v_2 and hydrodynamics

~ Has ideal hydrodynamical limit reached at RHIC ? ~

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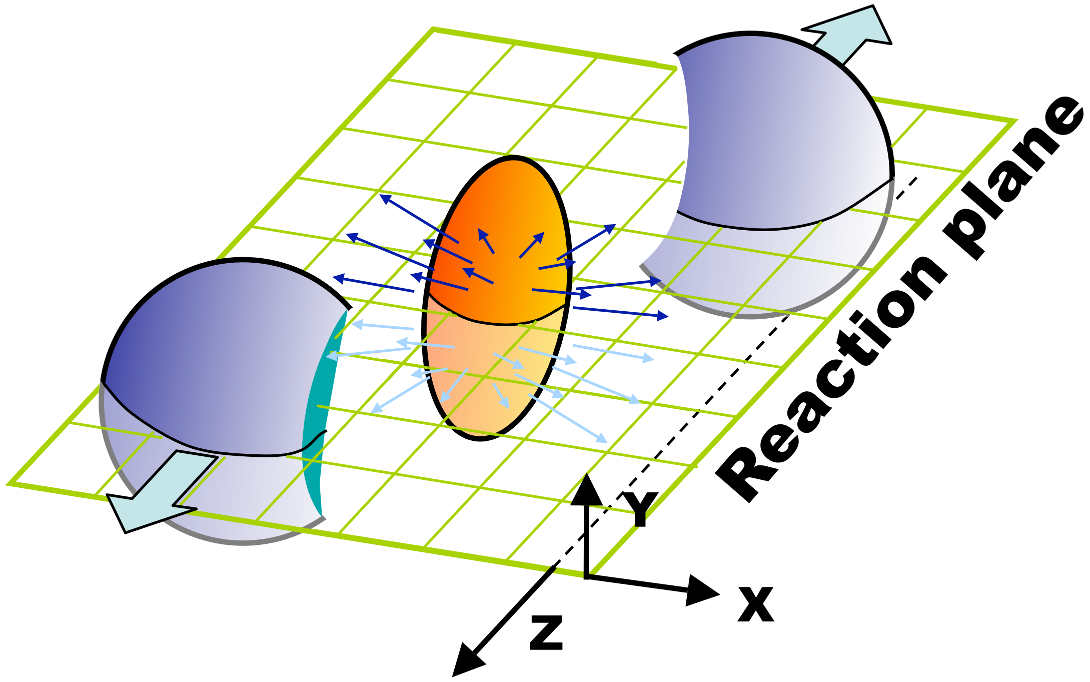
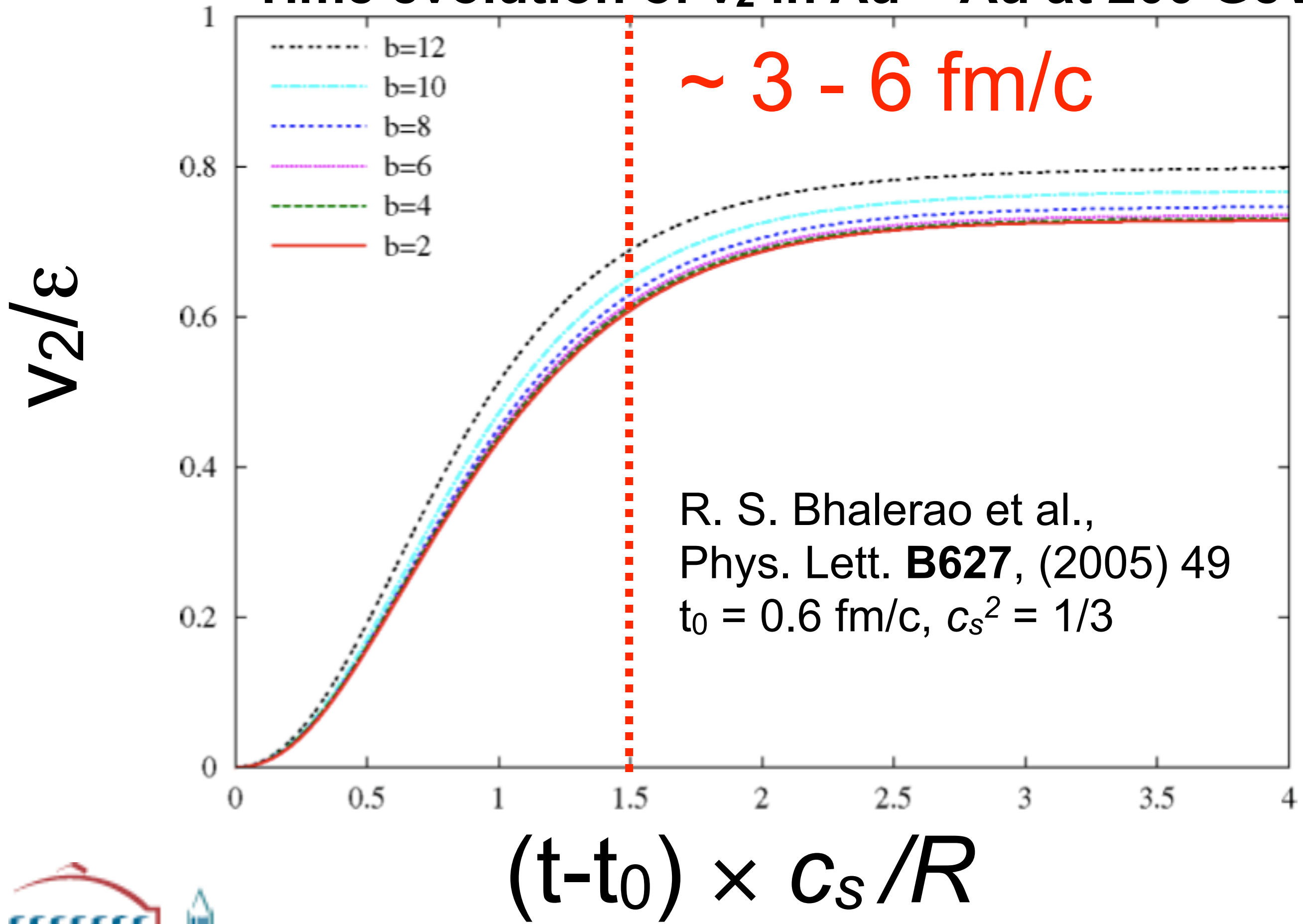
ATHIC 2008, Oct. 13 - 15
Tsukuba, Japan

Thanks to
Y. Bai, Y. Lu, S. Shi, R. Snellings, A. Tang and N. Xu

Elliptic flow (v_2)



Time evolution of v_2 in Au + Au at 200 GeV



R. S. Bhalerao et al.,
Phys. Lett. **B627**, (2005) 49
t_0 = 0.6 fm/c, $c_s^2 = 1/3$

$$\frac{dN}{d(\phi - \Psi_{\rm RP})} = N\left(1 + 2\sum_{n=1}^{\infty} v_n \cos\left(n[\phi - \Psi_{\rm RP}]\right)\right)$$

ϕ : azimuthal angle of emitted particles

$\Psi_{\rm RP}$: azimuthal angle of the reaction plane

$$v_2 = \langle \cos\left(2[\phi - \Psi_{RP}]\right)\rangle$$

- Anisotropic “collective” particle emission in non-central *A*-*A* collisions
- Sensitive to early stage

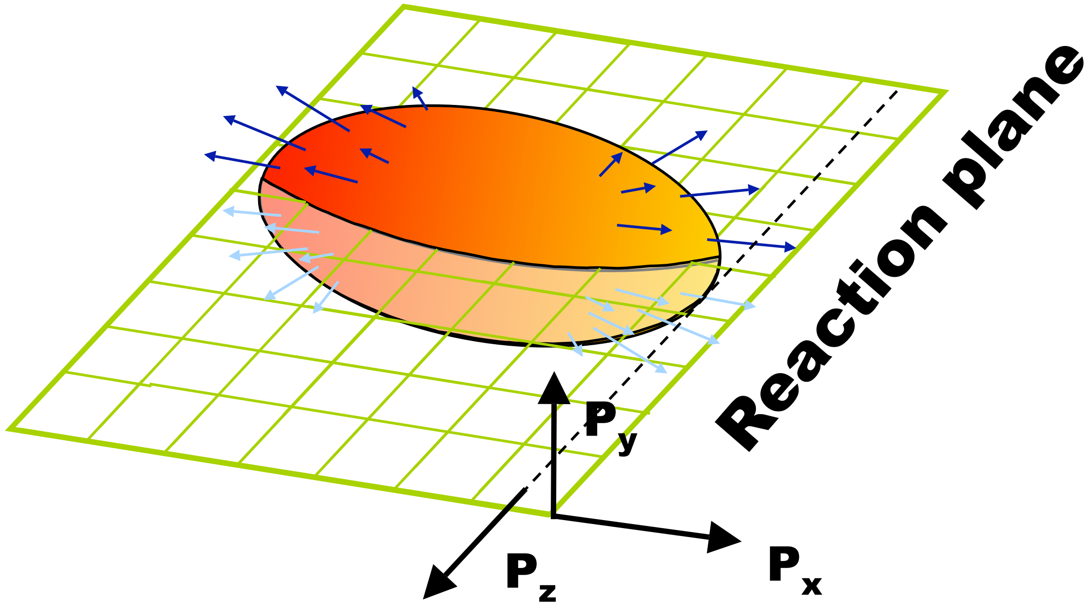
✓ Interactions among particles

➡ Pressure gradient + spatial anisotropy → v_2

✓ v_2 saturate within a few fm/c

➡ A probe to explore early time dynamics of heavy ion collisions

Elliptic flow (v_2)



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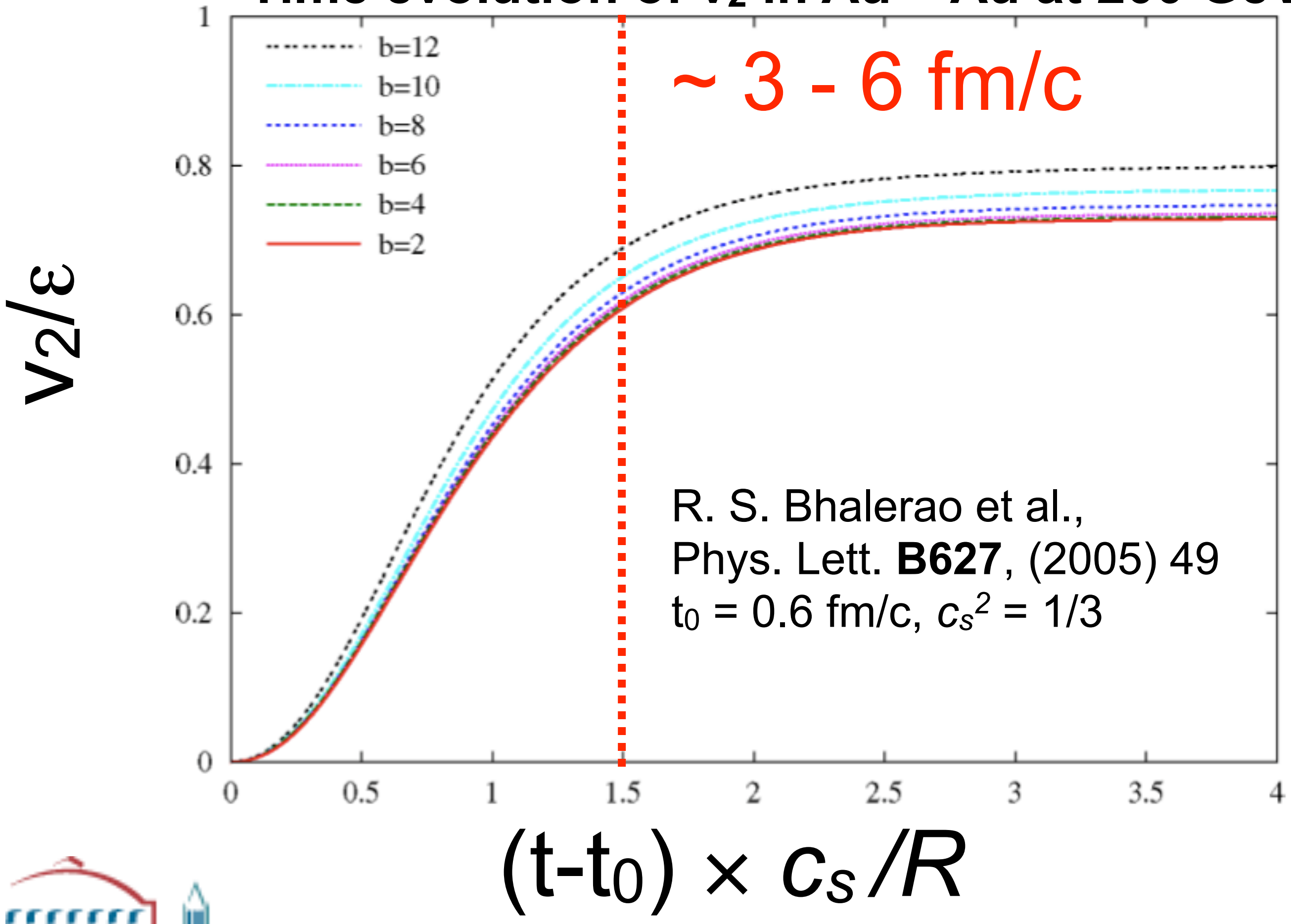

