

Fluctuation and HBT results

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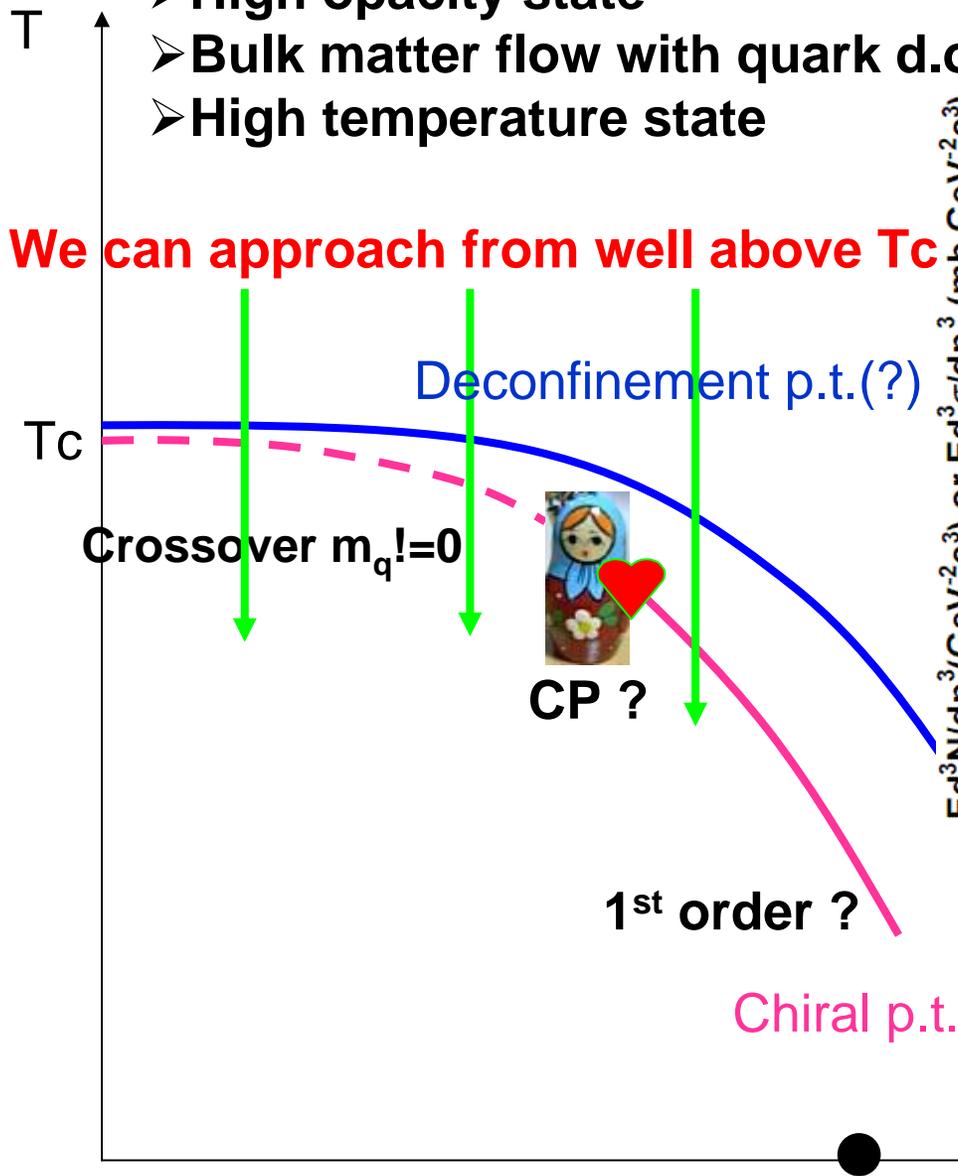
- 1. Approach and probes for critical phenomena relevant to QCD phase transitions**
- 2. HBT**
- 3. Two particle correlation via differential analysis on multiplicity fluctuations**
- 4. Summary**

@ATHIC08 on 15 Oct, 2008 in Tsukuba Univ., Japan

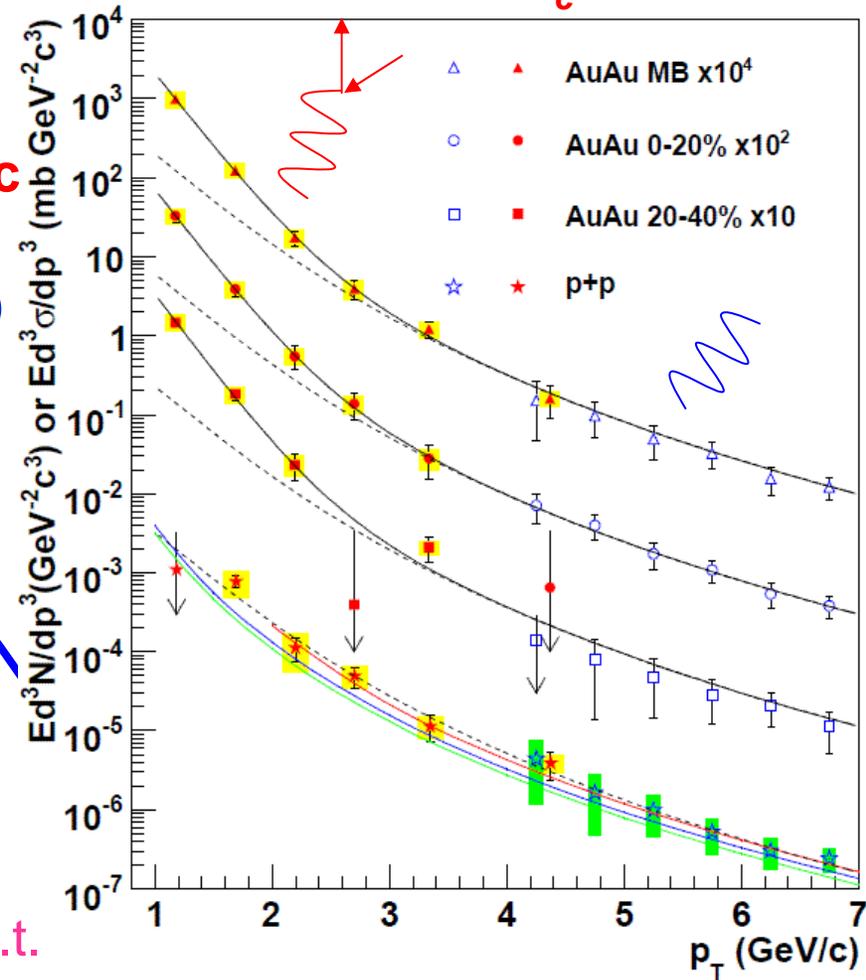
Approach to QCD phase diagram at RHIC

- High opacity state
- Bulk matter flow with quark d.o.f
- High temperature state

We can approach from well above T_c



$T=221 \pm 23(stat) \pm 18(sys)$
 Lattice result $T_c \sim 170 MeV$



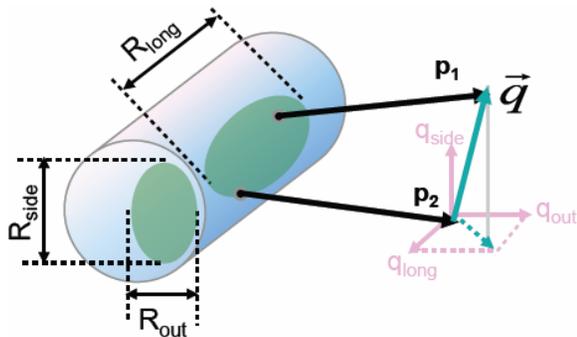
arXiv:0804.4168v1 [nucl-ex] 25 Apr 2008

Experimental approaches to critical phenomena

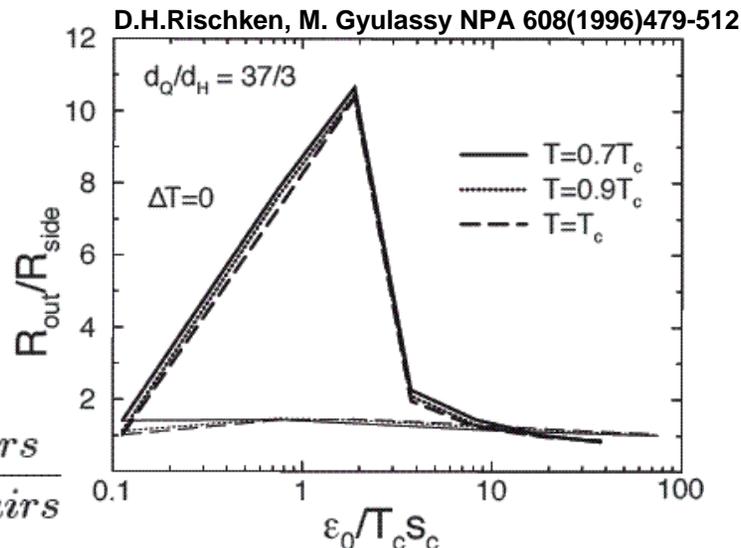
- $\langle qq\text{-bar} \rangle$:
- J/ψ suppression (deconfinement)
- low mass vector mesons and dilepton continuum (chiral)

- Bulk collective observables:
- Isothermal compressibility: Multiplicity fluctuations
- Heat capacity: Mean p_T fluctuations
- Particle ratio and fluctuations on particle compositions
- Duration time of particle emission: HBT
- Correlation length and the strength:
density-density correlation in longitudinal space
- Sound velocity via eccentricity scaling of v_2
- Viscosity to entropy ratio with v_2 and R_{AA}

HBT as a probe of 1st order P.T.

