Heavy quarkonia in AdS/QCD

Y. Kim (KIAS)

YK, J.-P. Lee, S. H. Lee, Phys. Rev. D75:114008, 2007.

YK, B.-H. Lee, C. Park, and S.-J. Sin, hep-th/0808l143.

Plan

- Why heavy quarkonia?
- Bottom-up AdS/QCD
- Heavy quarkonium in bottom-up
- Holographic heavy quark potential
- Summary

Why heavy quarkonium?

QCD (QGP):

- Reveals non-perturbative nature of QGP.
- Matsui, Satz (1986):

  J/psi will completely disappear just above T_c due to the color screening.
- Asakawa, Hatsuda(2003):

  J/psi will survive well above T_c up to ~ 2 T_c.

AdS/QCD:

- Due to HPT, AdS BH is not stable below T_c .
- No T dependence of hadrons?
- Heavy quarkonia above T_c .

Bottom-up AdS/QCD

"In the bottom-up approach, one looks at QCD first and then attempts to guess its 5D-holographic dual."

AdS/CFT Dictionary

- 4D CFT (QCD) ←→ 5D AdS
- 4D generating functional ←→ 5D (classical) effective action
- Operator ←→ 5D bulk field
- [Operator] ←→ 5D mass
- Current conservation ←→ gauge symmetry
- Large Q ←→ small z
- Confinement ←→ Compactified z
- Resonances ←→ Kaluza-Klein states

★ 5D field contents

Operator → 5D bulk field

$$\bar{q}_R q_L \longrightarrow \Phi(x,z)$$
$$\bar{q}_L \gamma^\mu q_L \longrightarrow L_M(x,z)$$
$$\bar{q}_R \gamma^\mu q_R \longrightarrow R_M(x,z)$$