~ 3 - 6 fm/c

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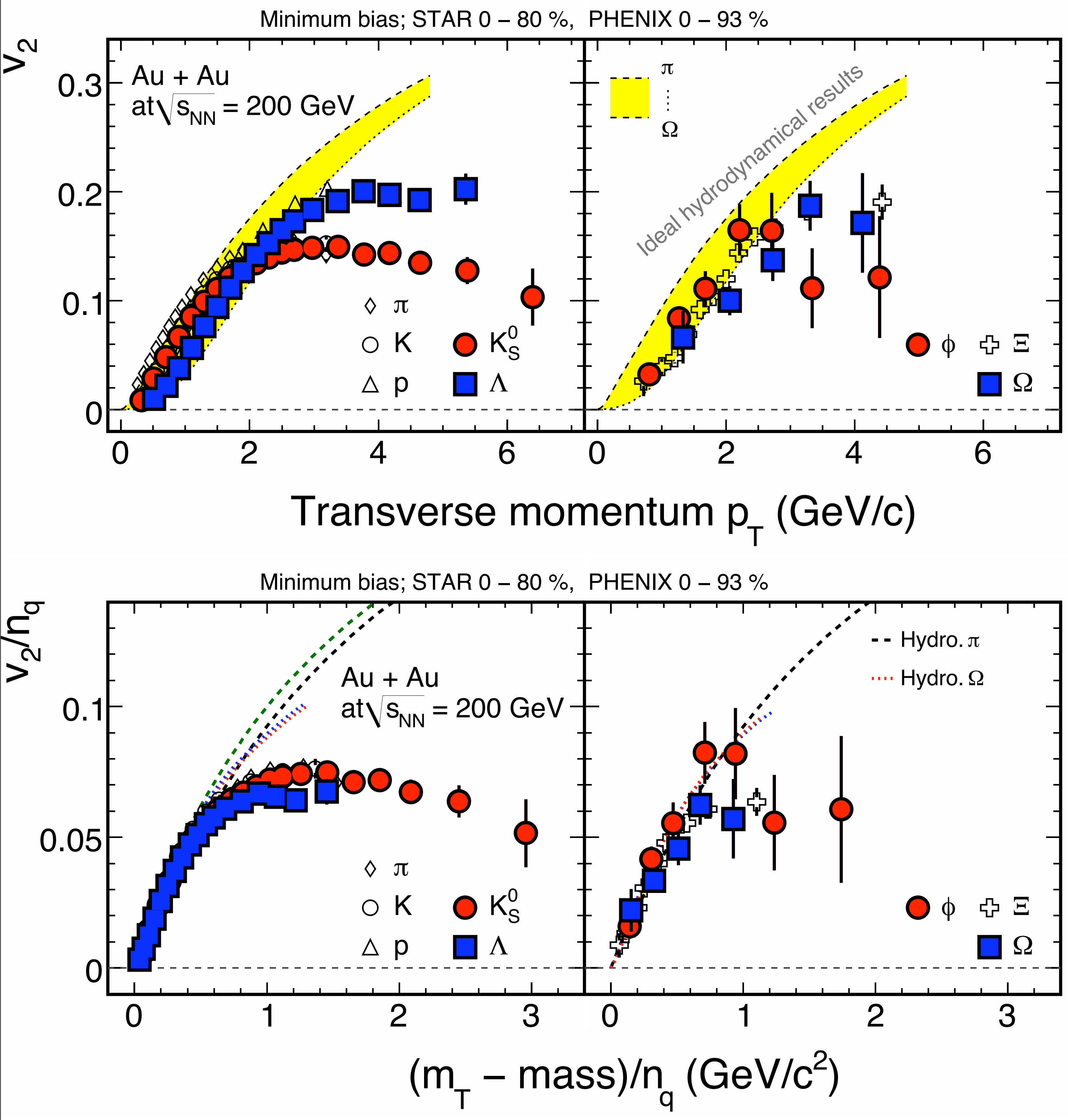
Partonic flow at RHIC



- Mass ordering of v_2
  - ✓ Radial flow
- Sizable v_2 for multi-strange hadrons
  - ✓ sensitive to partonic stage
- Number of Quark (NQ) scaling
  - ✓ vs $(m_T$ - mass$)/n_q$

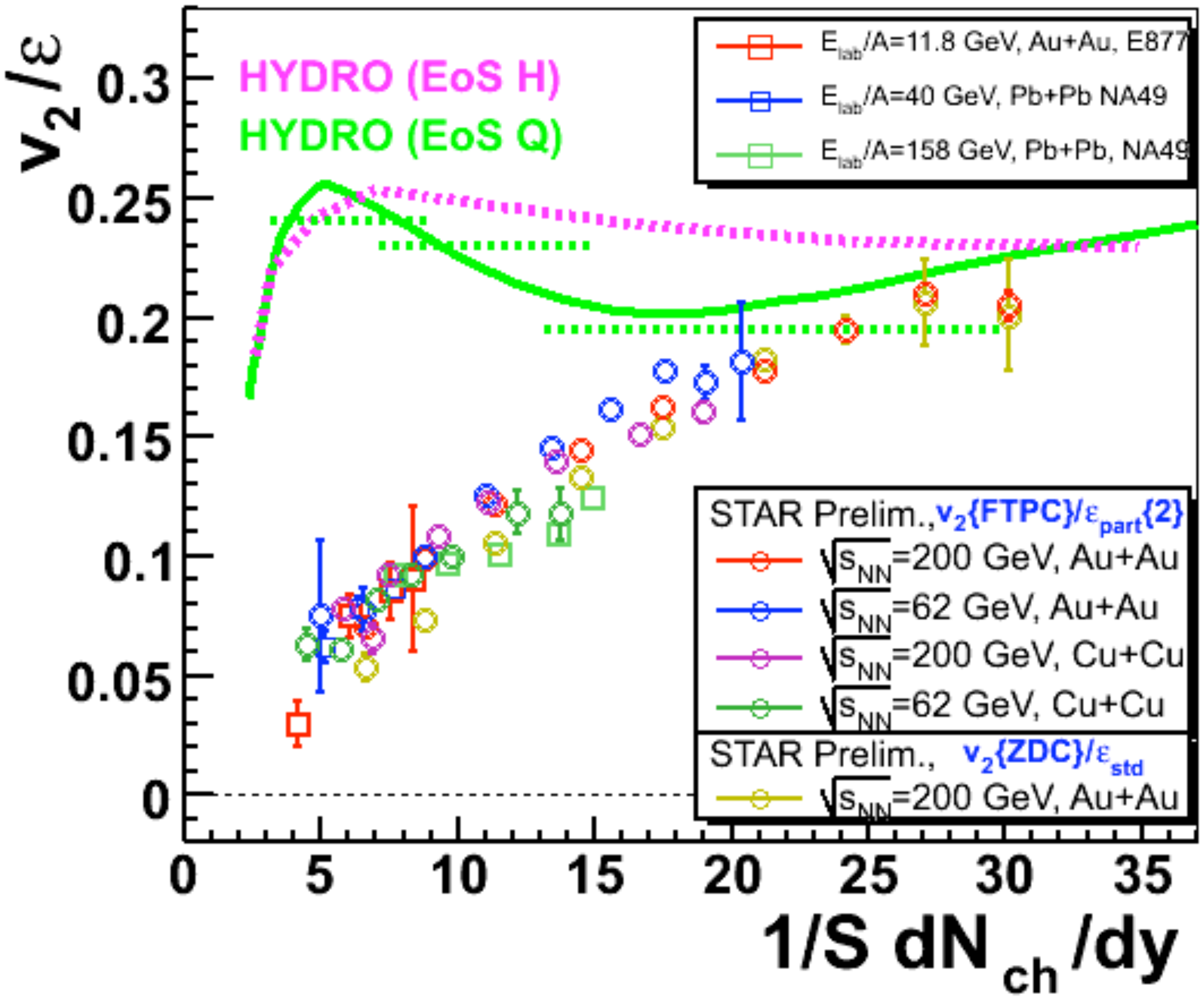
➡ Partonic collectivity

➡ Deconfinement

PHENIX: Phys. Rev. Lett. **98**, 162301 (2007)
STAR: Phys. Rev. Lett. 99, 112301 (2007),
Phys. Rev. **C77**, 054901 (2008)
Hydro results: Pasi Huovinen, private communication; P. Huovinen and P. V. Ruuskanen, Annu. Rev. Nucl. Part. Sci. **56**, 133 (2006)

Hydrodynamical limit ?

S. Voloshin, QM2006



- v_2/ε scales (1/S)dN/dy
  - ✓ scaling holds well from AGS, SPS to RHIC, among different systems and methods
- At RHIC, has v_2 reached hydrodynamical limit ?!

Hydro curves are obtained from calculations Kolb, Sollfrank, Heinz, PR**C62** (2000) 054909, made at **b=7 fm in Pb + Pb collisions** and rescaled by 'optical' eccentricity value.
The centrality dependence is not fully reflected by these curves, as it is more 'flat' at each given collision energy (very roughly indicated by strait lines)