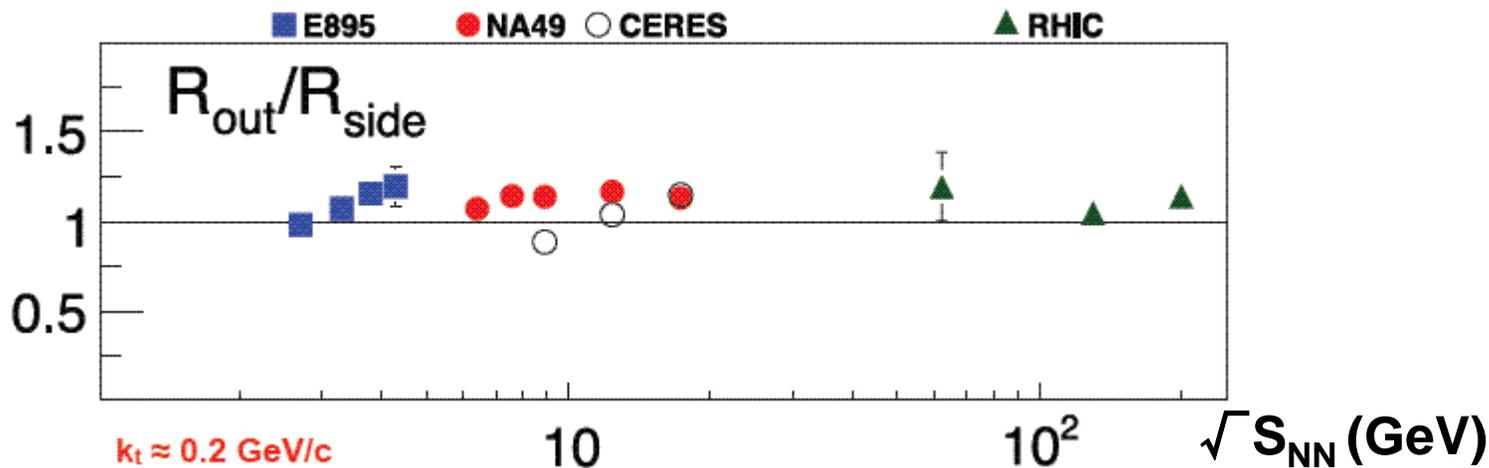


$$C_2(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} = \frac{\text{real event pairs}}{\text{mixed event pairs}}$$



$$\frac{s}{s_c}(T) = \left[\frac{T}{T_c} \right]^3 \left(1 + \frac{d_Q - d_H}{d_Q + d_H} \tanh \left[\frac{T - T_c}{\Delta T} \right] \right),$$

where $s_c = \text{const.} \times \frac{1}{2} (d_Q + d_H) T_c^3$ is the entropy density at T_c .



3-D imaging of source shape

Correlation moment

$$R(\mathbf{q}) = \sum_{l, \alpha_1, \dots, \alpha_l} R_{\alpha_1, \dots, \alpha_l}^l(\mathbf{q}) A_{\alpha_1, \dots, \alpha_l}^l(\Omega_{\mathbf{q}})$$

Encoding
FS interaction

Source function

$$S(\mathbf{r}) = \sum_l \sum_{\alpha_1, \dots, \alpha_l} S_{\alpha_1, \dots, \alpha_l}^l(\mathbf{r}) A_{\alpha_1, \dots, \alpha_l}^l(\Omega_{\mathbf{r}})$$

Cartesian surface-spherical
harmonic basis

$$A_{\alpha_1 \dots \alpha_l}^l$$

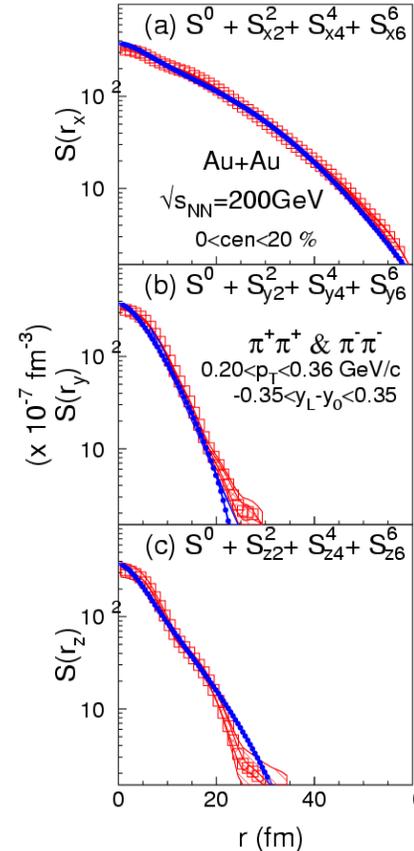
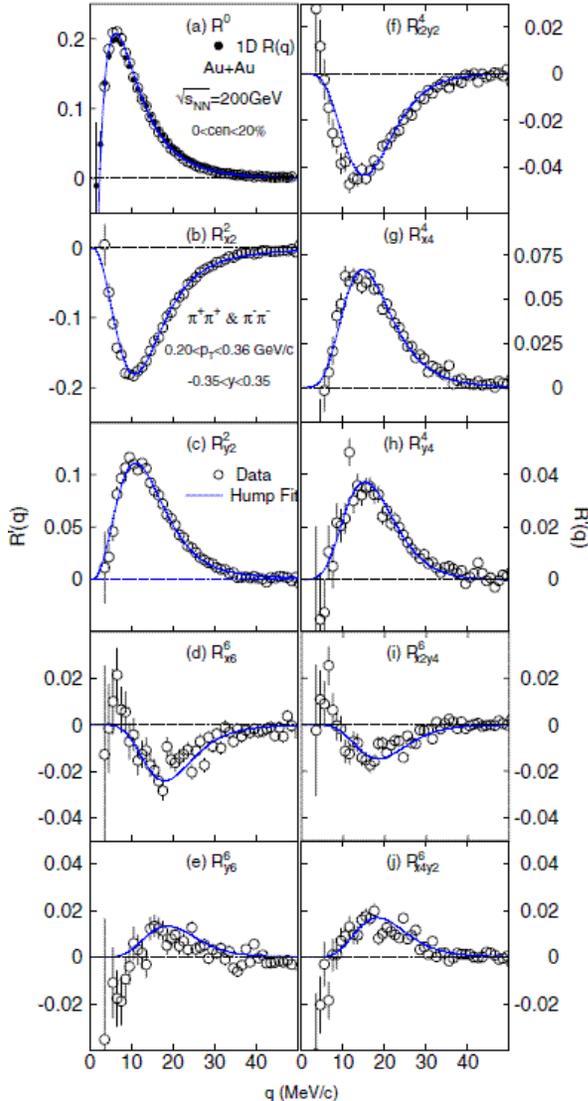
l : index from 0 to 6 here
 α : spatial index

$$R_{\alpha_1, \dots, \alpha_l}^l(\mathbf{q}) = 4\pi \int dr r^2 K_l(\mathbf{q}, \mathbf{r}) S_{\alpha_1, \dots, \alpha_l}^l(\mathbf{r})$$



Inversion from
 $R^l(\mathbf{q})$ to $S^l(\mathbf{r})$

No assumption on
source shape

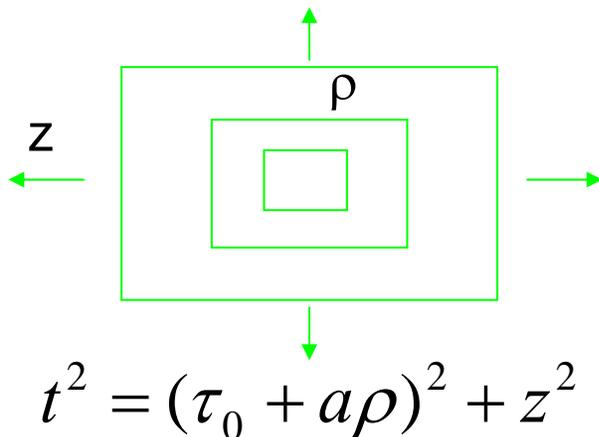


In PCMS

3-D imaging results

Therminator Model

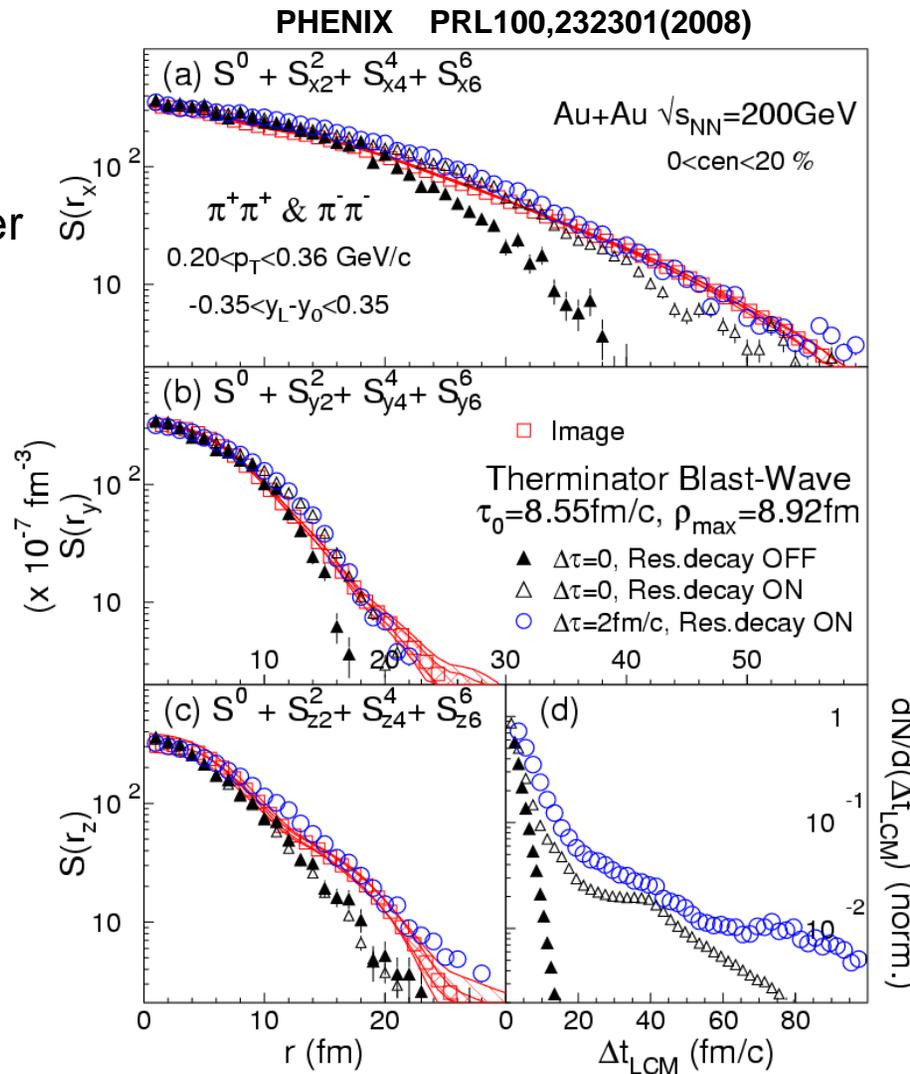
- Bjorken type longitudinal boost invariance
- Blast-wave transverse expansion
- Thermal emission from a longitudinal cylinder