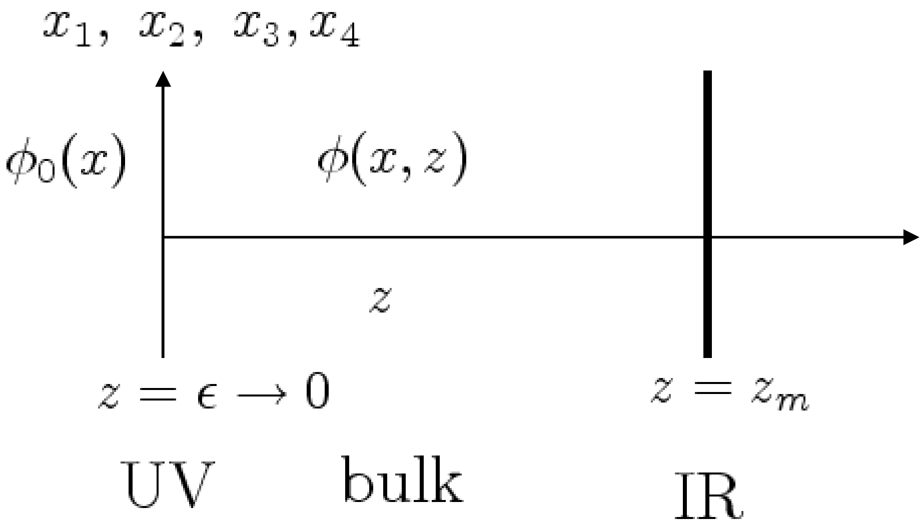
[Operator] → 5D mass

$$(\Delta - p)(\Delta + p - 4) = m_5^2 \qquad m_\phi^2 = -3$$

★ Confinement

Polchinski & Strassler, 2000

Confinement → IR cutoff in 5th direction



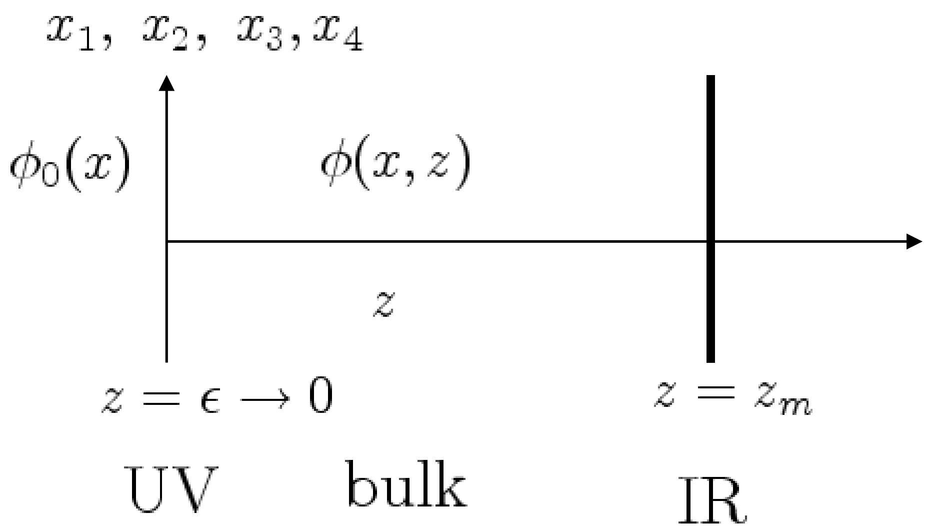
Hard wall model

J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95**, 261602 (2005)

L. Da Rold and A. Pomarol, Nucl.Phys. **B721**, 79 (2005)

$$S_I = \int d^4x dz \sqrt{g} \mathcal{L}_5 \,,$$

$$\mathcal{L}_5 = \mathrm{Tr}\left[-\frac{1}{4g_5^2}(L_{MN}L^{MN} + R_{MN}R^{MN}) + |D_M\Phi|^2 - M_\Phi^2|\Phi|^2 \right],$$



$$ds_5^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$

$$\begin{aligned}
&V_M \sim L_M + R_M\,, \quad A_M \sim L_M - R_M\,, \\
&\Phi = Se^{iP/v(z)}, \;\; v(z) \equiv \langle S \rangle\,, \\
&\Phi \leftrightarrow X, \quad v(z) \leftrightarrow X_0\,.
\end{aligned}$$

The model describes $\rho,\ a_1,\ \pi,\ \sigma,\ \ldots$.

Example: 4D vector meson mass

$$V(x,z) = \sum f_v(z)\tilde{V}(x)$$

$$[\partial_z^2 - \frac{1}{z}\partial_z + q^2]f_v(z) = 0,\ \ q^2 = m_n^2$$

$$m_n \simeq (n - \frac{1}{4})\frac{\pi}{z_m}$$

$$m_1 = m_\rho\ , \quad \frac{1}{z_m} \simeq 320\ \mathrm{MeV.}$$

Soft wall model

$$S_{\mathrm{hQCD-II}} = \int d^4x dz e^{-\Phi} \mathcal{L}_5 \,, \quad \Phi = cz^2.$$

A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys.Rev.D74:015005,2006

- Mode equation

$$\partial_z \left(e^{-B} \partial_z v_n \right) + m_n^2 e^{-B} v_n = 0,$$

where $B = \Phi(z) - A(z)$, with $e^{A(z)} = z^{-1}$. Substitute $v_n = e^{B/2}\psi_n$

$$-\psi_n'' + V(z)\psi_n = m_n^2 \psi_n, \qquad V(z) = \frac{1}{4}(B')^2 - \frac{1}{2}B''.$$

- With $\Phi = z^2$: $V = z^2 + \frac{3}{4z^2}$ — 2d harmonic oscillator (radial, $m = 1$).

$$m_n^2 = 4(n+1)$$

Deconfinement tempreature:

Hawking-Page transition in a cut-off AdS_5

E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998),
C. P. Herzog, Phys. Rev. Lett.98, 091601 (2007)

$$I = -\frac{1}{2\kappa^2}\int d^5x\sqrt{g}\left(R + \frac{12}{L^2}\right) \ .$$

$$\kappa \sim 1/N_c, \quad F_\pi^2 \sim N_c$$

Gravitational action:~N_c^2 , Meson action:~N_c

1. thermal AdS:

$$ds^2 = L^2\left(\frac{dt^2 + d\vec{x}^2 + dz^2}{z^2}\right)$$

β' : the periodicity in the timedirection, (undetermined)