Caveats

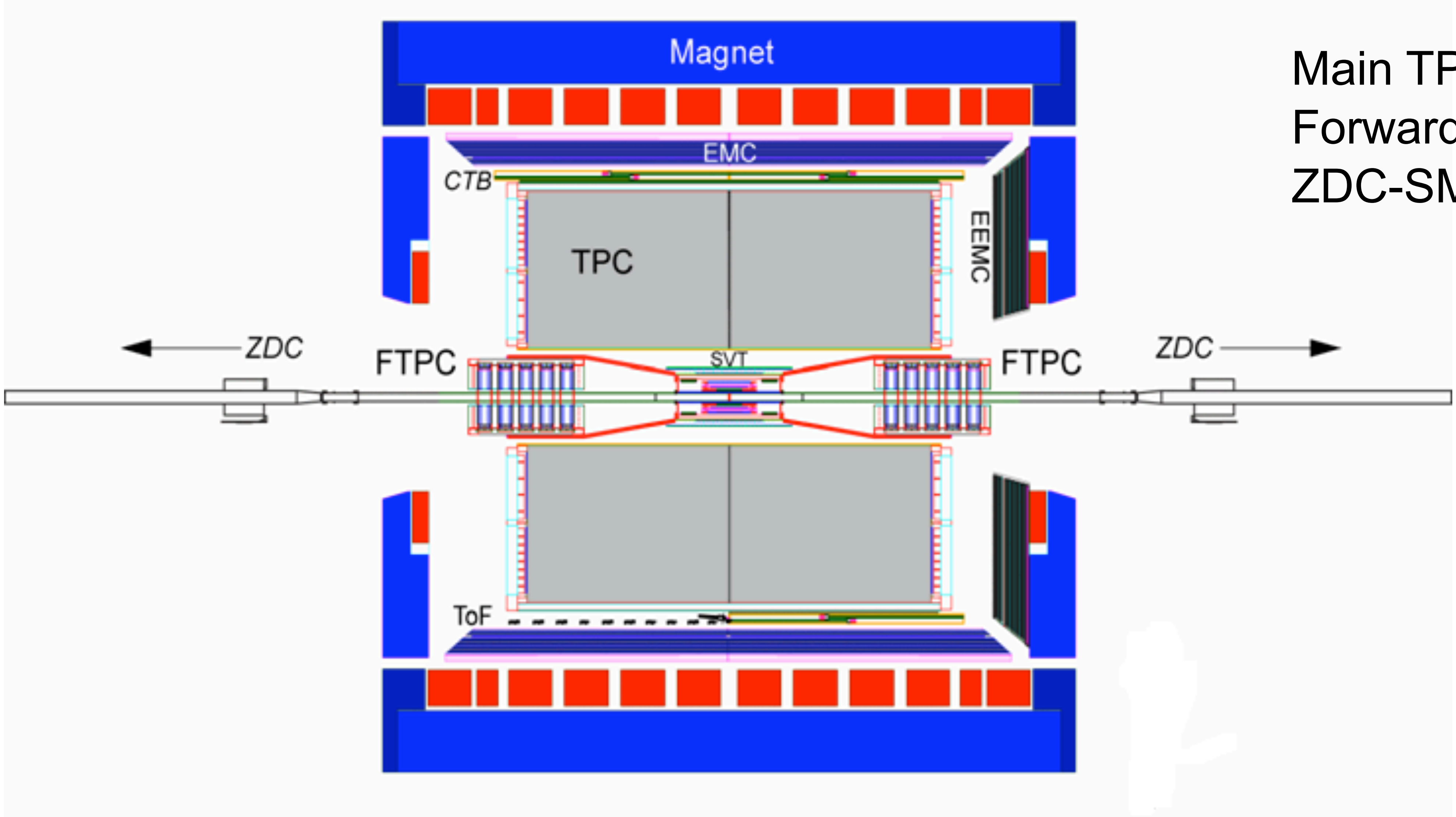
- Ideal hydrodynamical calculations
  - ✓ for pions in Pb + Pb collisions

- Particle composition could change v_2 for inclusive charged hadrons
  - ✓ Heavier hadrons have larger v_2 due to radial flow

- Compare v_2 for identified hadrons
- Extract hydrodynamical limit from the data

➡ A transport model approach, extract hydrodynamical limit from identified hadron v_2 measurements

STAR experiment

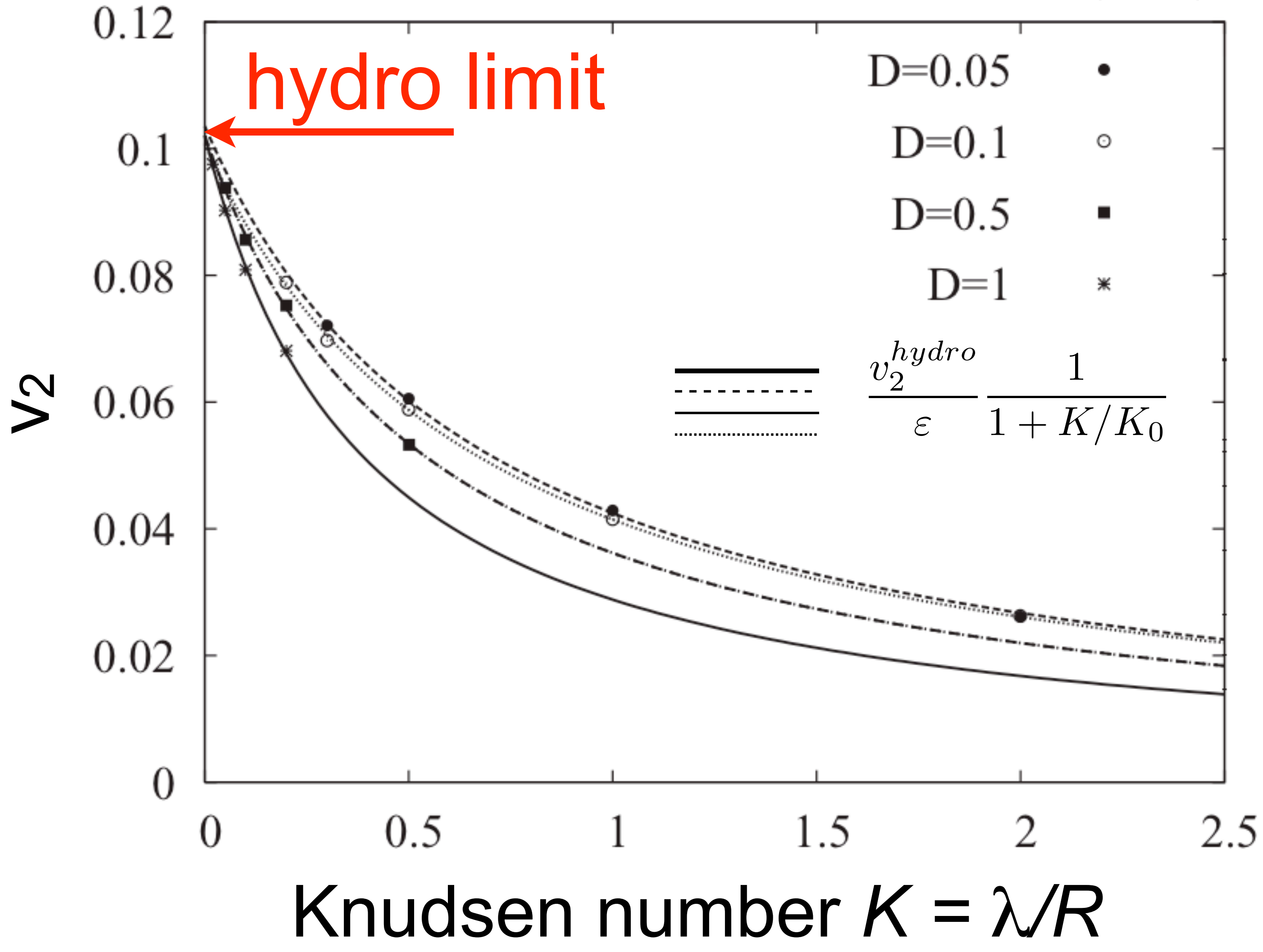


Main TPC $|\eta| < 1$
Forward TPC $|\eta| = 2.5 - 4$
ZDC-SMD $|\eta| > 5$

- Different methods to measure v_2
  - ✓ 4-particle cumulant $v_2\{4\}$, ZDC-SMD event plane from directed flow $v_2\{\text{ZDC-SMD}\}$ for unidentified charged hadrons
  - ✓ For identified hadrons, standard event plane method at the main or forward TPC were used

Transport model approach

C. Gombeaud and J.-Y. Ollitrault, PR**C77**, 054904 (2008)



- Knudsen number
  - ✓ degrees of equilibration
- Transport model
  - ✓ Reach hydro limit when $K \rightarrow 0$
- Simple formula to describe $v_2(K)$
  - ✓ Reduces hydro limit ($K \rightarrow 0$) and low density limit ($K >> 1$)
  - ✓ Reproduce transport model calculations

$$\frac{v_2}{\varepsilon} = \frac{v_2^{hydro}}{\varepsilon} \frac{1}{1+K/K_0} = \begin{cases} v_2^{hydro}/\varepsilon & (K \rightarrow 0) \\ v_2^{hydro}/(\varepsilon \times K/K_0) & (K \gg 1) \end{cases}$$

Fitting procedure

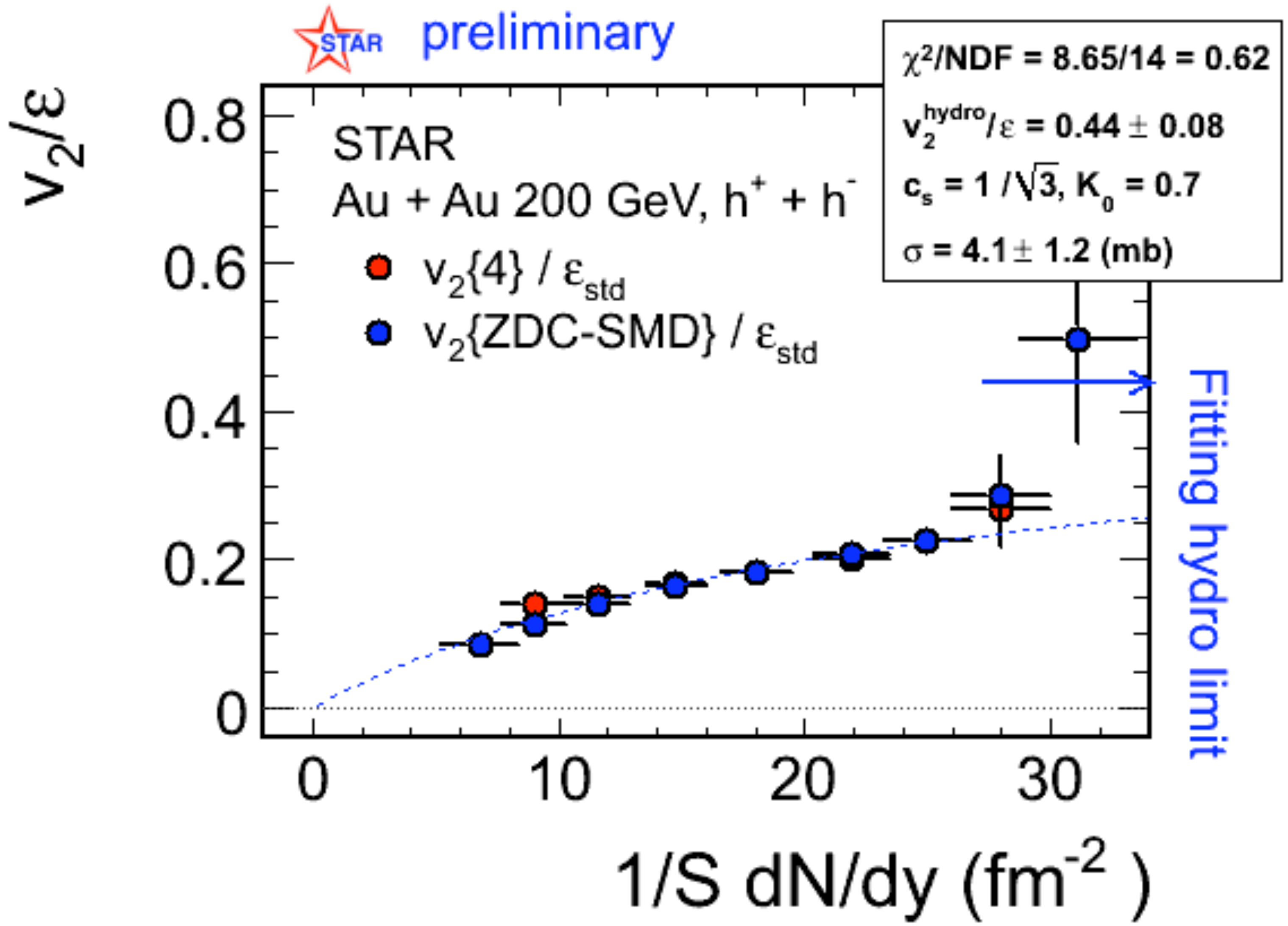
$$\frac{v_2}{\varepsilon} = \boxed{\frac{v_2^{hydro}}{\varepsilon}} \frac{1}{1 + 1/(\boxed{K_0 \sigma c_s}(1/S)dN/dy)}$$

σ : partonic cross section
c_s : speed of sound
S : transverse area

$$\frac{1}{K} = \frac{R}{\lambda} = \frac{\sigma c_s}{S}\frac{dN}{dy}, \quad \left(\lambda = \frac{1}{n\sigma},\ n(\tau) \sim \frac{1}{\tau S}\frac{dN}{dy},\ \tau \sim \frac{R}{c_s}\right)$$