τ_0 is proper breakup time at $\rho=0$

$\tau = \tau_0 + a\rho$ is proper breakup time at ρ
with $a=-0.5$ (burn outside in)

τ is replaced by τ' with probability of
 $dN/d\tau' = \theta(\tau' - \tau) / \Delta\tau * \exp[-(\tau' - \tau) / \Delta\tau]$

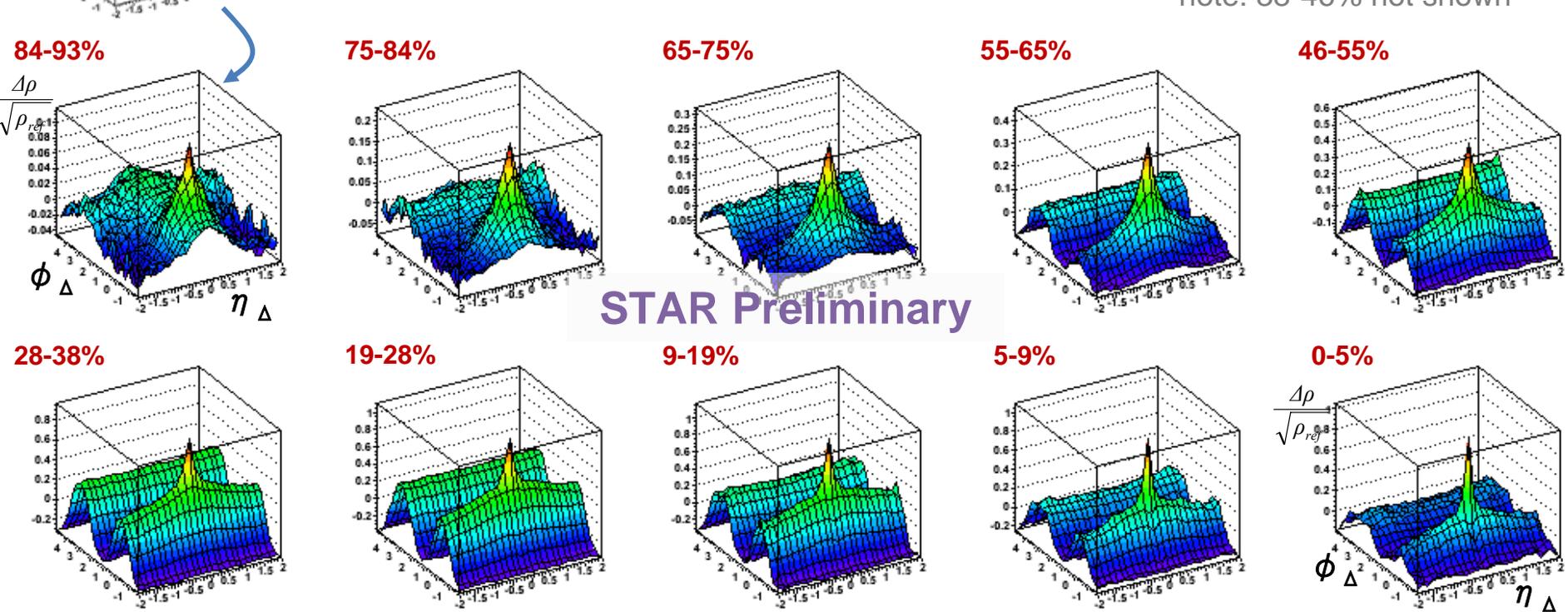


Mean proper emission time
 $\Delta\tau = 2 \text{ fm}/c$ is small but finite !
 $(\tau_0 \sim 9 \text{ fm}/c \quad \langle |\Delta t_{\text{LCM}}| \rangle \sim 12 \text{ fm}/c)$

Low pT two particle correlation in STAR

Analyzed 1.2M minbias 200 GeV Au+Au events, and 13M 62 GeV minbias events (not shown) Included all tracks with $p_T > 0.15$ GeV/c, $|\eta| < 1$, full ϕ

note: 38-46% not shown



STAR Preliminary

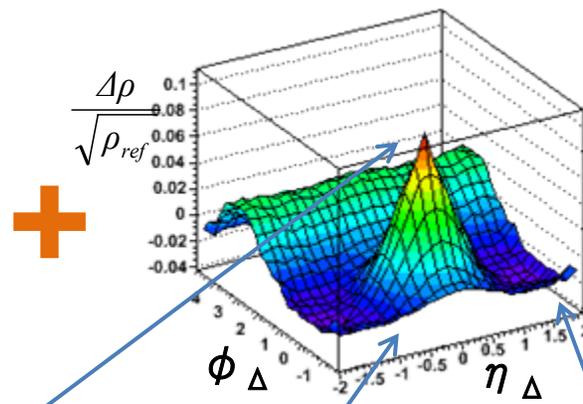
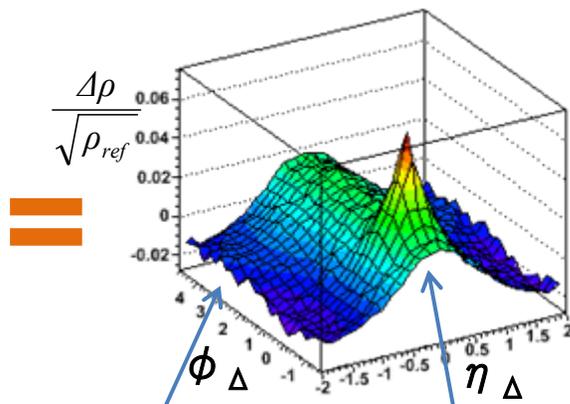
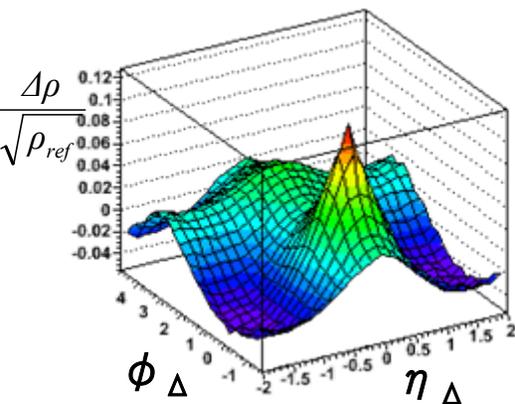
We see the evolution of correlation structures from peripheral to central Au+Au

Slide from M. Daugherty, STAR Collaboration presented at QM08

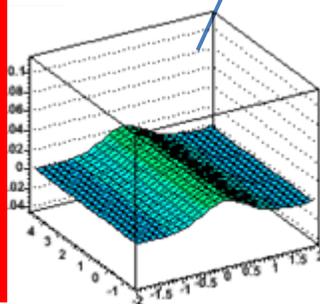
Fit Function (in 5 Easy Pieces)

Proton-Proton fit function

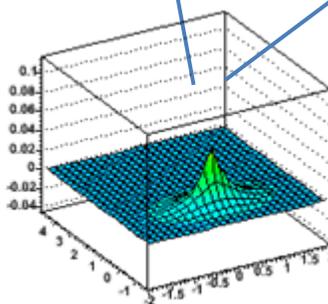
STAR Preliminary



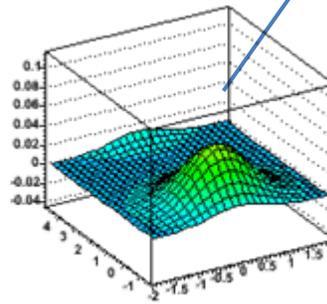
Slide from
M. Daugherty, STAR
Collaboration
presented at QM08



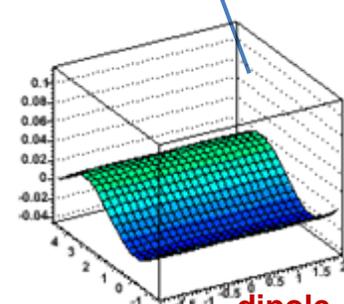
longitudinal fragmentation
1D gaussian



HBT, res., e+e-
2D exponential



Minijet Peak
2D gaussian



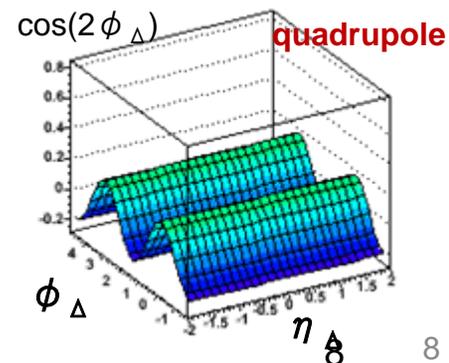
Away-side
-cos(φ)

dipole

Au-Au fit function

Use proton-proton fit function + $\cos(2\phi_\Delta)$ quadrupole term ("flow").
This gives the **simplest possible** way to describe Au+Au data.