2. AdS black hole: $f(z) = 1 - \frac{z^4}{z_h^4}$ $T = \frac{1}{\pi z_h}$

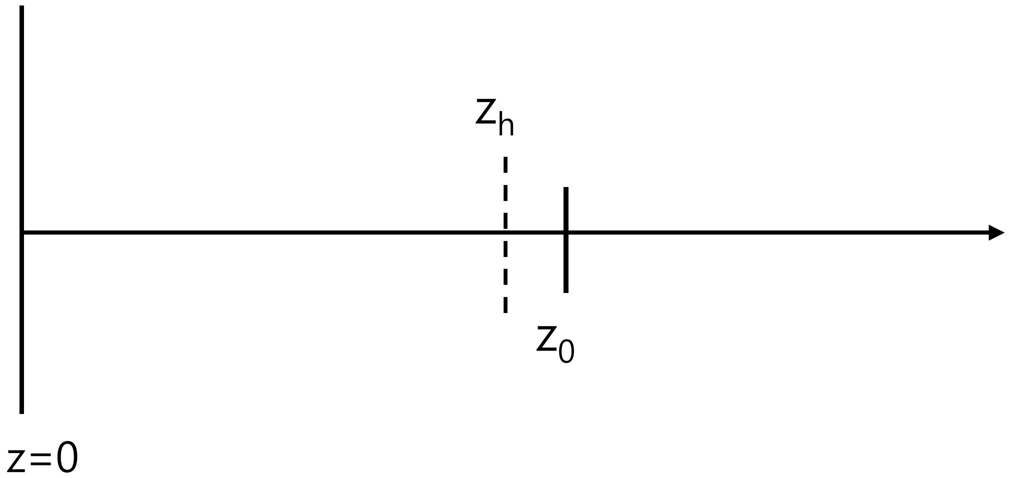
$$ds^2 = \frac{L^2}{z^2}\left(f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right) \qquad 0 \leq t < \pi z_h$$

Transition between two backgrounds ⟷ (De)confinement transition

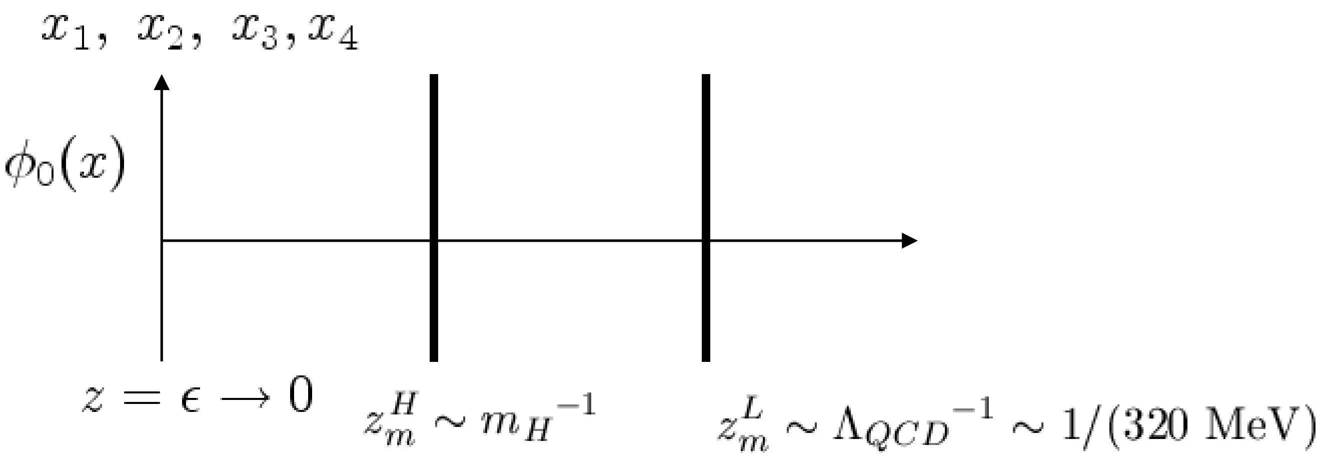
$$\beta' = \pi z_h \sqrt{f(\epsilon)}$$

$$\begin{aligned} \Delta V &= \lim_{\epsilon \longrightarrow 0} (V_2(\epsilon) - V_1(\epsilon)) \\ &= \begin{cases} \frac{L^3 \pi z_h}{\kappa^2} \frac{1}{2 z_h^4} & z_0 < z_h \\ \frac{L^3 \pi z_h}{\kappa^2} \left(\frac{1}{z_0^4} - \frac{1}{2 z_h^4} \right) & z_0 > z_h \end{cases} \end{aligned}$$