- Fit unidentified charged hadrons
  - ✓ Extract parton cross section
    - Note: one can only extract the product $K_0 \sigma c_s$
- Fit all identified hadrons simultaneously
  - ✓ σ is fixed from fitting result for unidentified hadron
- $K_0 = 0.7$ and $c_s^2 = 1/3$ (fixed)
  - ✓ K_0 value is determined so as to reproduce the transport model calculation ($K_0 = 0.7 \pm 0.03$)* *C. Gombeaud and J.-Y. Ollitrault, PR**C77**, 054904 (2008)
- Only Glauber initial condition is considered

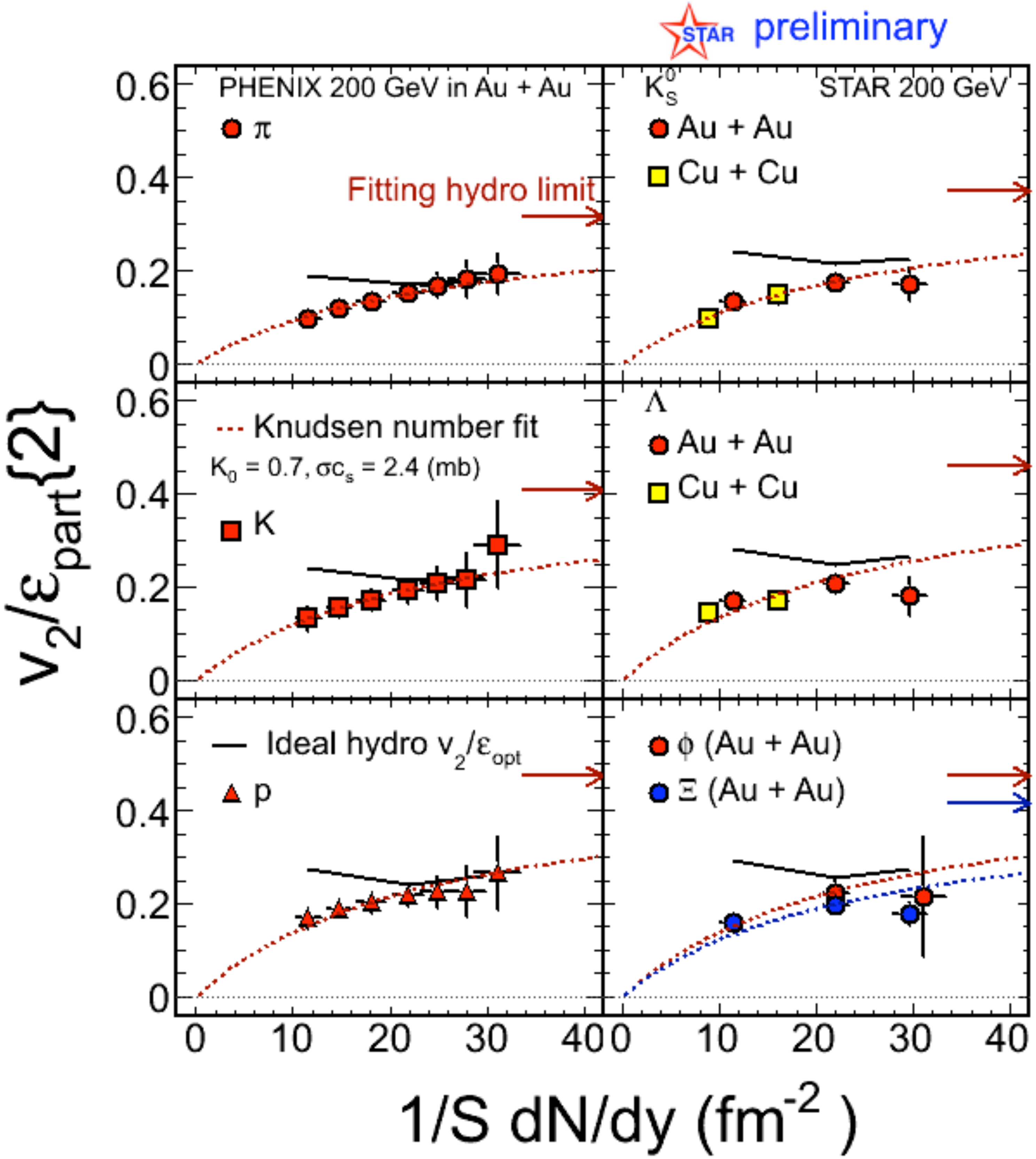
$v_2\{4\}$ & $v_2\{ZDC\text{-}SMD\}$



- $v_2\{4\}$ and $v_2\{ZDC\text{-}SMD\}$
  - ✓ sensitive to the true reaction plane
  - ✓ scaled by standard eccentricity
- Parton cross section
  - ✓ σ = 4.1 ± 1.2 mb

➡ Extract hydro limit for identified hadrons with K_0 = 0.7, c_s^2 = 1/3 and σ = 4.1 mb

Centrality dependence of $\langle v_2 \rangle$



- Transport model fit well describe the data
- "Fitting" hydro limits (arrows) increase as a function of mass
- Ideal hydro v_2/ε
  - ✓ ε from "optical" glauber model

PHENIX π, K and p: preliminary, nucl-ex/0604011v1
STAR K^0_S, Λ, Ξ : Phys. Rev. **C77**, 054901 (2008)
STAR ϕ : Phys. Rev. Lett. **99**, 112301 (2007)
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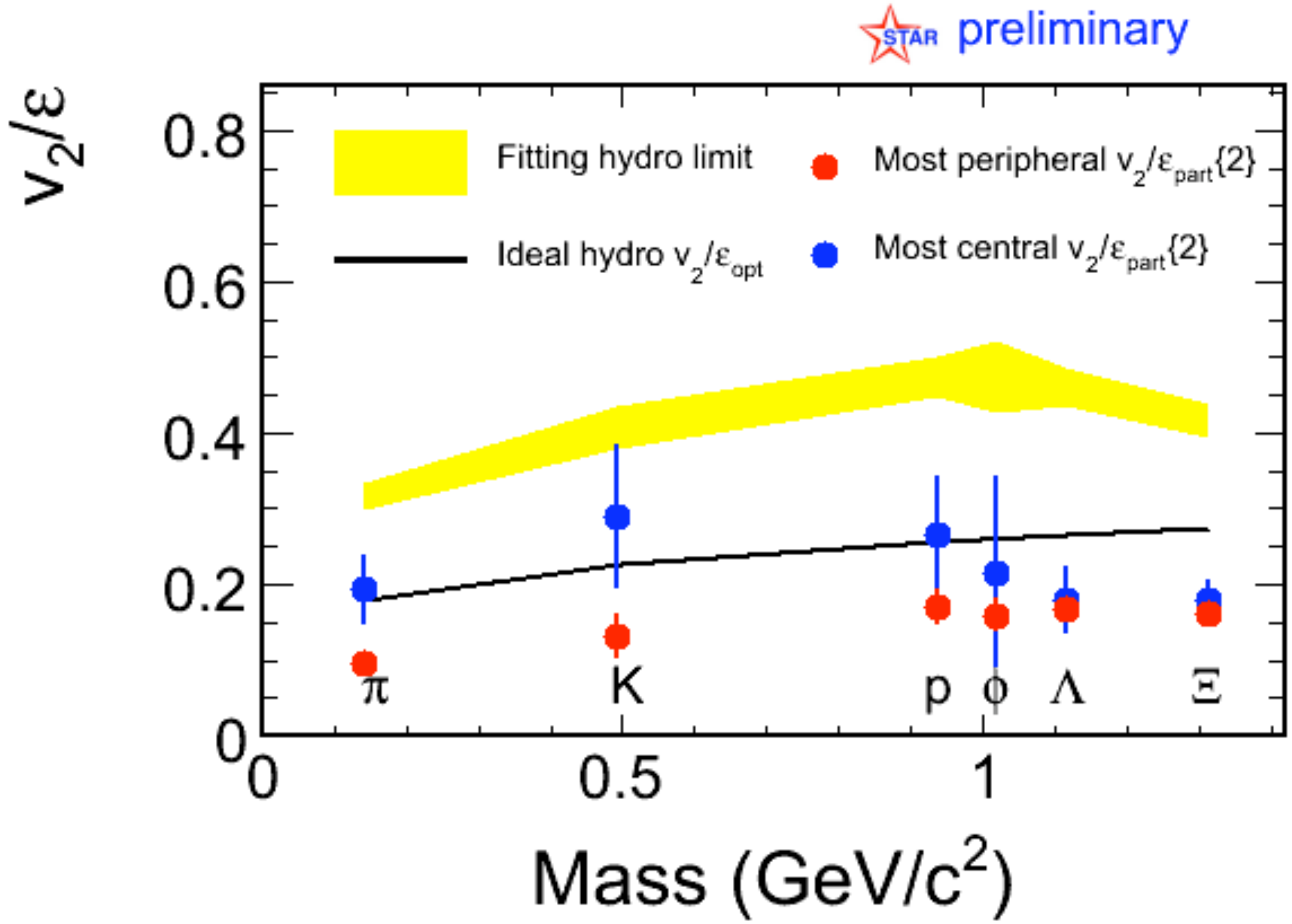
Hydro limit vs mass



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Ideal hydro : Average over all centrality bins

- "Fitting" hydro limits increase as a function of mass
  - ✓ Mass dependence is similar to that from ideal hydro
  - ➡ Radial flow at partonic stage
  - ✓ Early decoupling for (multi-)strange hadrons ?

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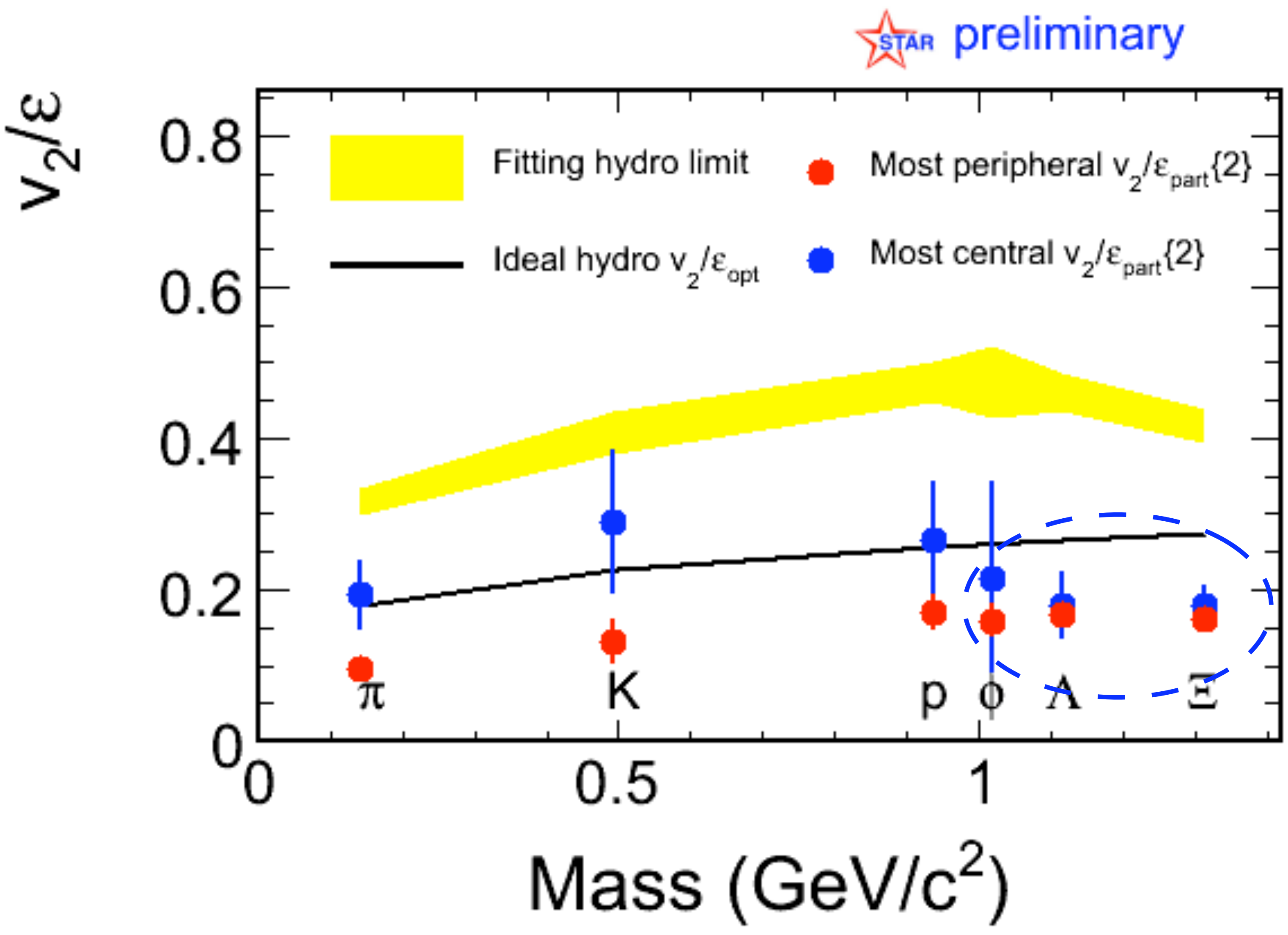