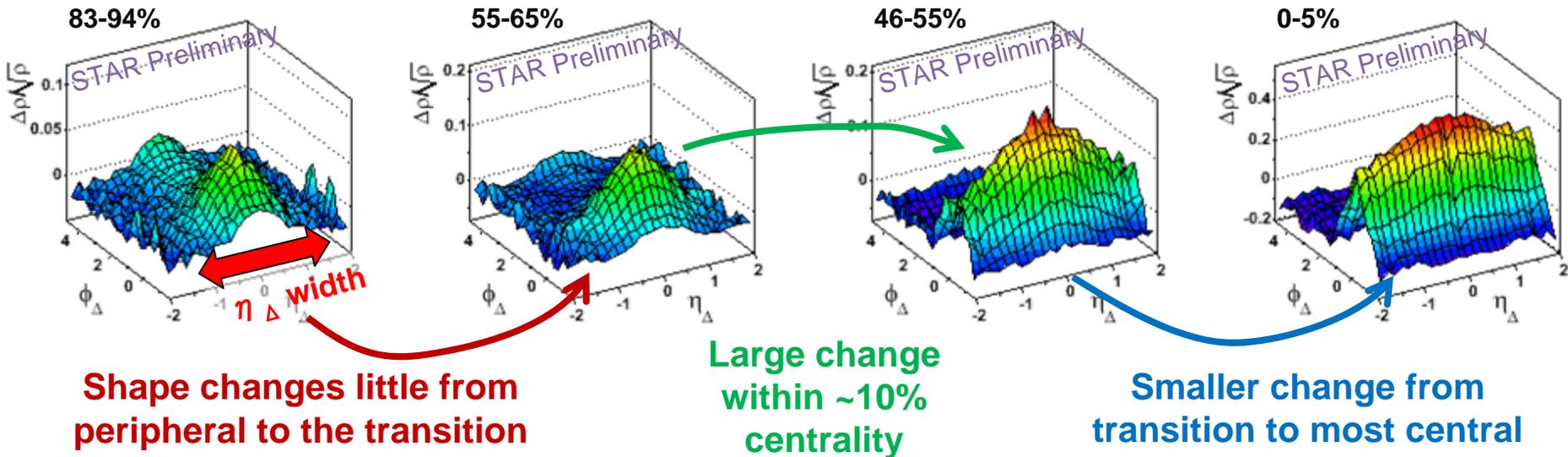
Note: from this point on we'll include entire momentum range
instead of using soft/hard cuts



Transition

Does the transition from narrow to broad η_{Δ} occur quickly or slowly?

data - fit (except same-side peak)



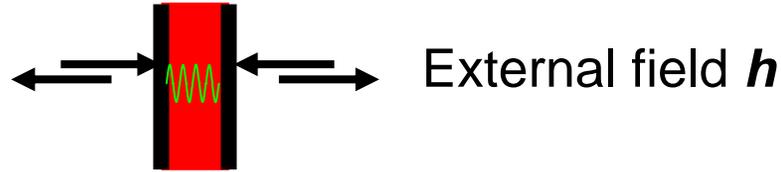
Low- p_T manifestation of the “ridge”

The transition occurs quickly

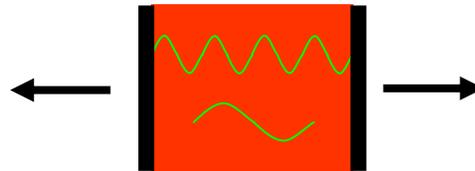
Slide from M. Daugherty, STAR Collaboration presented at QM08

A picture of expanding medium in early stage

Initial stage

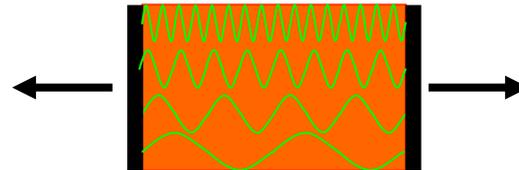


$T > T_c$



Longitudinal field
density fluctuations from
the mean density is a
natural order parameter

$T = T_c$

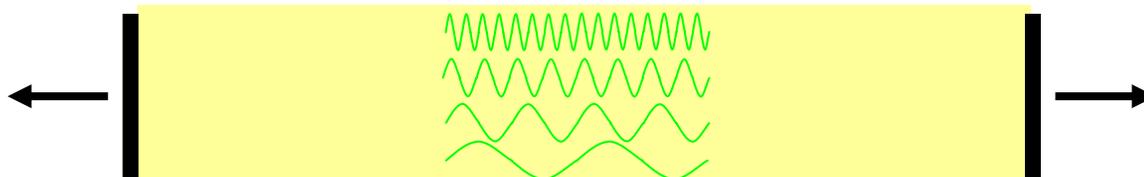


$$\phi(z) = \rho(z) - \langle \rho \rangle$$



Many modes from small to large appear at $T = T_c$.
A typical length scale disappears i.e. transition to power law

$T < T_c$



We may expect freeze of initially embedded fluctuation
due to rapid dilution of medium in the longitudinal direction

Direct observable for Tc determination

GL free energy density g with $\phi \sim 0$ from high temperature side is insensitive to transition order, but it can be sensitive to Tc

$$g(T, \phi, h) = g_0 - \frac{1}{2} A(T) (\nabla \phi)^2 + \frac{1}{2} a(T) \phi^2 + \frac{1}{4} b \phi^4 + \frac{1}{6} c \phi^6 \dots - h \phi$$

spatial correlation ϕ disappears at Tc $\rightarrow a(T) = a_0(T - T_c)$

Fourier analysis on

$$G_2(\mathbf{y}) = \langle \phi(\mathbf{0}) \phi(\mathbf{y}) \rangle$$

$$\langle |\phi_k|^2 \rangle = Y \int G_2(\mathbf{y}) e^{-i\mathbf{k} \cdot \mathbf{y}} d\mathbf{y}$$

$$\langle |\phi_k|^2 \rangle = \frac{NT}{Y} \frac{1}{a(T) + A(T)k^2}$$

Susceptibility

$$\chi_k = \frac{\partial \phi_k}{\partial h} \propto \left(\frac{\partial^2 (g - g_0)}{\partial \phi_k^2} \right)^{-1} = \frac{1}{a_0(T - T_c)(1 + k^2 \xi^2)}$$

Susceptibility in long wavelength limit

$$\chi_{k=0} = \frac{1}{a_0(T - T_c)} \propto \frac{\xi}{T} G_2(0)$$

1-D two point correlation function

$$G_2(y) = \frac{NT}{2Y^2 A(T)} \xi(T) e^{-|y|/\xi(T)}$$

Correlation length

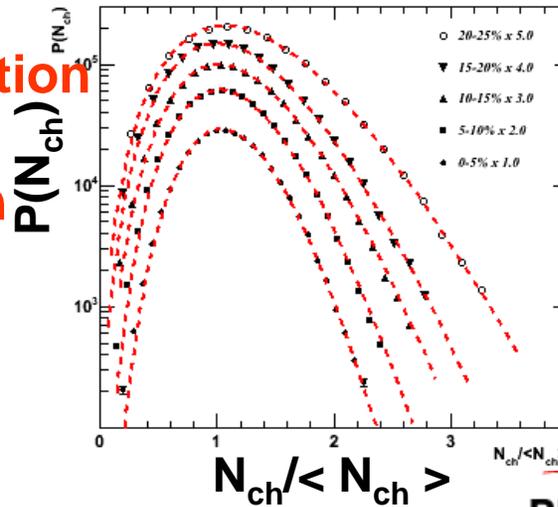
$$\xi(T)^2 \equiv \frac{A(T)}{a_0(T - T_c)}$$

Product between correlation length and amplitude can also be a good indicator for T~Tc

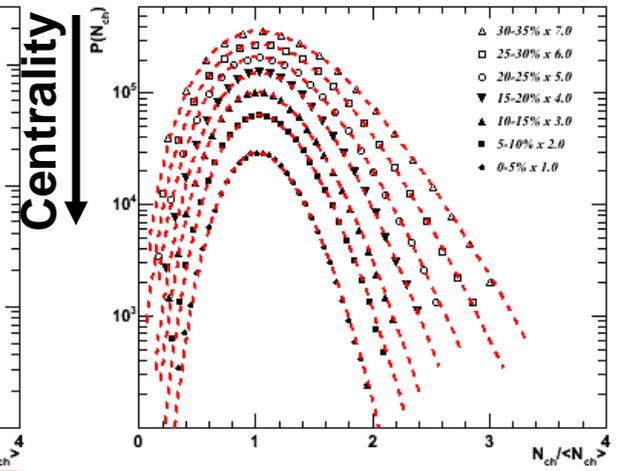
Density measurement : $dN_{ch}/d\eta$

Negative Binomial Distribution (NBD) perfectly describes multiplicities in all collision systems and centralities at RHIC.