$$T_c = 2^{1/4}/(\pi z_0)$$



Heavy quarkonium in bottom-up

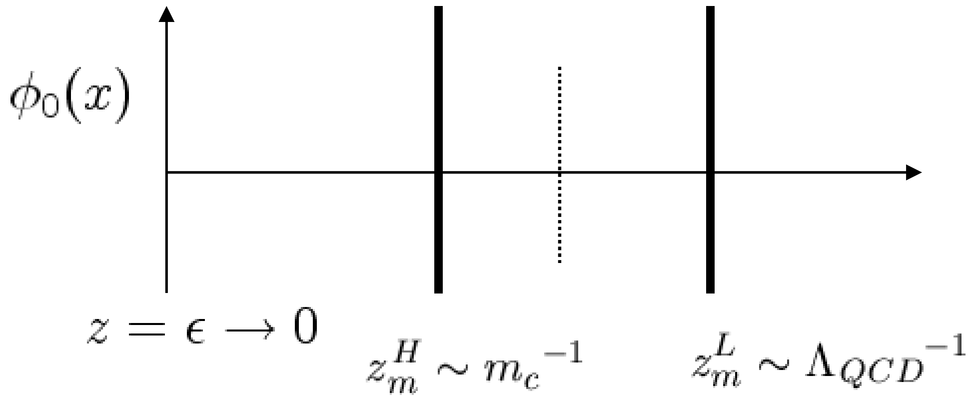


Soft wall model

$$m_n^2 = 4(n+1)c$$

where $\sqrt{c} \sim 1/z_m^H$. Again the lowest mode $(n=0)$ is used to fix c, $\sqrt{c} \simeq 1.55$ GeV. Then the mass of the second resonance is $m_1 \simeq 4.38$ GeV and the third one $m_3 \sim 5.36$ GeV.