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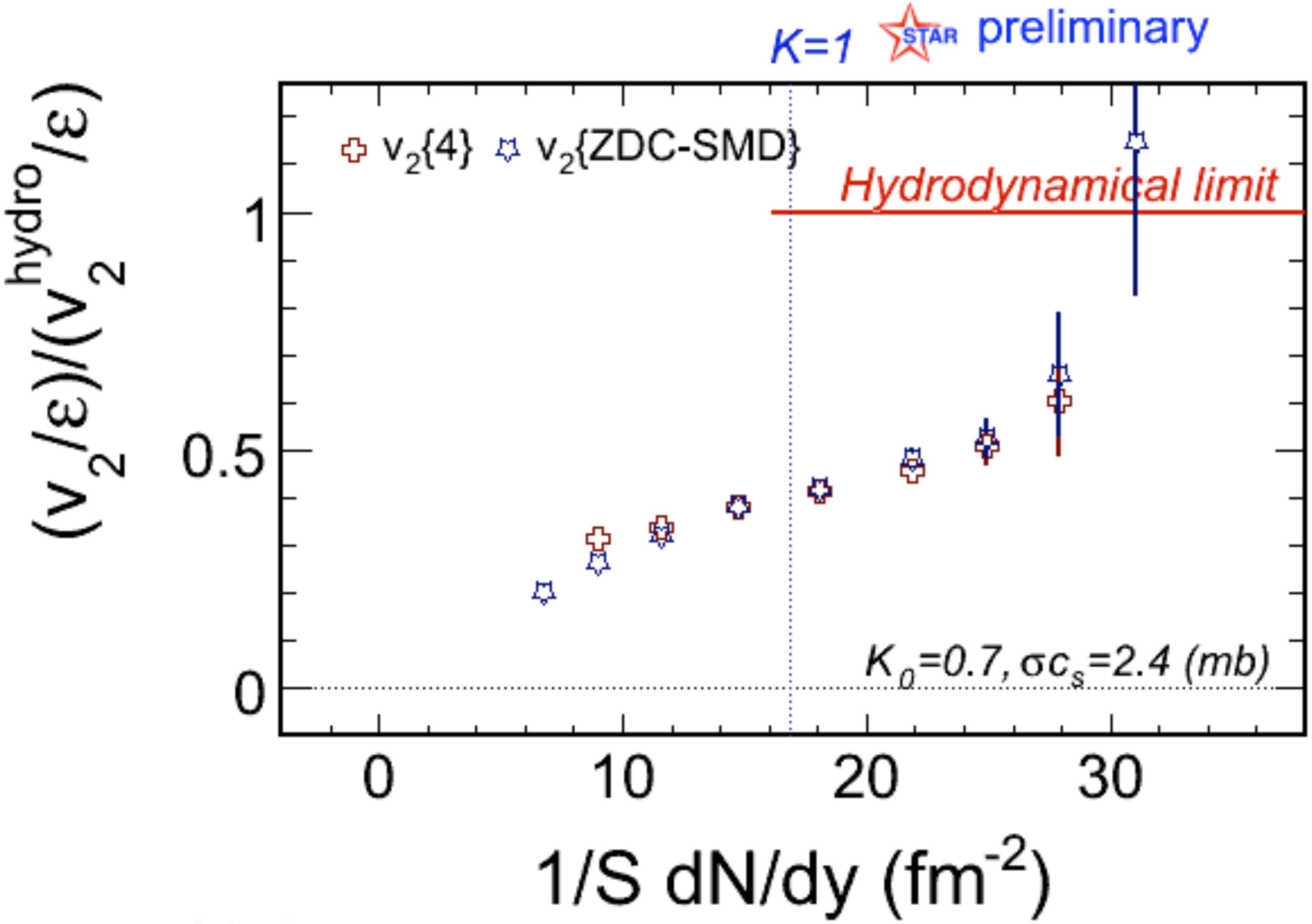
Hydrodynamical limit



~ 10 % errors on hydro limit from the fit

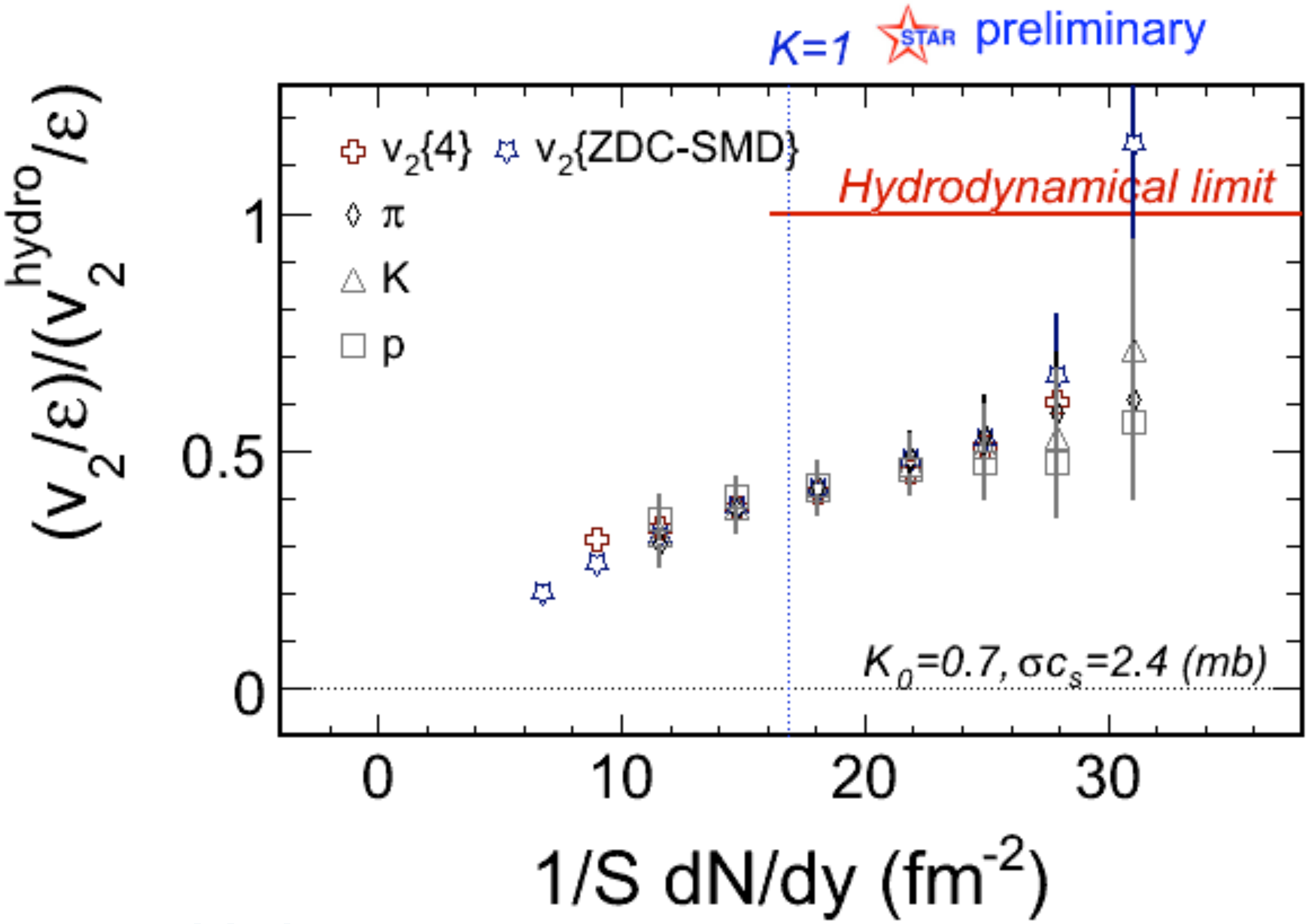
$K = 1 \rightarrow (1/S)\ dN/dy \sim 17$
$K = 0.6 \rightarrow (1/S)\ dN/dy \sim 30$

- Universal curve

✓ different particles, methods, systems and experiments

➡ Hydrodynamical limit has not been reached at RHIC within the transport model approach

Hydrodynamical limit



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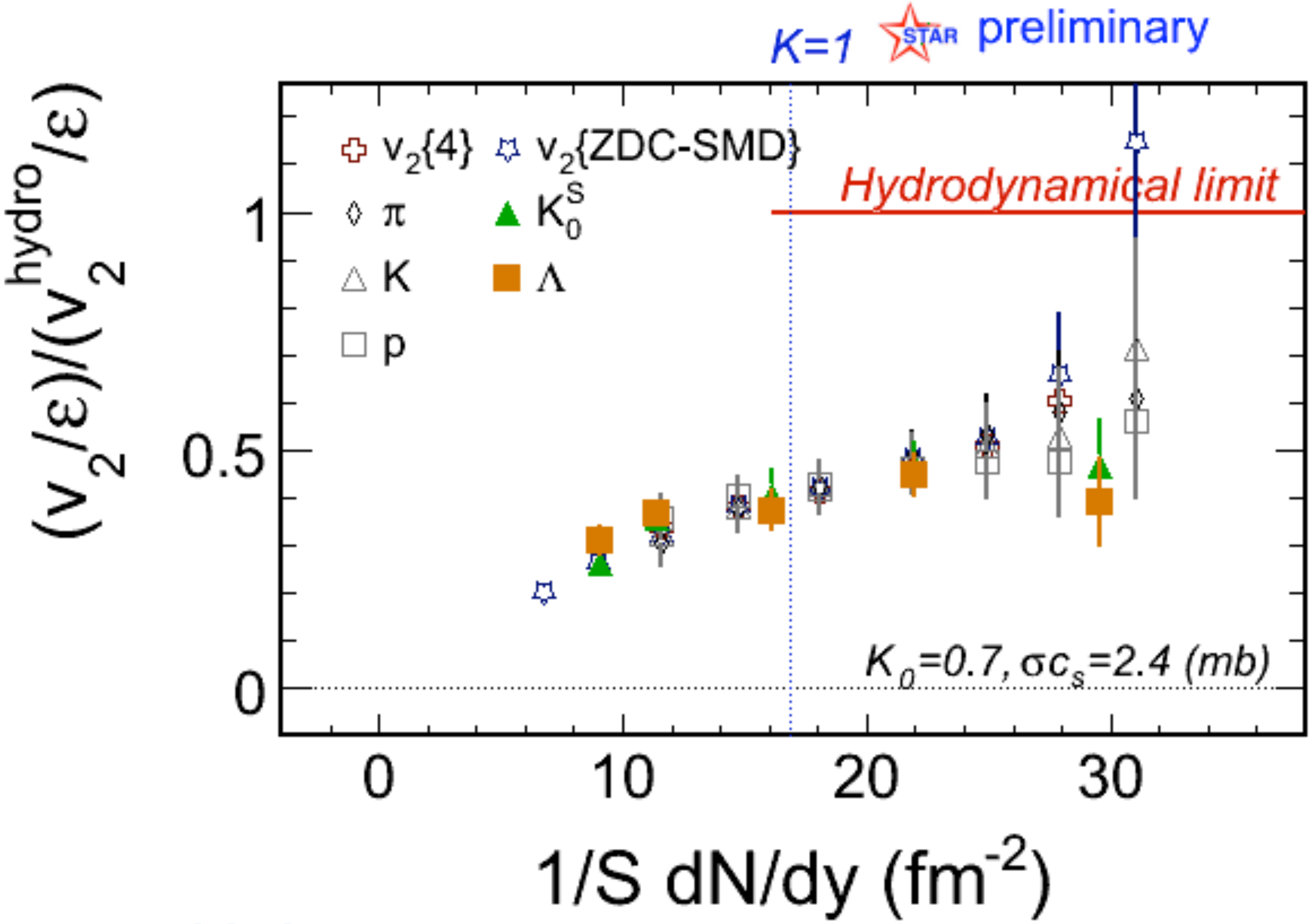