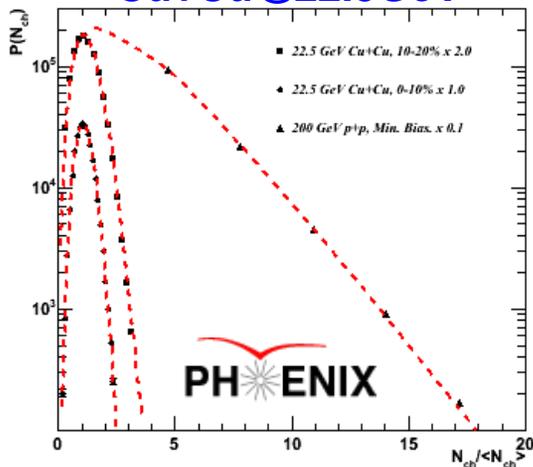
Cu+Cu@62.4GeV



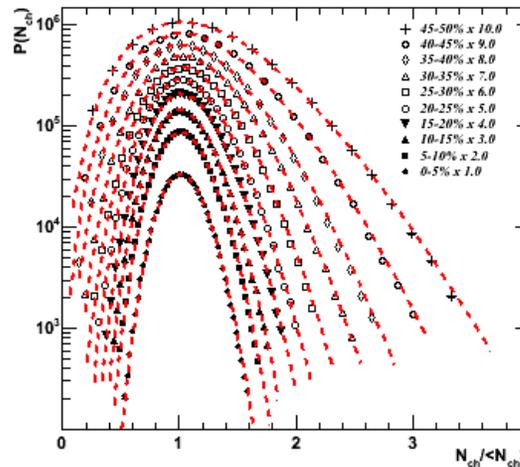
Cu+Cu@200GeV



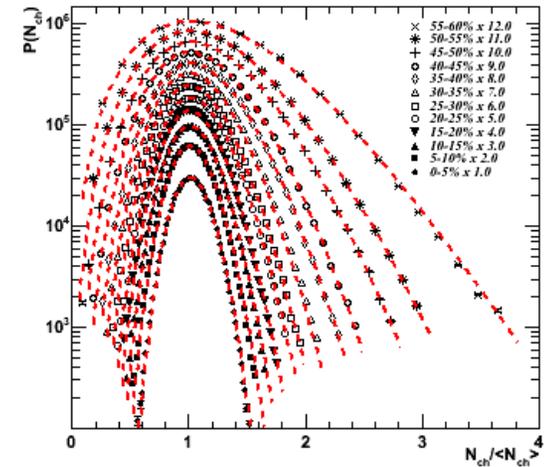
**p+p@200GeV
Cu+Cu@22.5GeV**



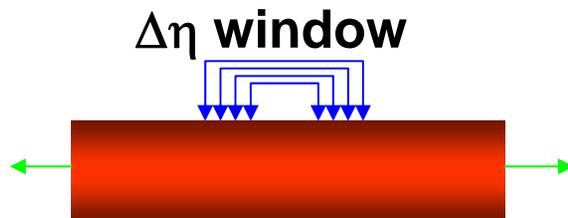
Au+Au@62.4GeV



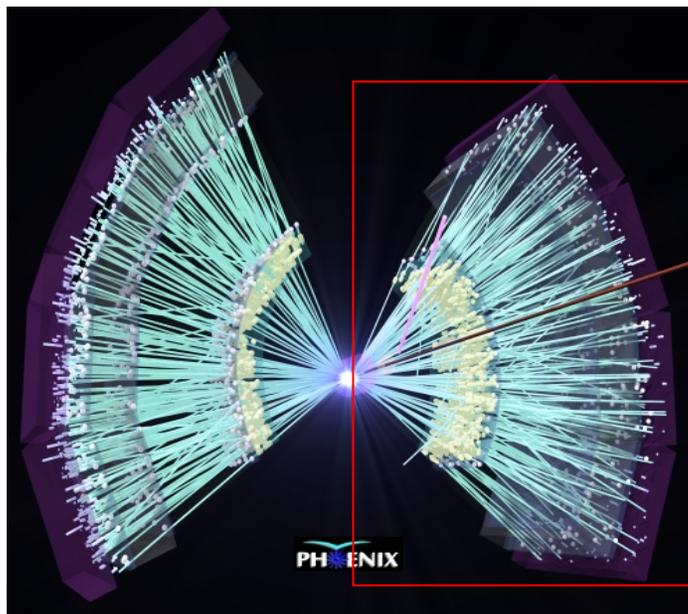
Au+Au@200GeV



Differential multiplicity fluctuations



$\Delta\eta < 0.7$ integrated over $\Delta\phi < \pi/2$ and $p_T > 0.1 \text{ GeV}$



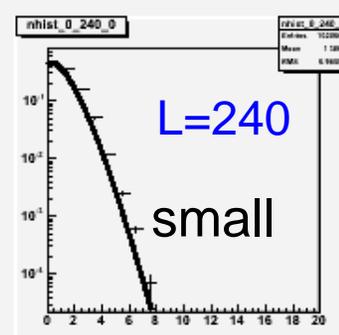
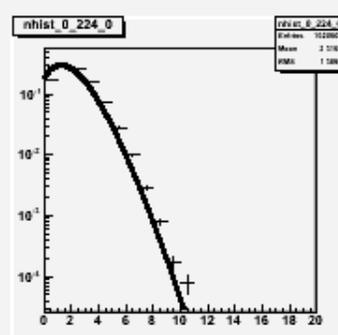
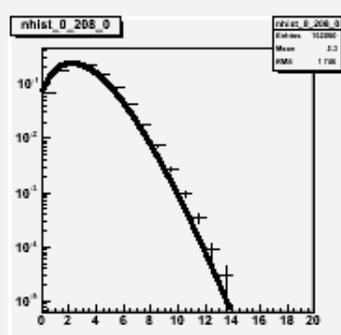
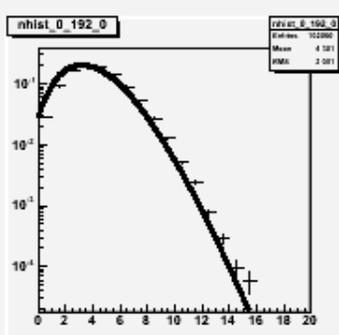
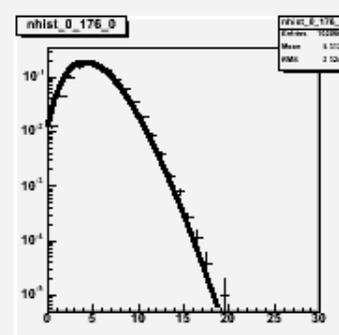
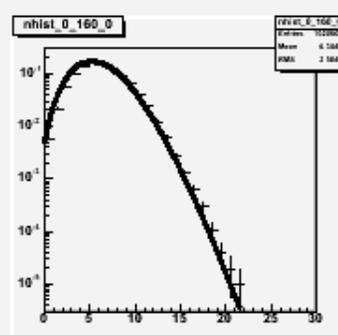
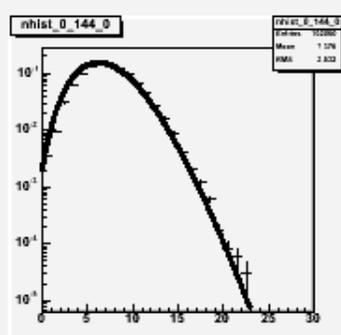
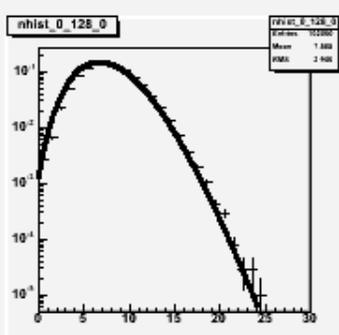
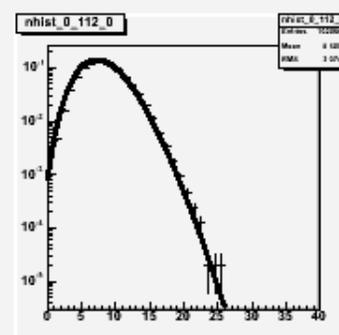
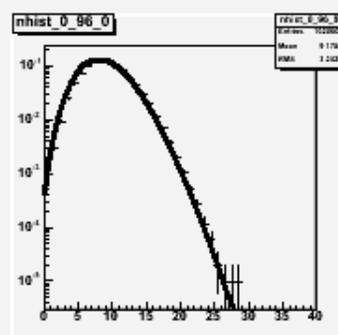
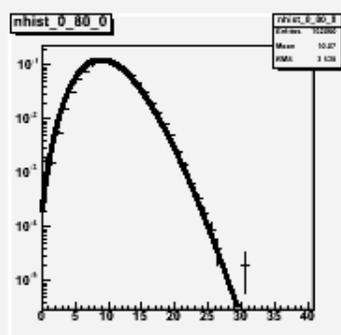
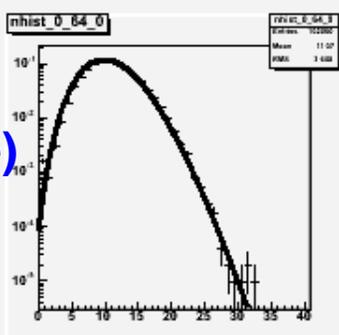
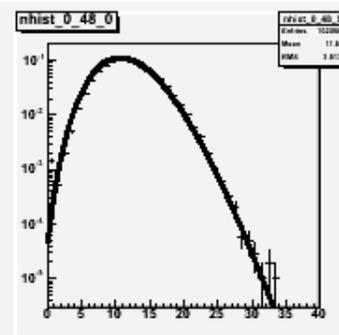
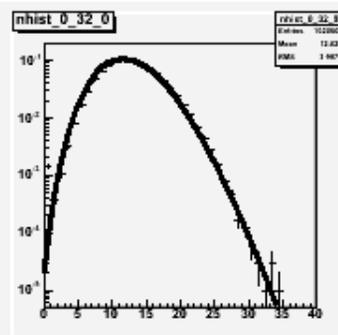
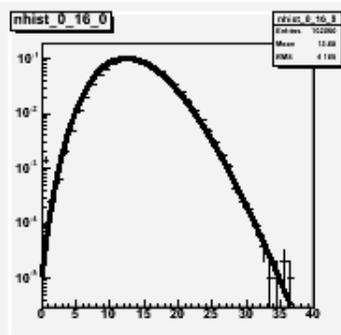
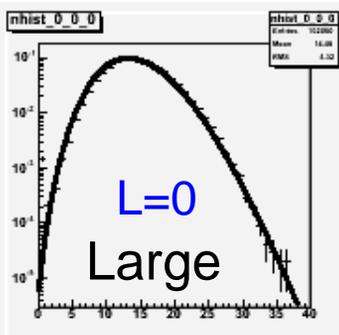
**Zero magnetic field to
enhance low p_T statistics
per collision event.**