Dissociation temperature



$$z_m^H < z_h < z_m^L$$

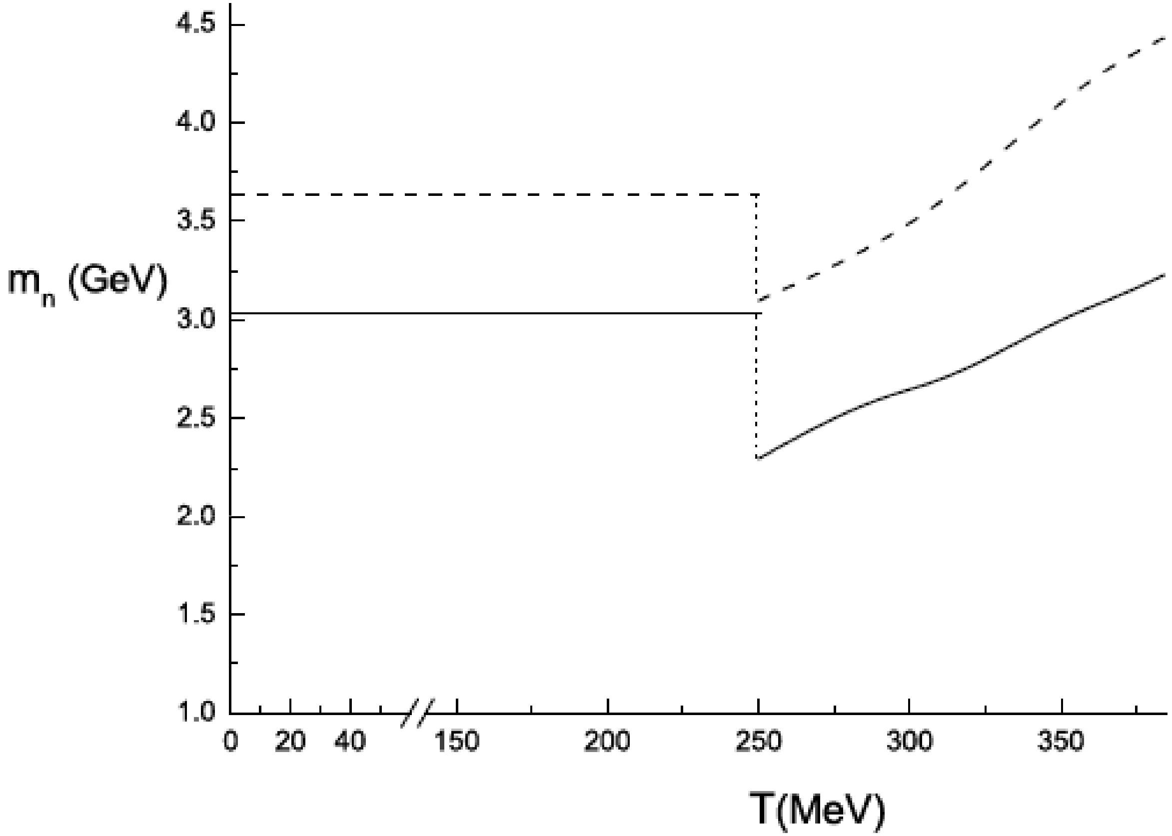
$$T_{\rm D} \simeq 1/(\pi z_m^H)$$

so the predicted dissociation temperature in the soft wall model is ~ 494 MeV.

The equation of motion for the vector field at finite temperature in the soft-wall model is given by

$$[\partial_z^2 - (2cz + \frac{4-3f}{zf})\partial_z + \frac{m^2}{f^2}]V_i = 0\,.$$

Prediction from the bottom-up AdS/QCD model



Deconfinement + temperature effects

YK, J.-P. Lee, and S. H. Lee, PRD (2007)

Holographic heavy quark potential

1. Gluon condensation and heavy quarkonium
   are both telling us about the non-perturbative nature of QGP.

2. Temperature dependence of gluon condensation is conveyed into
   the temperature dependence of heavy quarkonium in QCD sum rule
   [K. Morita and S. H. Lee, PRL (2008)]

3. So there should be a close relation between the two.

A deformed AdS due to the gluon condensate

- The dilaton couples to the gluon operator $\mathrm{tr}G^2$:

non-zero gluon condensate in QCD → the dilaton will have a non-trivial background.

A. Kehagias, K. Sfetsos, Phys.Lett. **B454**, 270 (1999), hep-th/9902125.

S. S. Gubser, hep-th/9902155.

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^5x\sqrt{g}\left(-\mathcal{R} + \frac{12}{R^2} + \frac{1}{2}g^{MN}\partial_M\phi\partial_N\phi\right).$$

Einstein equation and the dilaton EoM with the following Ansatz:

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$
$$\phi = \phi(y).$$

$$4A'^2 - A'' = 4R^2$$
$$A'^2 = \frac{\phi'^2}{24} + \frac{1}{R^2}$$
$$\phi'' = 4A'\phi'.$$

$$\begin{aligned} \mathrm{d}s^2 &= \left(\frac{R}{z}\right)^2 \left(\sqrt{1-\left(\frac{z}{z_c}\right)^8}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}z^2\right) \\ \phi(z) &= \sqrt{\frac{3}{2}}\log\left(\frac{1+\left(\frac{z}{z_c}\right)^4}{1-\left(\frac{z}{z_c}\right)^4}\right) + \phi_0. \end{aligned}$$

:dAdS

* Here, z_c is nothing but the gluon condensate via AdS/CFT:

Klebanov and Witten '99

For small z (d=4)

$$\phi(z, x) \to z^{d-\Delta}[\phi_0(x) + O(z^2)] + z^{\Delta}[A(x) + O(z^2)],$$

$$A(x) = \frac{1}{2\Delta - d}\langle\mathcal{O}(x)\rangle.$$

$$: \rightarrow \langle \mathrm{Tr} G^2 \rangle = 4\sqrt{3}\sqrt{\frac{R^3}{\kappa^2}}\frac{1}{z_c^4}$$

$$\frac{R^3}{\kappa^2} = \frac{4(N^2-1)}{\pi^2}.$$

$$\langle \mathrm{Tr} G^2 \rangle = \frac{8}{\pi z_c^4}\sqrt{3(N^2-1)}.$$

AdS black hole type solution

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{g}\left[\mathcal{R} + \frac{12}{R^2} - \frac{1}{2}\partial_M\phi\partial^M\phi\right].$$