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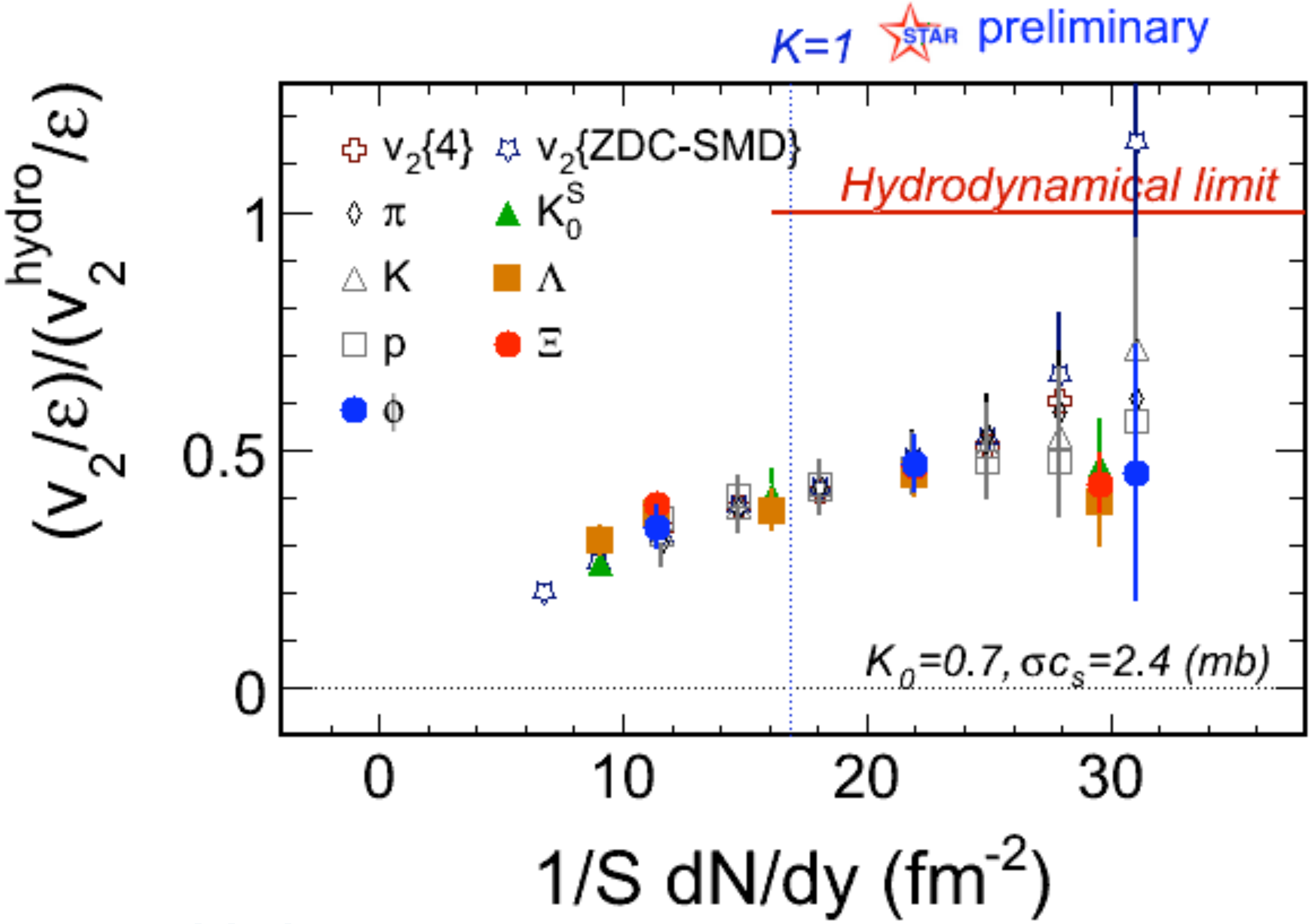

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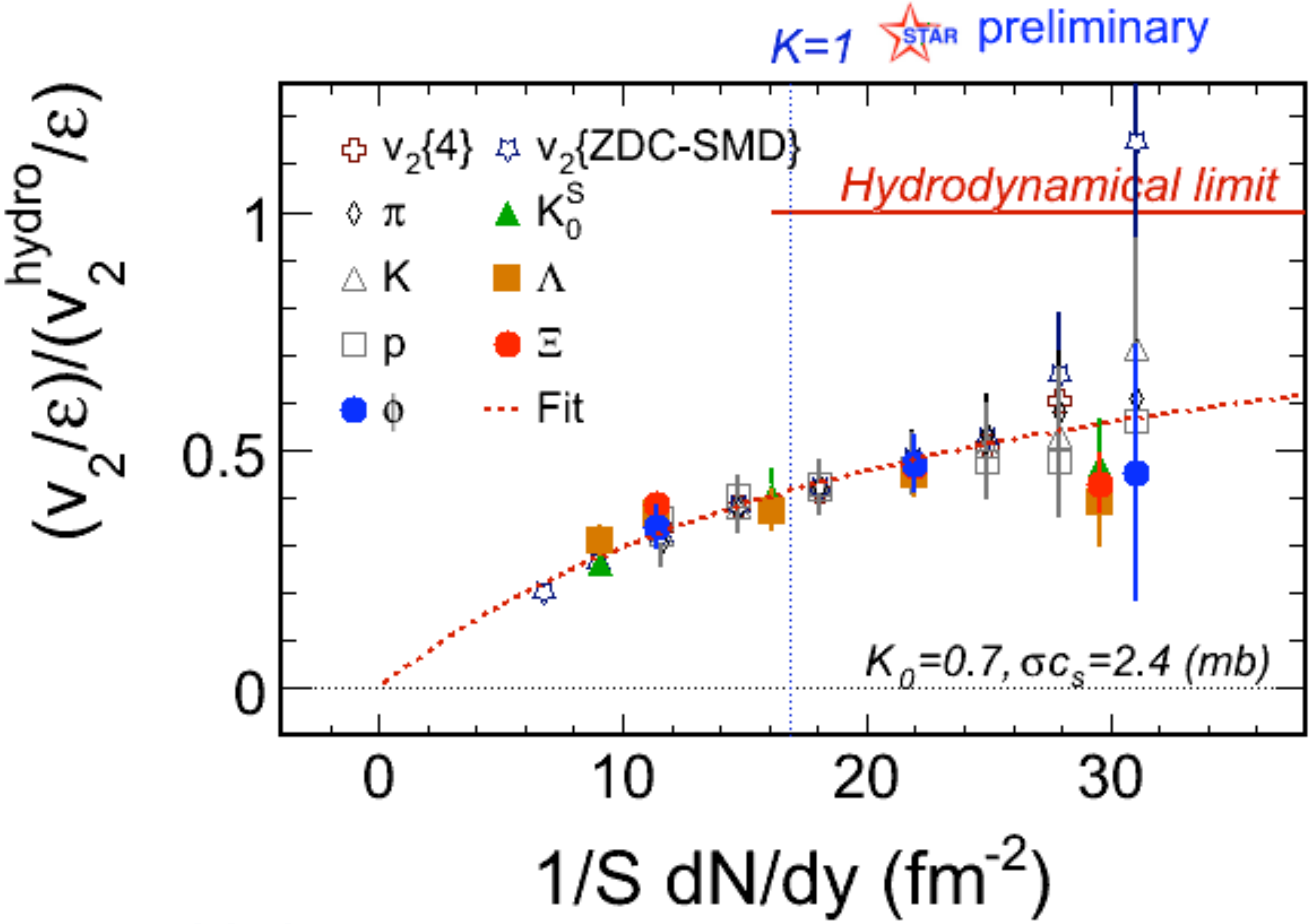

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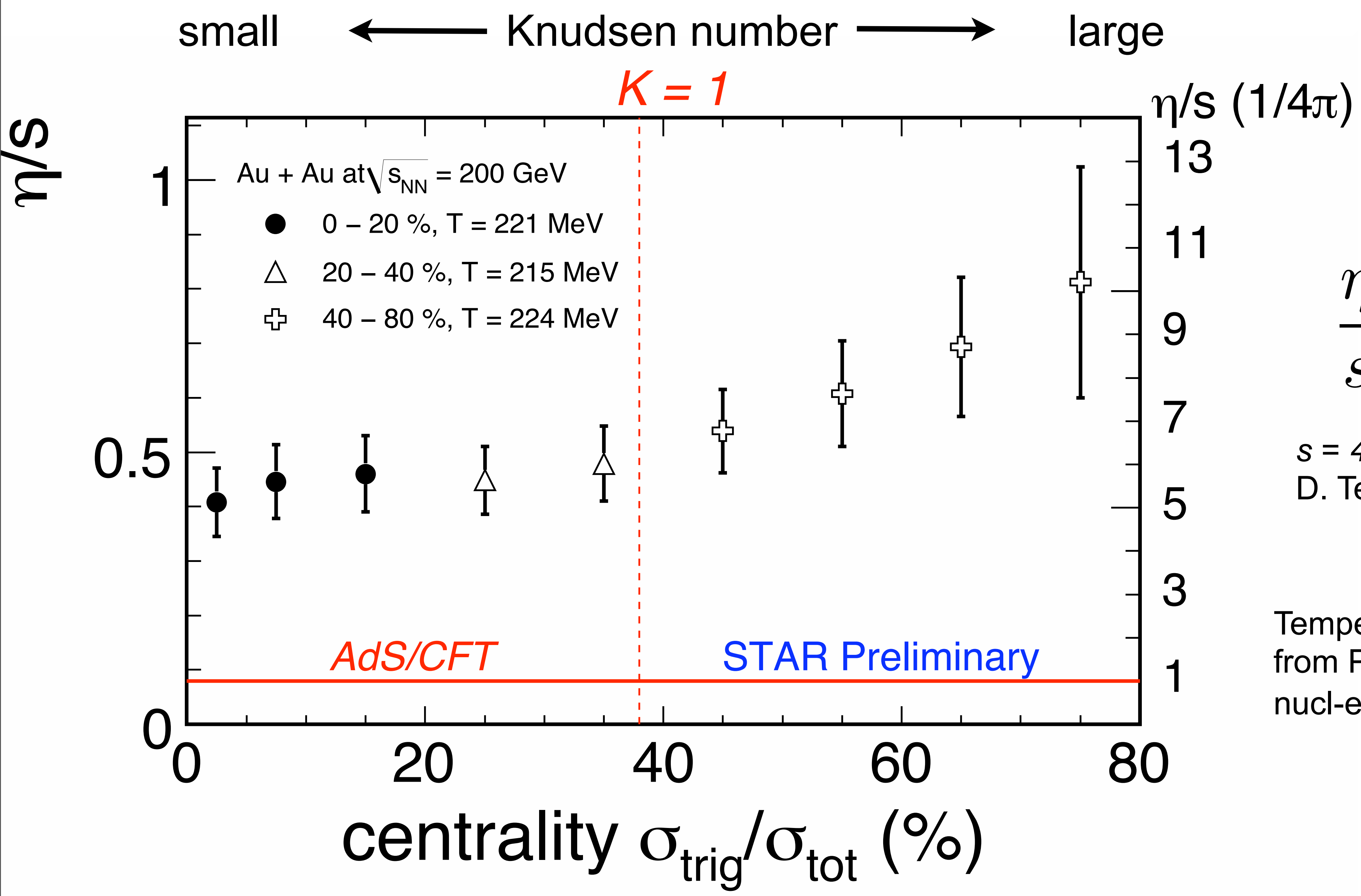
Constrain shear viscosity



$$\frac{\eta}{s} = 0.316 \frac{T}{\sigma c_s n}$$

s = *4n* for a classical relativistic gas
D. Teaney, PR**C68**, 034913 (2003)

