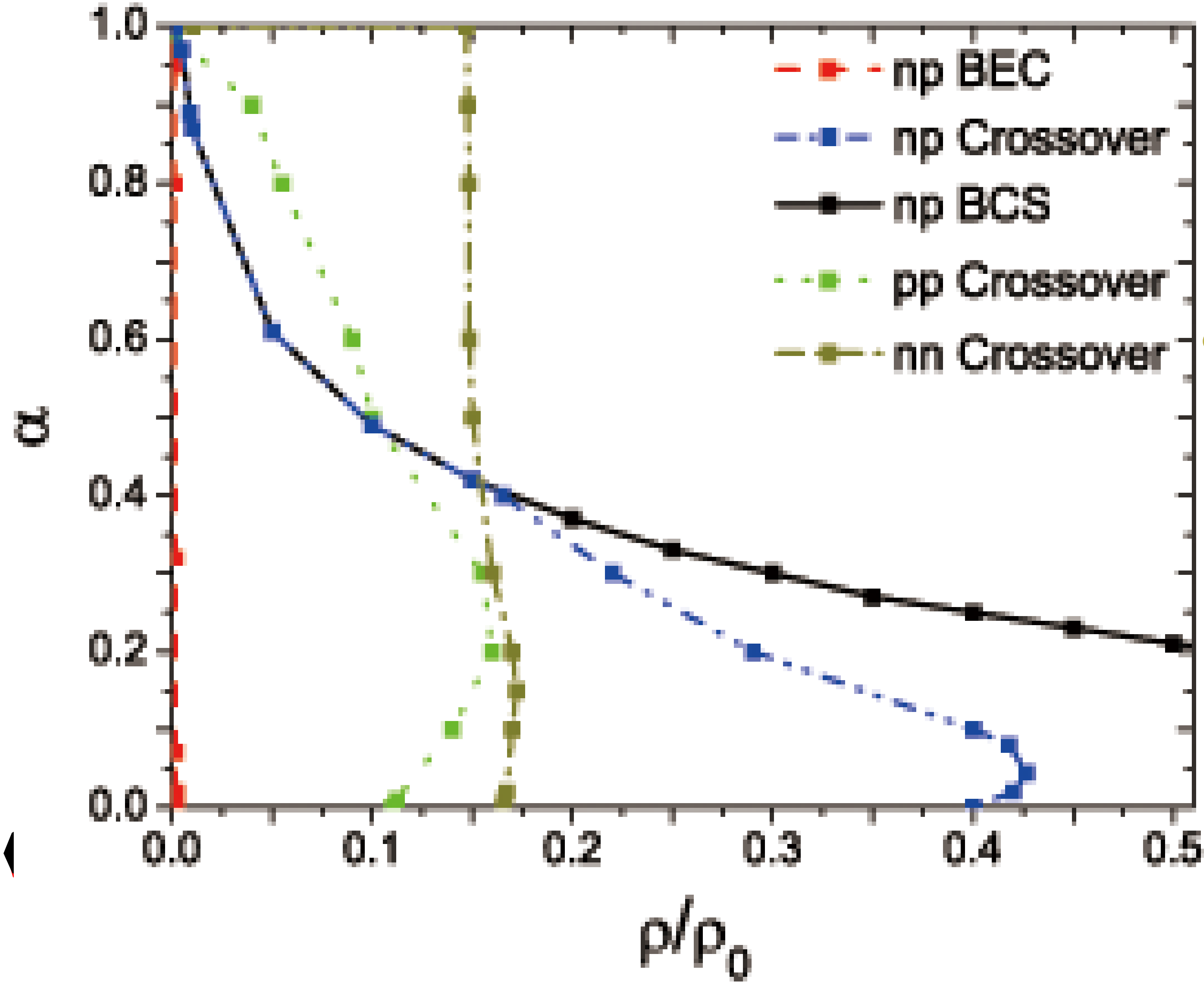
***np* BEC** ***np* crossover** ***np* BCS**

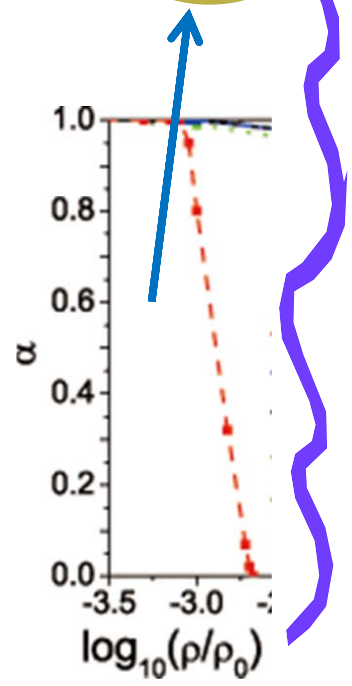
0 → ρ

nn, pp relatively strong correlated

nn, pp BCS

Ⅲ. $\rho - \alpha$ phase diagram





4. Summary:

☆ For asymmetric nuclear matter, we consistently consider both n-p and n-n, p-p Cooper pairs and obtain ρ-α phase diagram at T=0;

☆ A possible signature of BCS-BEC Crossover is Friedel Oscillation, which weakens as density decreases;

☆ As density decreases, np-BCS-BEC Crossover occurs and deuteron BEC is reached, while no BEC state for nn,pp pair.


Thank you!
谢谢！