NBD fits at each window size in

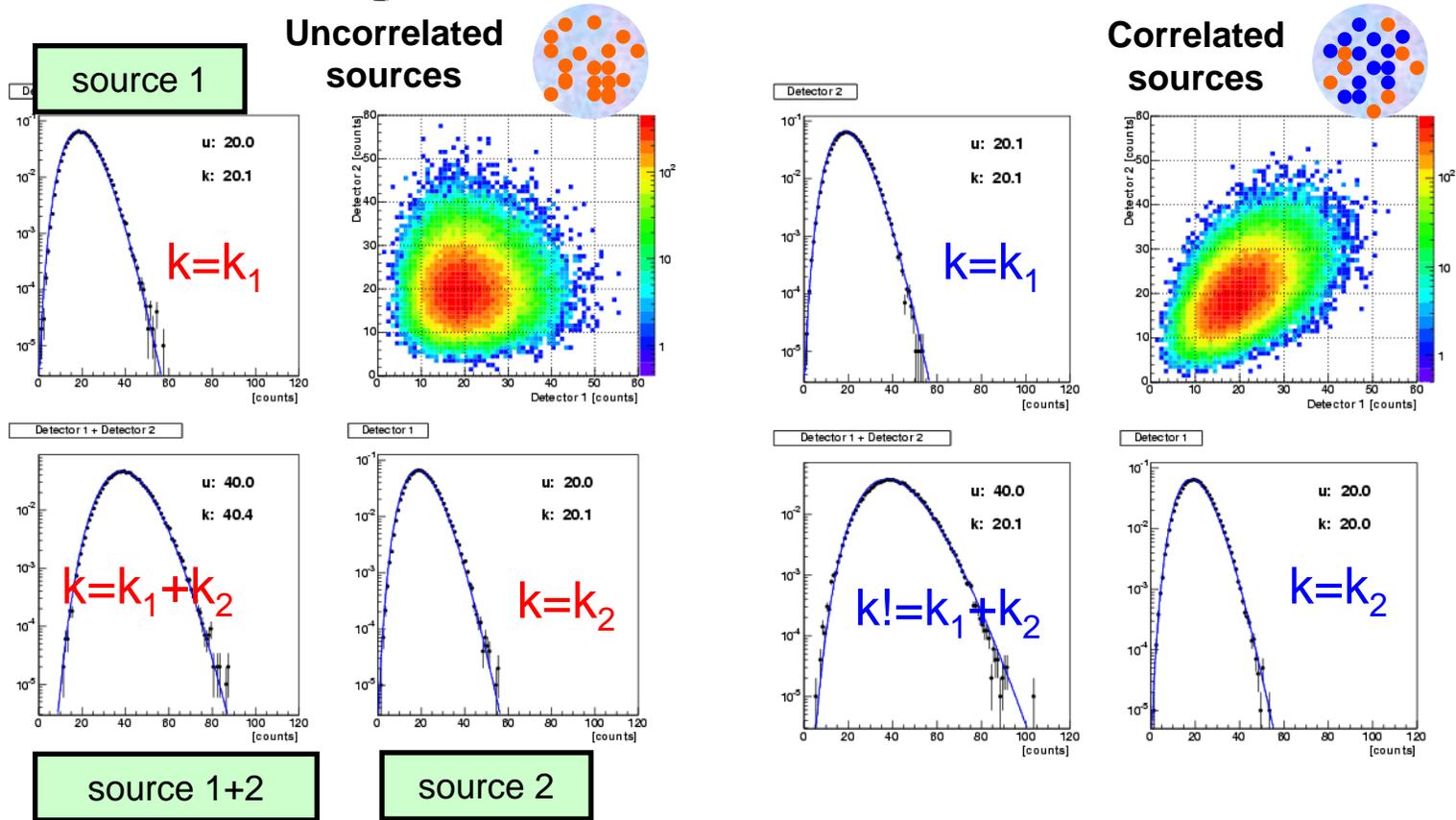
CuCu@200

Level (window size)
 $L=2^8(1-\delta\eta/\Delta\eta_{\text{PHENIX}})$

16 fit examples in
most left edge in
top 10% events
out of $2^8/2*(1+2^8)$
times NBD fits



Two point correlation via NBD



NBD $P_n^{(k)} = \frac{\Gamma(n+k)}{\Gamma(n-1)\Gamma(k)} \left(\frac{\mu/k}{1+\mu/k} \right)^n \frac{1}{(1+\mu/k)^k}$

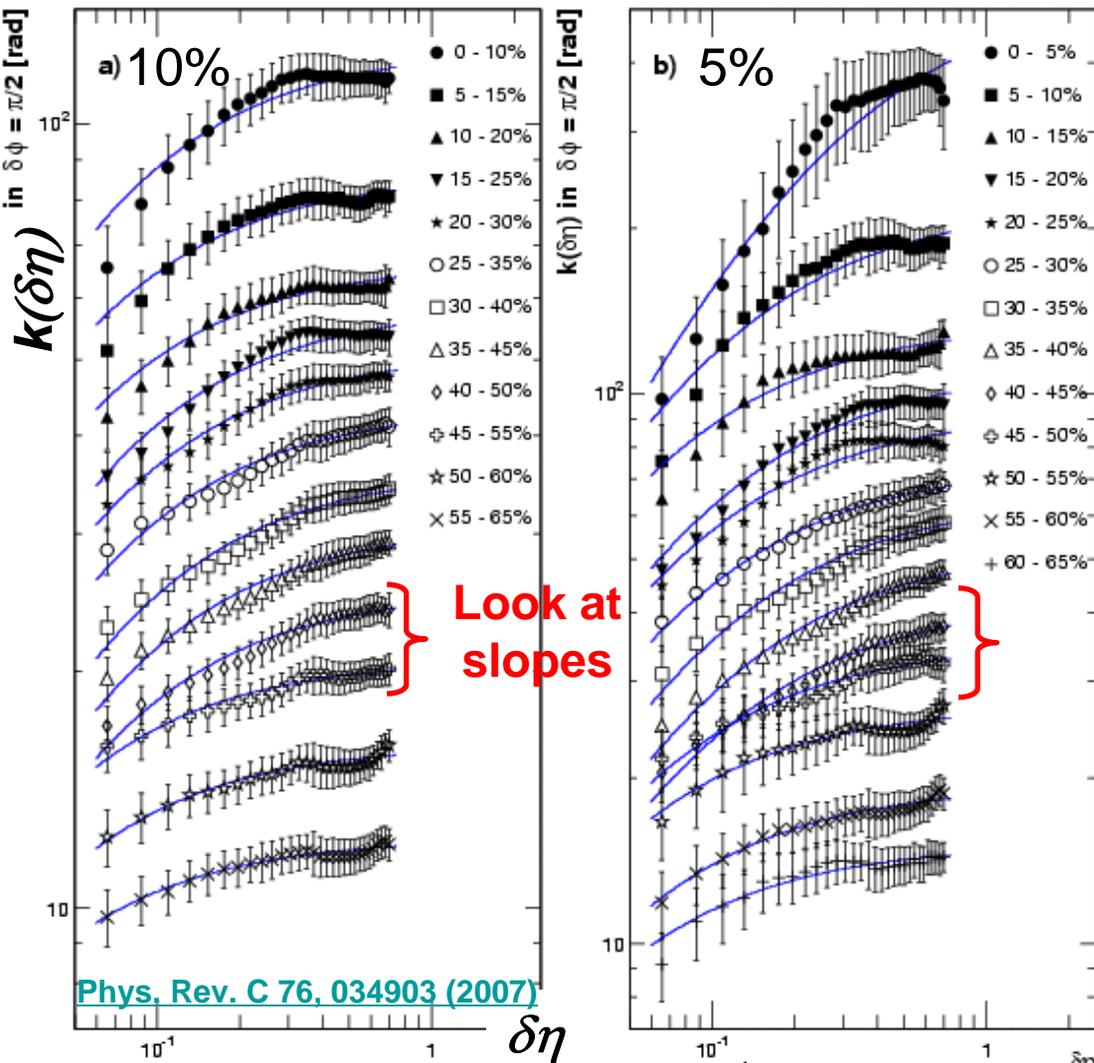
$k=1$ Bose-Einstein
 $k=\infty$ Poisson

$$\frac{\sigma^2}{\mu^2} = \frac{1}{\mu} + \frac{1}{k} \quad \mu \equiv \langle n \rangle$$

1/k corresponds to integral of two point correlation

Extraction of $\chi_{k=0}^* \mathbf{T}$

Fit with approximated functional form



Phys. Rev. C 76, 034903 (2007)

Approximated functional form

$$k(\delta\eta) = \frac{1}{2\alpha\xi/\delta\eta + \beta} \quad (\xi \ll \delta\eta)$$

Parametrization of two particle correlation

$$C_2(\eta_1, \eta_2) \equiv \rho_2(\eta_1, \eta_2) - \rho_1(\eta_1)\rho_1(\eta_2)$$

$$\frac{C_2(\eta_1, \eta_2)}{\bar{\rho}_1^2} = \alpha e^{-\delta\eta/\xi} + \beta$$

β absorbs rapidity independent bias: Npart fluctuation and reaction plane rotation and v2

Exact relation with NBD k

$$\begin{aligned} k^{-1}(\delta\eta) &= \frac{\langle n(n-1) \rangle}{\langle n \rangle^2} - 1 \\ &= \frac{\int_0^{\delta\eta} \int_0^{\delta\eta} C_2(\eta_1, \eta_2) d\eta_1 d\eta_2}{\delta\eta^2 \bar{\rho}_1^2} \\ &= \frac{2\alpha\xi^2 (\delta\eta/\xi - 1 + e^{-\delta\eta/\xi})}{\delta\eta^2} + \beta \end{aligned}$$