Temperatures
from PHENIX direct γ measurements,
nucl-ex/0804.4168

- σc_s ↔ from fit, *n* (density) ↔ centrality
  - ✓ η/s ~ 5/4π in 0 - 40 % centrality
  - ✓ Closer to ideal hydro in more central collisions

Conclusions

- Extract hydrodynamical limits for identified hadrons

✓ Fitting results well describe the centrality dependence of v_2

  - Simultaneous fit by the simple formula motivated from the transport model

✓ Universal curve is obtained

  - for different methods, particle species, systems and experiments

✓ “Fitting” hydrodynamical limits increase as a function of mass

  - similar mass dependence to that from ideal hydro calculation

✓ Simultaneous fit for identified hadrons gives a constraint on the shear viscosity as a function of centrality

  - $\eta/s \sim 5/4\pi$ at in 0 - 40 % centrality, close to hydro limit in more central collisions

➡ Ideal hydrodynamical limit has not been reached within the transport model approach (with Glauber initial conditions)

Extra slides

What ε we should use ?

2-particle correlation

- Event plane method : $v_2\{EP_2\}$, $v_2\{BBC\}$, $v_2\{FTPC\}$, $v_2\{etaSub\}$
- Two particle cumulant : $v_2\{2\}$
- Scalar product method : $v_2\{uQ\}$

➡ Sensitive to “Participant Plane”
➡ Scaled by participant eccentricity ε_{part}

Multi-particle correlation

- ZDC-SMD event plane : $v_2\{ZDC\text{-}SMD\}$
- q-distribution method: $v_2\{q\}$
- 4 or higher order cumulant : $v_2\{n\}$, $n \geq 4$
- Lee-Yang zero method : $v_2\{LYZ\}$
- Bessel-Transform method : $v_2\{BT\}$

➡ Sensitive to “Reaction Plane”
➡ Scaled by standard eccentricity ε_{std} or $\varepsilon\{4\}$

$$\varepsilon_{part} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2} \rightarrow \frac{\{y^2\} - \{x^2\}}{\{y^2\} + \{x^2\}} = \varepsilon_{std} \ (\{x\} = \{y\} = \{xy\} = 0)$$

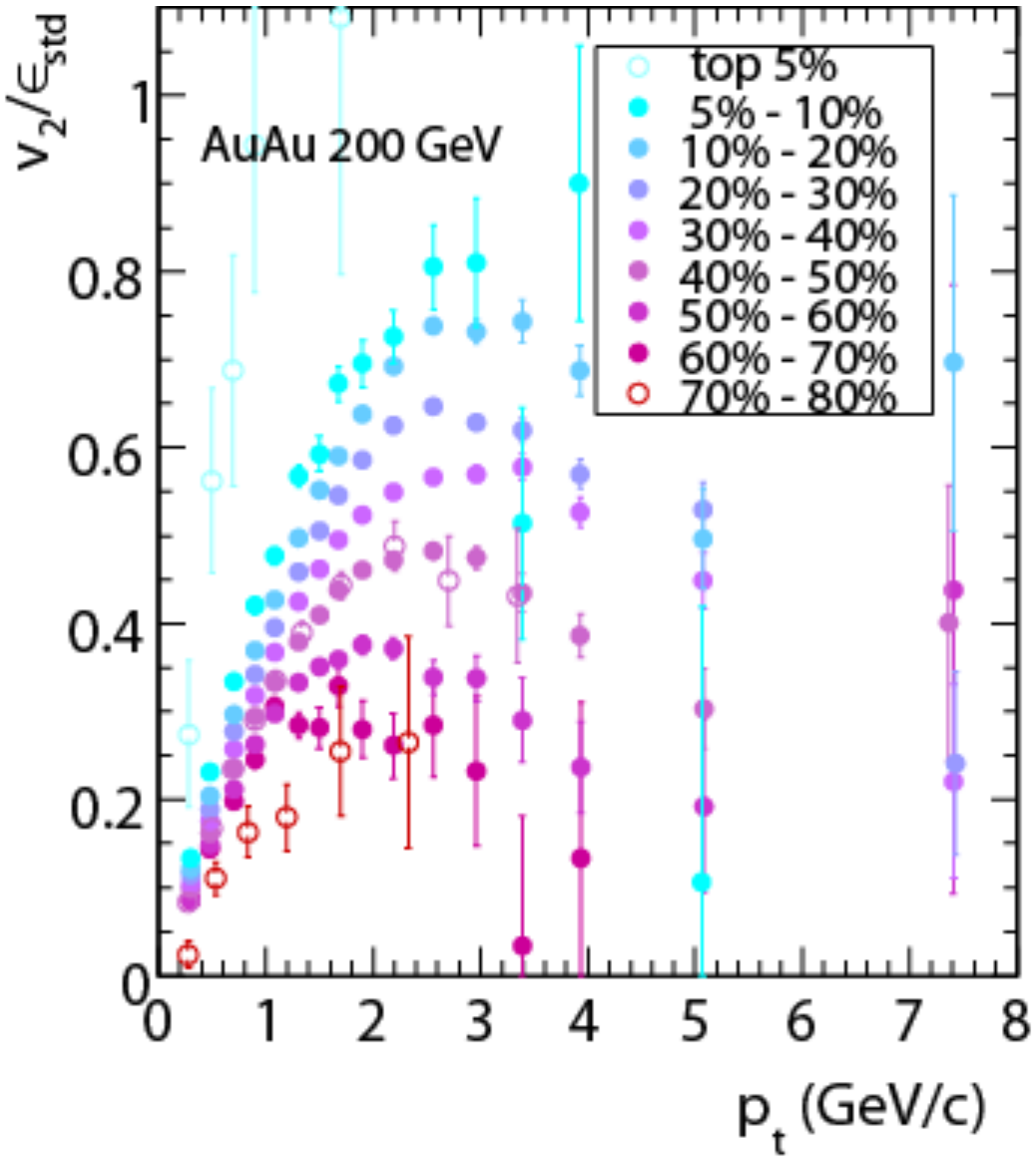
$$\sigma_x^2 = \{x^2\} - \{x\}^2, \quad \sigma_y^2 = \{y^2\} - \{y\}^2, \quad \sigma_{xy} = \{xy\} - \{x\}\{y\}$$

- Methods based on multi-particle correlation are sensitive to the true reaction plane

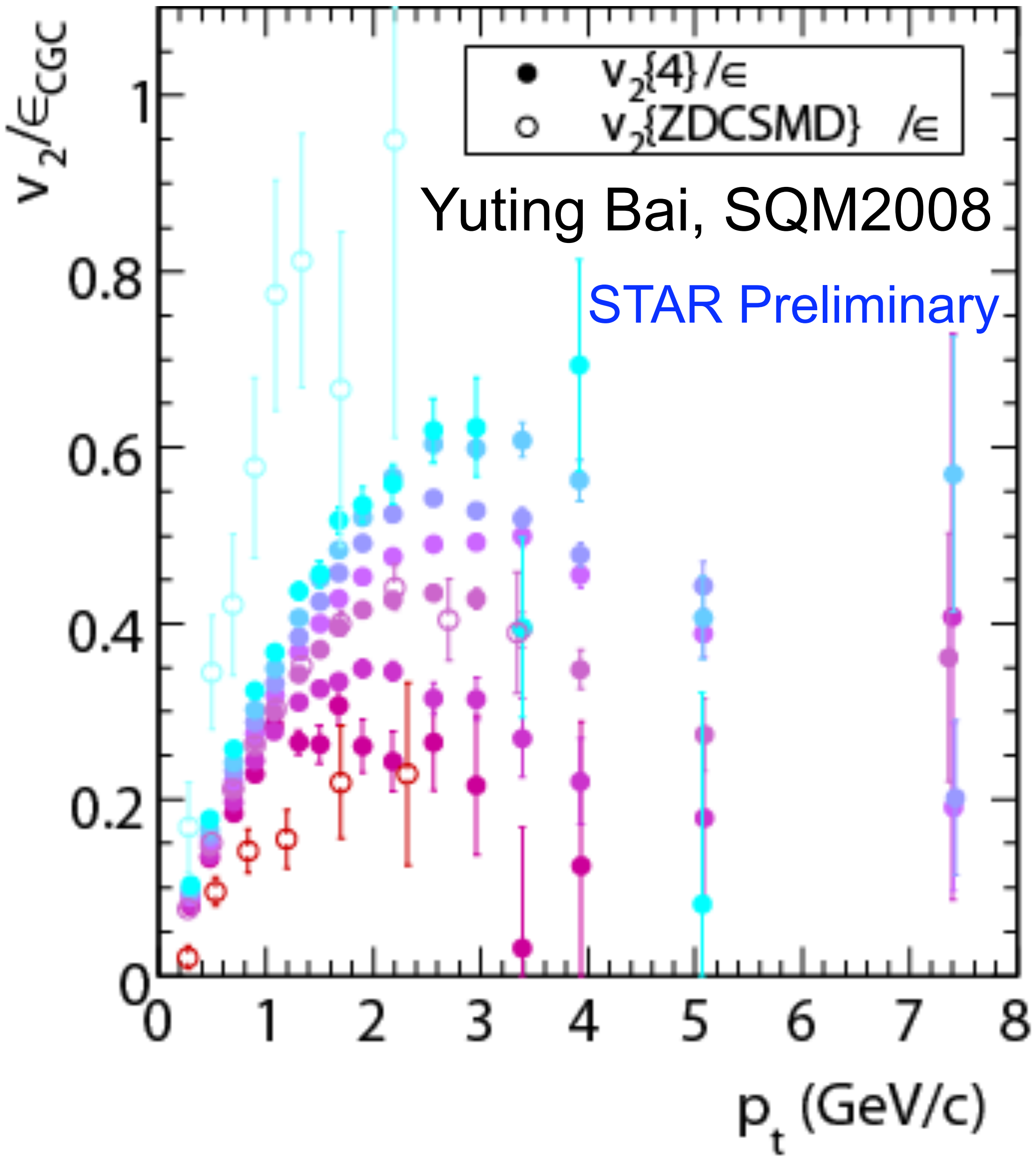
✓ should be scaled by standard eccentricity ε_{std}