Ansatz,

$$\begin{aligned} ds^2 &= \frac{R^2}{z^2}dz^2 + e^{2A(z)}(d\vec{x}^2 - e^{2B(z)}dt^2) \\ \phi &= \phi(z). \end{aligned}$$

S. Nojiri and S. D. Odintsov, Phys. Rev. D **61**, 024027 (2000)

YK, B.-H. Lee, C. Park, and S.-J. Sin, JHEP (2007)

dBH

$$ds^2 = \frac{R^2}{z^2}\left[dz^2 + \left(1 - \frac{f^2 z^8}{R^8}\right)^{1/2}\left(\frac{1+\frac{fz^4}{R^4}}{1-\frac{fz^4}{R^4}}\right)^{a/2f}\left(d\vec{x}^2 - \left(\frac{1-\frac{fz^4}{R^4}}{1+\frac{fz^4}{R^4}}\right)^{2a/f} dt^2\right)\right].$$

$$\phi(z) \quad = \quad \phi_0 + \frac{c}{2f}\log\left(\frac{1+f\frac{z^4}{R^4}}{1-f\frac{z^4}{R^4}}\right)$$

$$f^2 = a^2 + \frac{c^2}{6}\,.$$

For $a = 0,\ f = \sqrt{c^2/6} = z_c^{-4},$ → dilaton solution

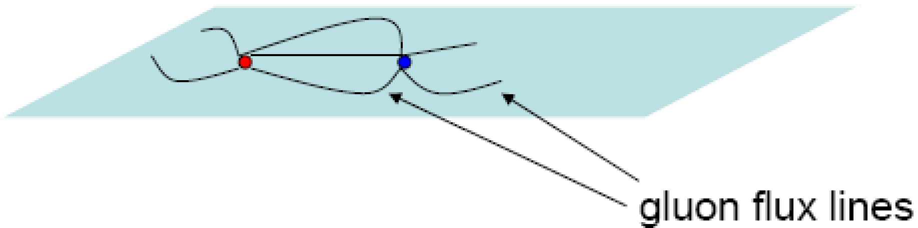
For $c = 0,\ f = a,$ → AdS-Schwarzschild Black hole.

HQ potential in the deformed AdS

Let's see how the gluon condensate affects the HQp.