Correlation functions and correlation length

Used in E802

$$C_2 = 1 + R(0,0)e^{-|y_1 - y_2|/\xi}$$

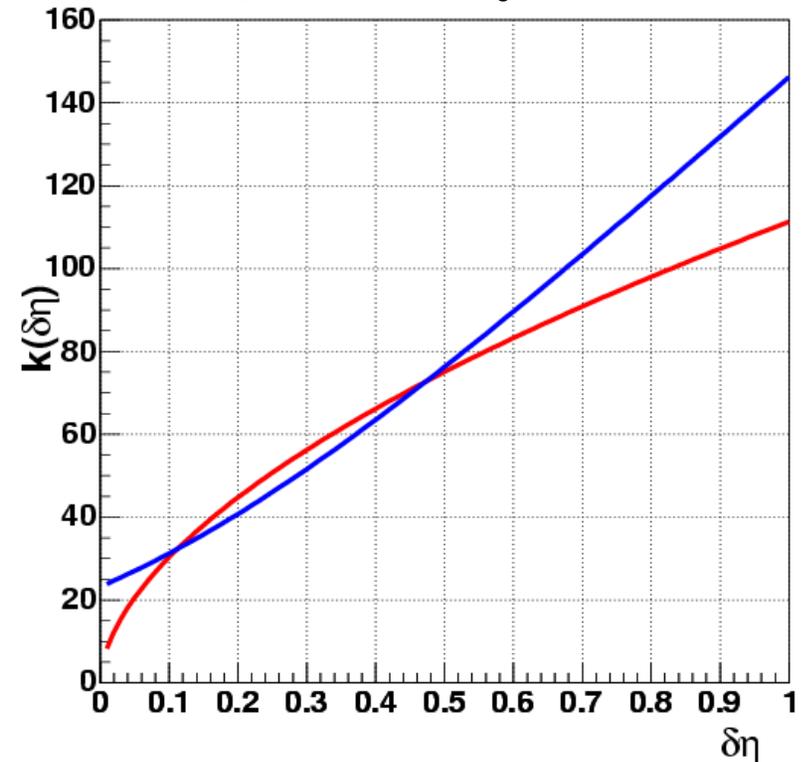
$$k(\delta\eta) = \frac{1}{R_0} \frac{\delta\eta / 2\xi}{[1 - (\xi / \delta\eta)(1 - e^{-\delta\eta/\xi})]}$$

General correlation function

$$C_2 = 1 + \frac{R_0}{|y_1 - y_2|^\alpha} e^{-|y_1 - y_2|/\xi}$$

$$k(\delta\eta) = \frac{\delta\eta}{\int_0^{\delta\eta} \frac{R_0}{y^\alpha} e^{-y/\xi} dy}$$

Using arbitrary R_0 , ξ and α .

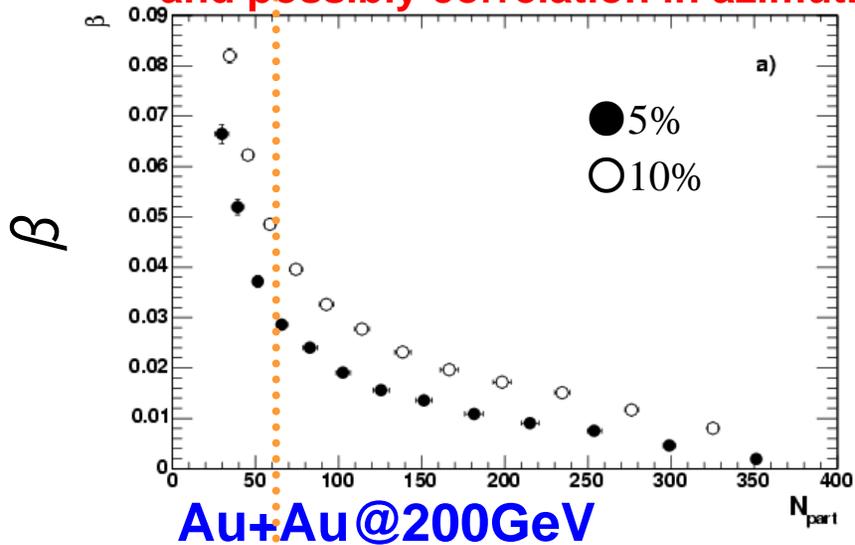


One may discuss an effective potential form

ξ : correlation length, α : critical exponent

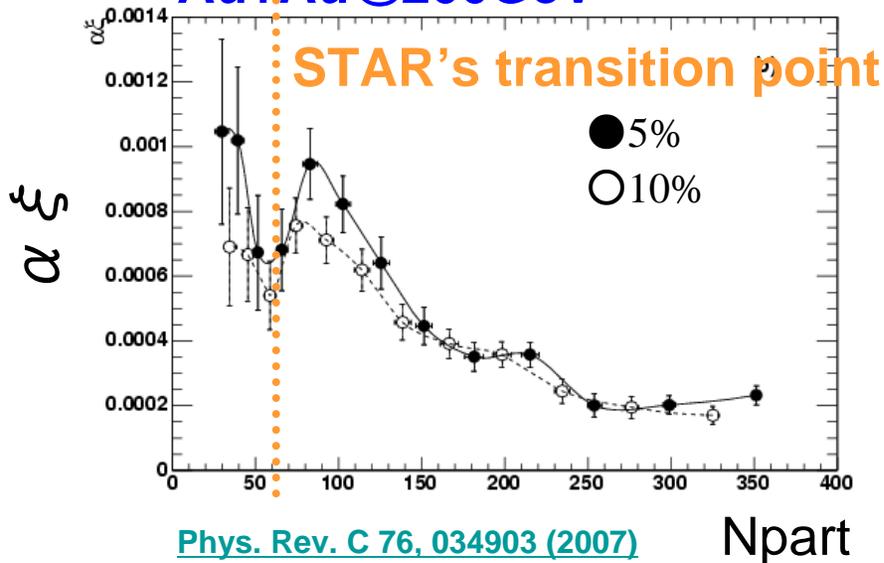
$\alpha \xi, \beta$ vs. N_{part}

Dominantly N_{part} fluctuations and possibly correlation in azimuth



β is systematically shift to lower values as the centrality bin width becomes smaller from 10% to 5%. This is understood as fluctuations of N_{part} for given bin widths

$\alpha \xi$ product, which is monotonically related with $\chi_{k=0}$ indicates the non-monotonic behavior around $N_{part} \sim 90$.



$$\alpha \xi = \chi_{k=0} T / \bar{\rho}_1^2 \propto \bar{\rho}_1^{-2} \frac{T}{|T - T_C|}$$

Significance with Power + Gaussian:
3.98 σ (5%), 3.21 σ (10%)

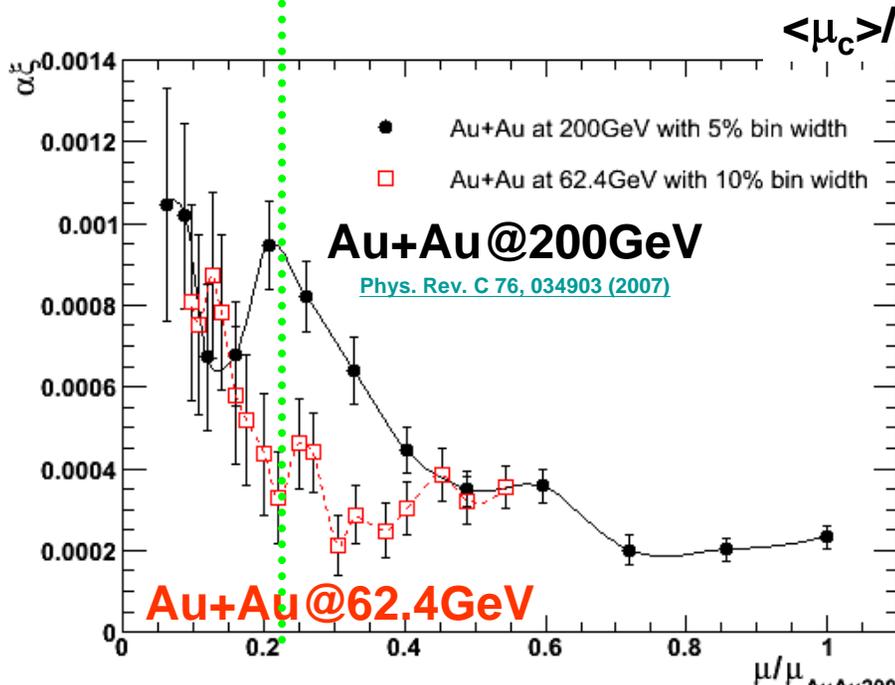
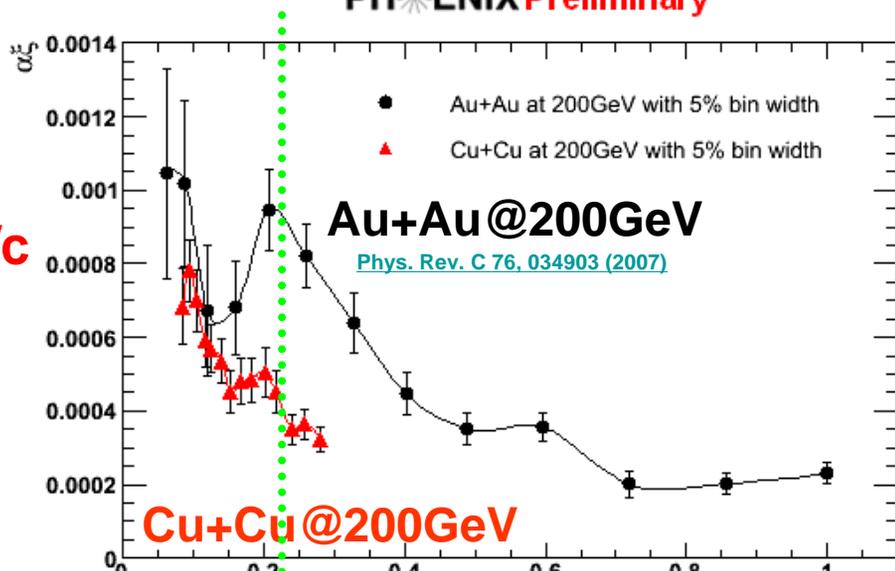
Significance with Line + Gaussian:
1.24 σ (5%), 1.69 σ (10%)

Comparison of three collision systems

PHENIX Preliminary

$N_{part} \sim 90$ in
AuAu@200GeV
 $\epsilon_{BJT} \sim 2.4 \text{ GeV}/\text{fm}^2/c$