Stronger flow in central collisions

Eccentricity from Glauber MC

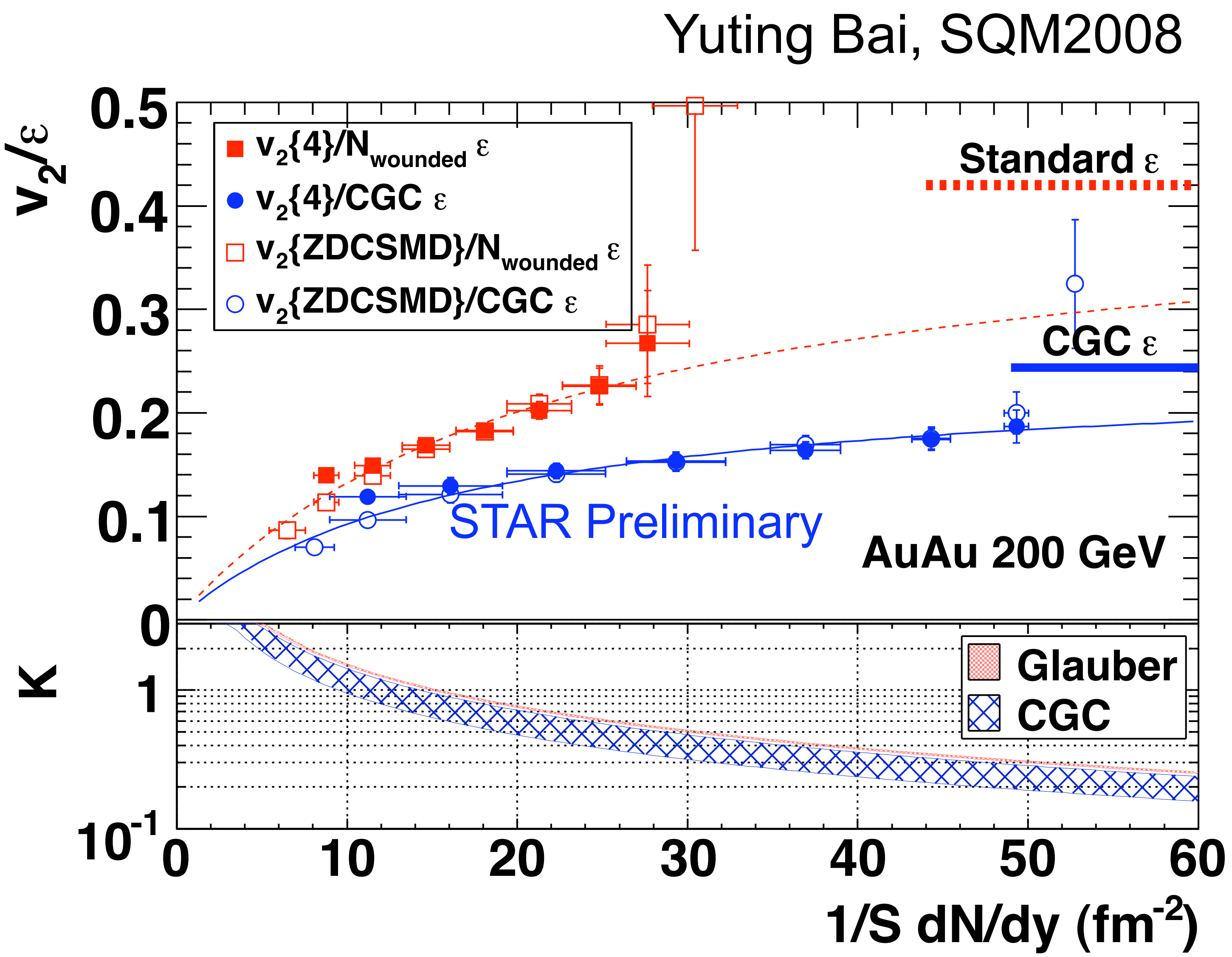


Eccentricity from CGC, nucl-th/0605012



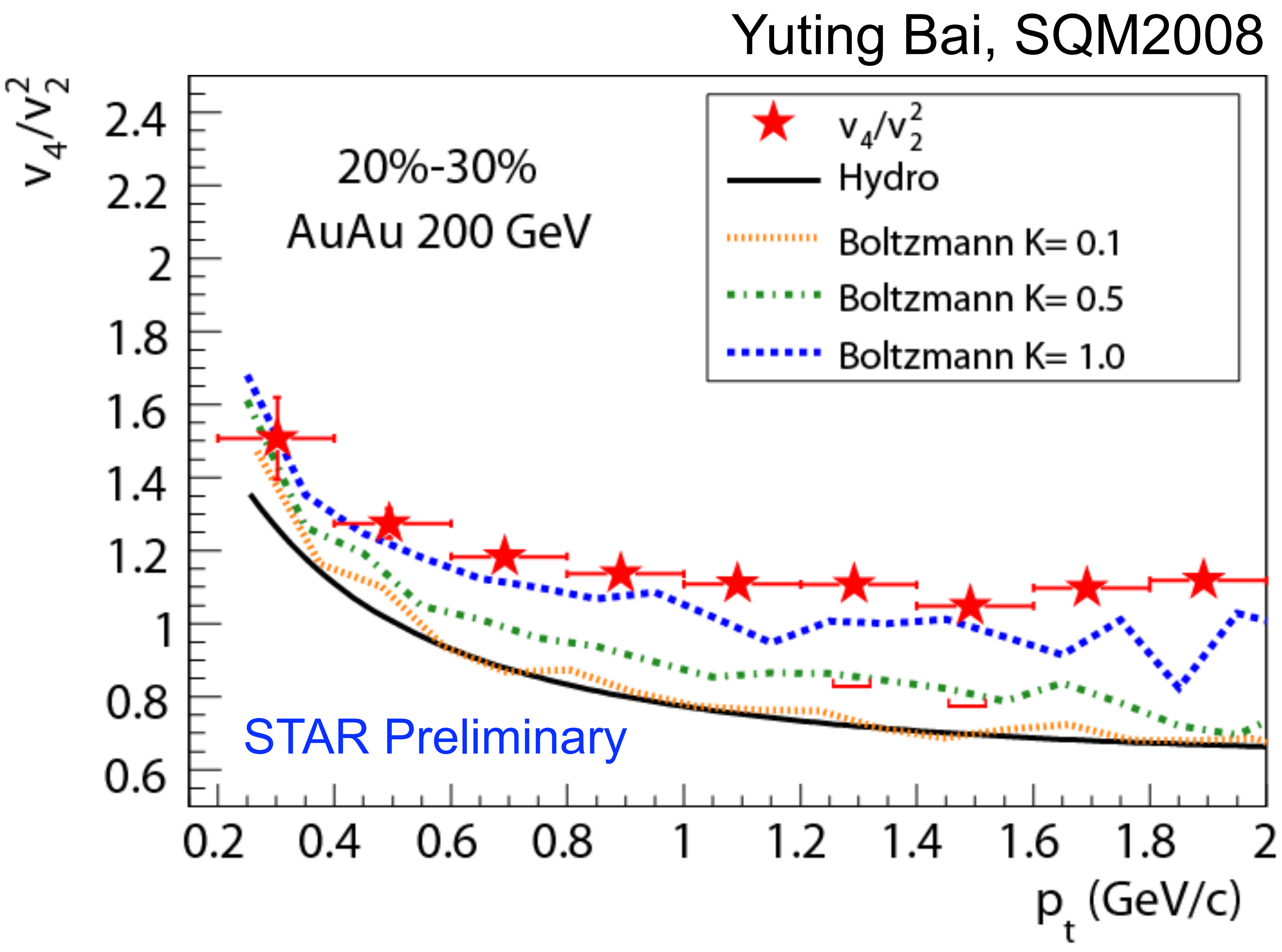
- In more central collisions
  - ✓ Stronger v_2/ε and peak position shift toward higher p_T
  - ✓ Applicable range for hydro extends to higher p_T

Knudsen number



- $v_2\{4\}$ and $v_2\{ZDCSMD\}$
  - ✓ sensitive to reaction plane
    - scaled by standard eccentricity
- Knudsen number is constrained by simultaneous fit for charged hadron $v_2\{4\}$, $v_2\{ZDCSMD\}$ as well as identified hadrons
  - ✓ *K* ~ 0.3 - 1 from central to peripheral collisions

$v_4/(v_2)^2$ vs p_T



Hydro, Boltzmann calculations:J-Y. Ollitrault

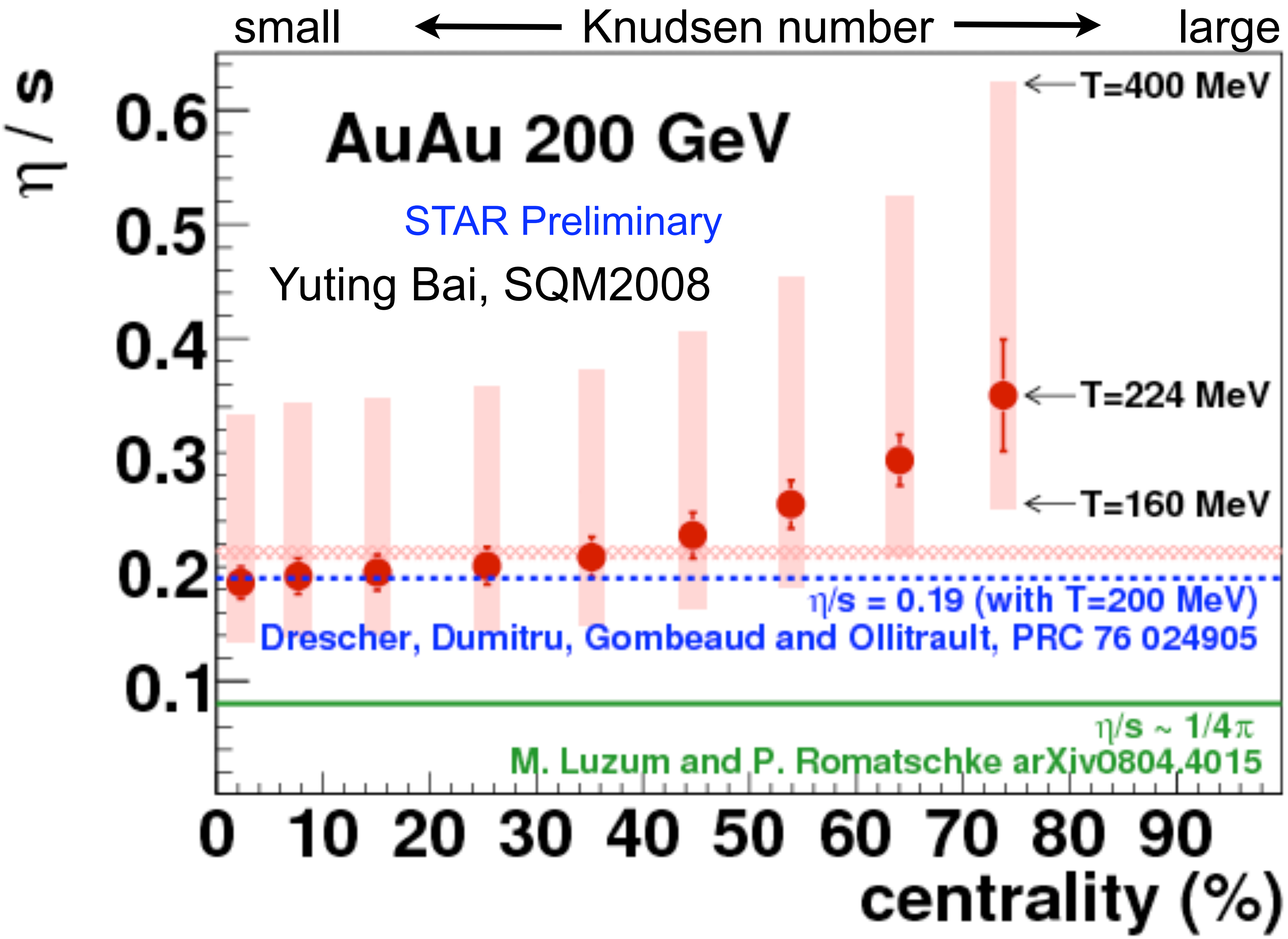
$$v_2 \propto K^{-1}, \quad v_4 \propto K^{-1} \text{ for small } K^{-1}$$

$$\rightarrow \frac{v_4}{(v_2)^2} \textit{ decreases} \text{ with } K^{-1}$$

- $v_4/(v_2)^2$ reach a minimum when $K \rightarrow 0$
- Comparison with a transport model
  - ✓ Model calculations approach hydro limit when $K \rightarrow 0$
  - ✓ The data is consistent with the transport model calculation when $K > 0.5$
    - Knudsen number is consistent with that from identified hadron v_2

➡ Hydro limit has not reached at RHIC !

Constrain shear viscosity



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from PHENIX direct γ measurements, nucl-ex/0804.4168

- σc_s ↔ from fit, *n* (density) ↔ centrality
  - ✓ Average η/s ~ 0.22 for T = 224 MeV
  - ✓ η/s decrease in more central collisions → stronger flow