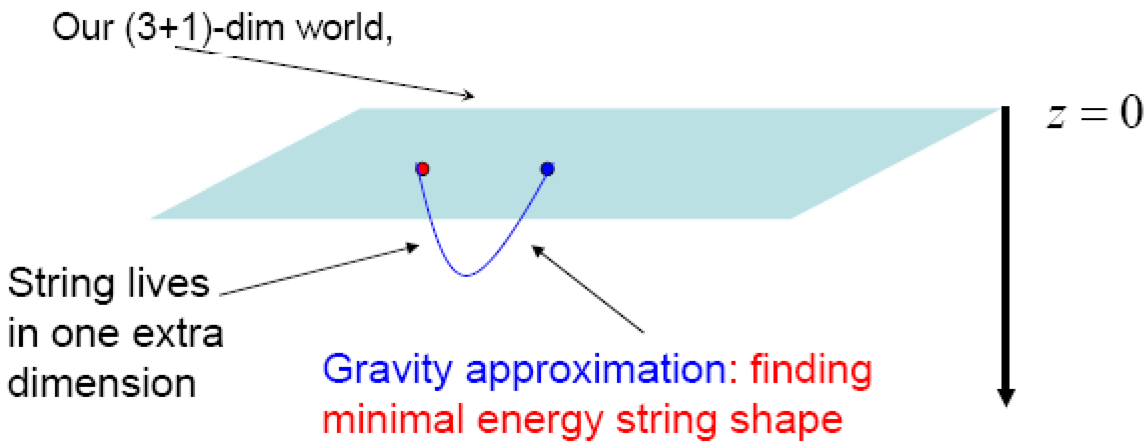
Static quark potential

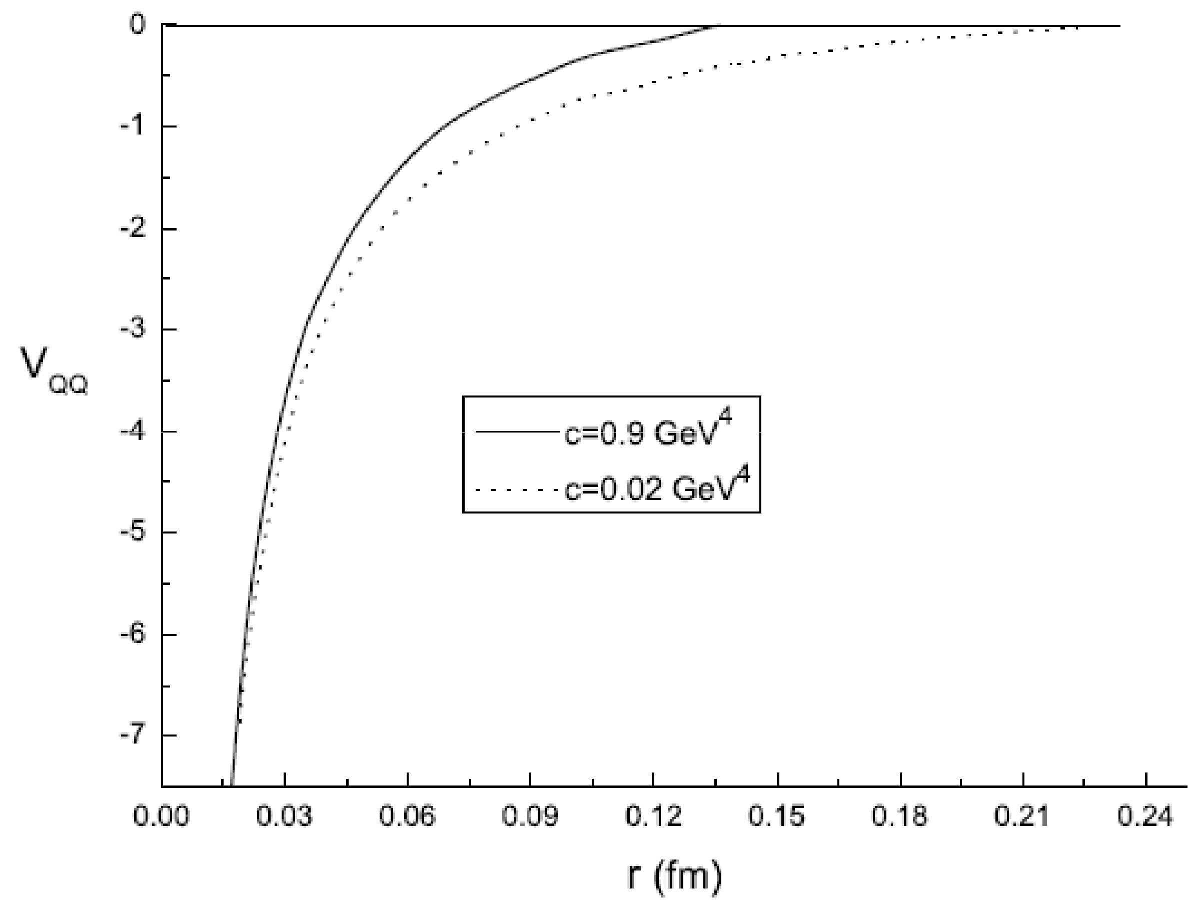
Gauge theory description:



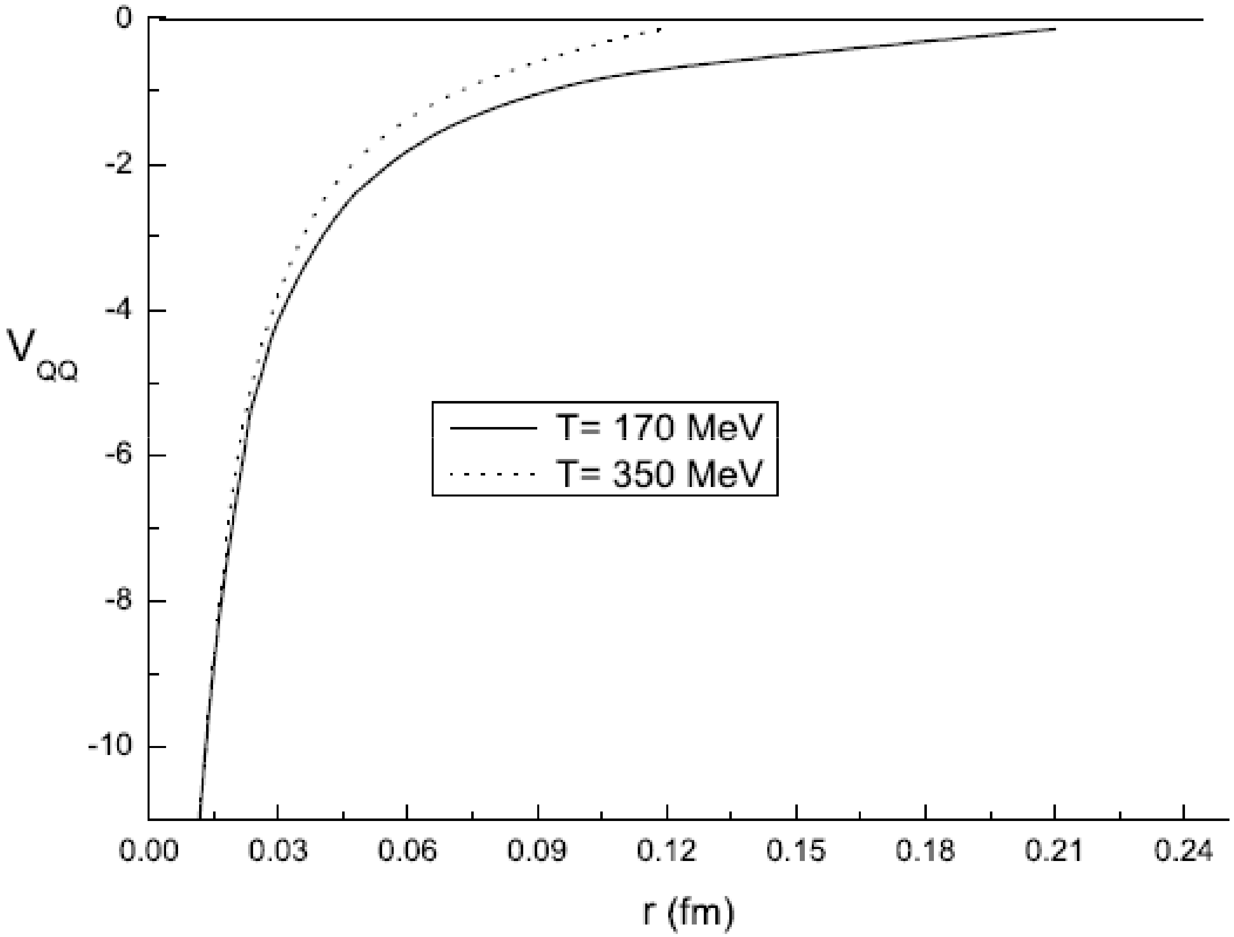
String theory description:

Malddacena; Rey, Yee





0
-1
-2
-3
-4
-5
-6
-7
V_QQ
c=0.9 GeV^4
c=0.02 GeV^4
0.00
0.03
0.06
0.09
0.12
0.15
0.18
0.21
0.24
r (fm)



0
-2
-4
-6
-8
-10
V_{QQ}
T= 170 MeV
T= 350 MeV
0.00
0.03
0.06
0.09
0.12
0.15
0.18
0.21
0.24
r (fm)

Summary

- A prediction from the AdS/QCD model and the holographic potential study: the mass of heavy quarkonum drops at and/or very near Tc, but is increases afterwards with increasing temperature.
- Stringy set-up? D3/D7, etc