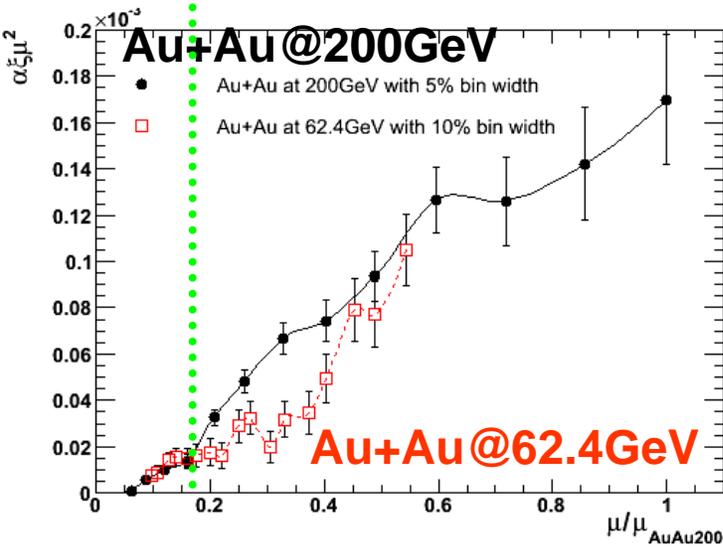
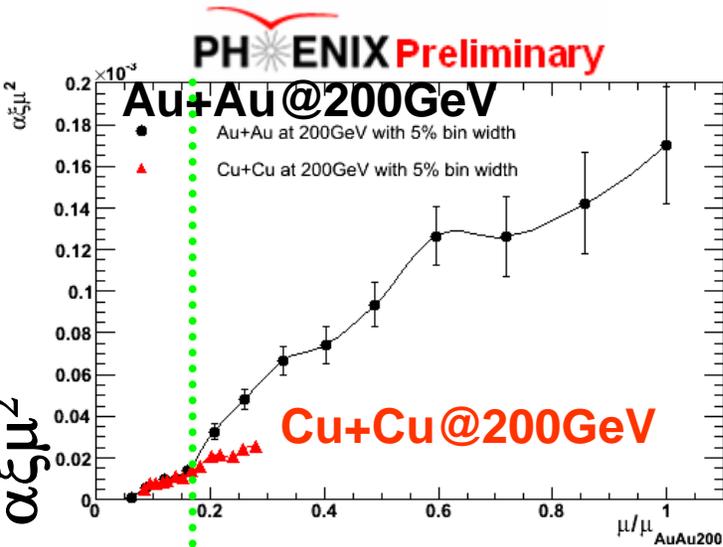
$\alpha \xi$



$\langle \mu_c \rangle / \langle \mu_c \rangle_{@AuAu200}$

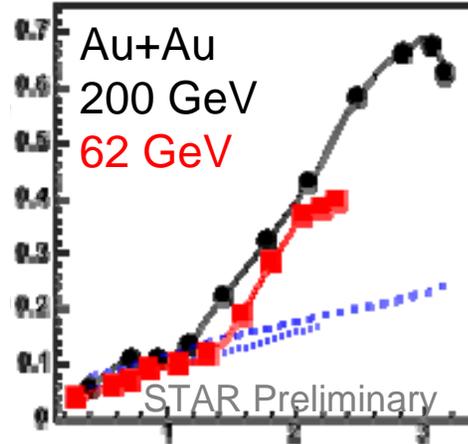
Normalized mean multiplicity to that of top 5% in Au+Au@200GeV

Similarity to STAR mini jet results at low p_T

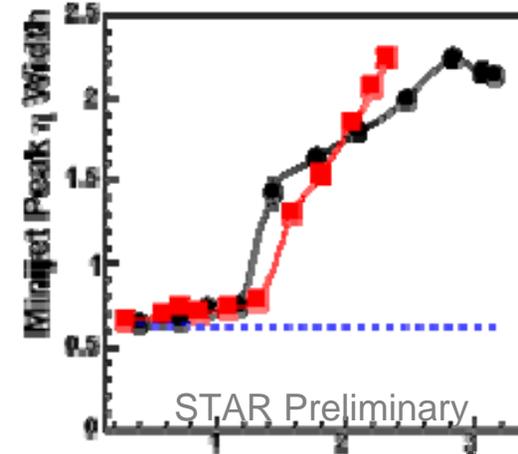


$$\langle \mu_c \rangle / \langle \mu_c \rangle @ AuAu200$$

Peak Amplitude



Peak η Width



X

ϵ_{BJ} M. Daugherty: QM2008 ϵ_{BJ}

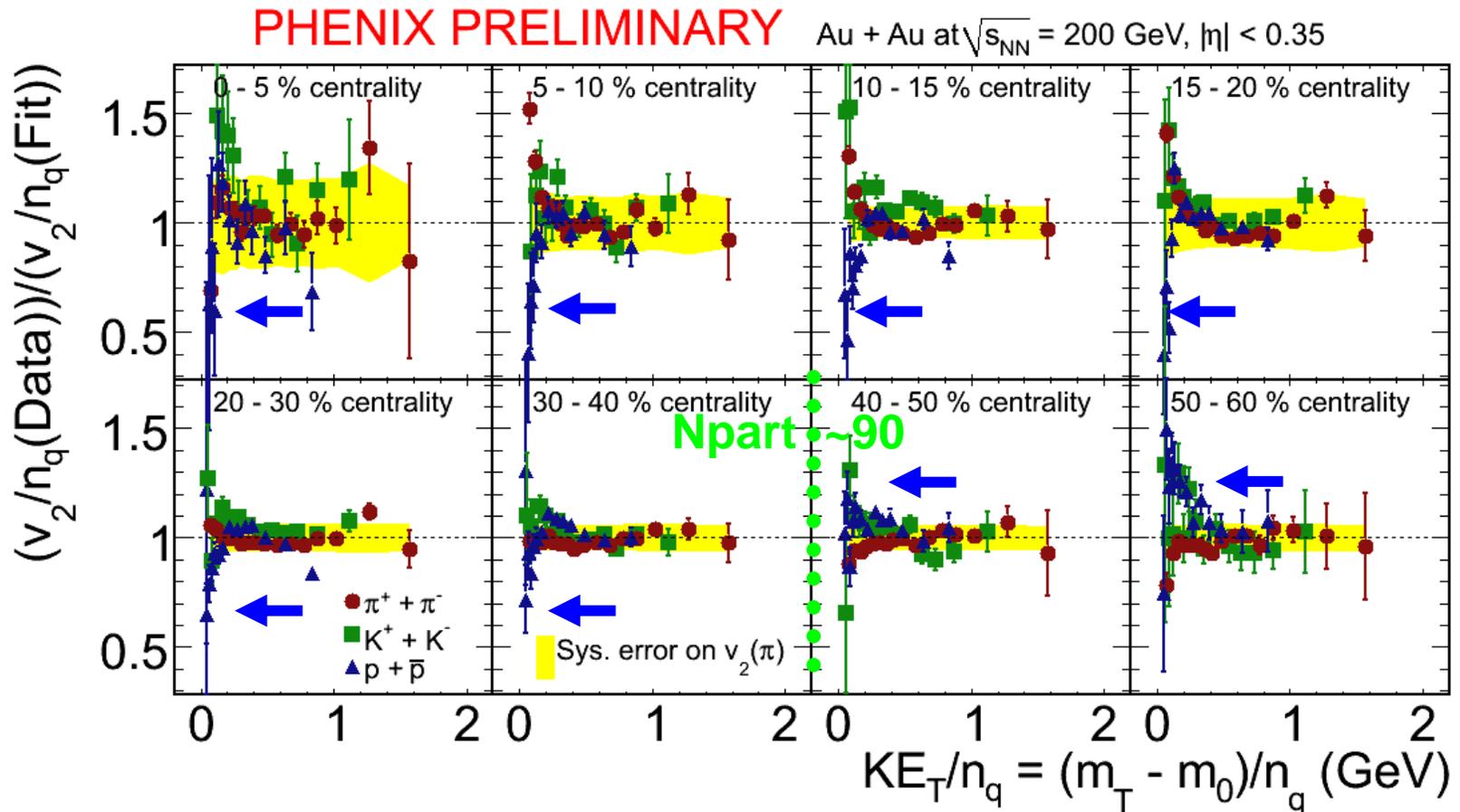
Equivalent quantity;

$$\chi T \propto \alpha \xi \mu^2 \propto \text{amplitude} \times \text{width}$$

shows similar trends to what STAR sees.

Is there other symptom?

- deviation from v2 scaling at low KE_T region -



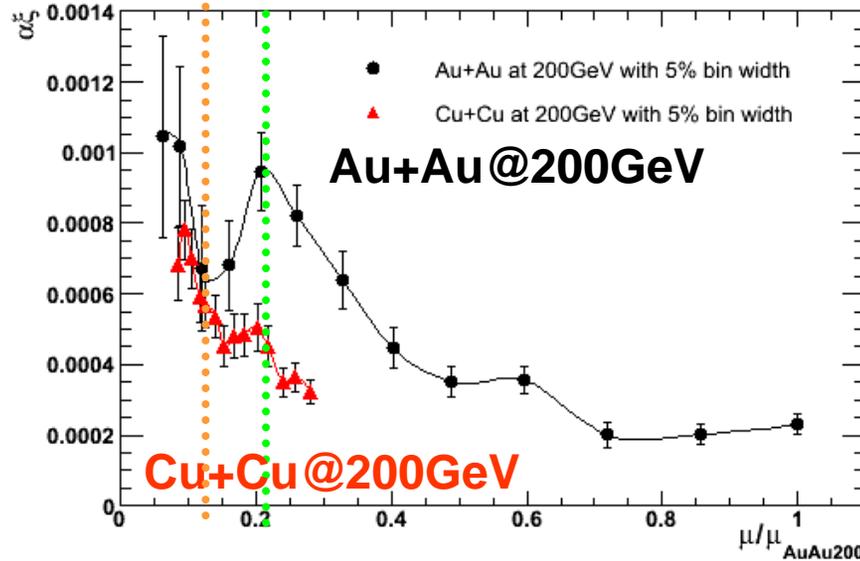
In lower KE_T, there seems to be different behaviors between baryon and mesons. The transition is at $N_{\text{part}} \sim 90$.

Low mass sigma field may repulse pions and attract protons according to hep-ph/0504048 by E.Shuryak .

Can this phenomenon be understood as such effects?

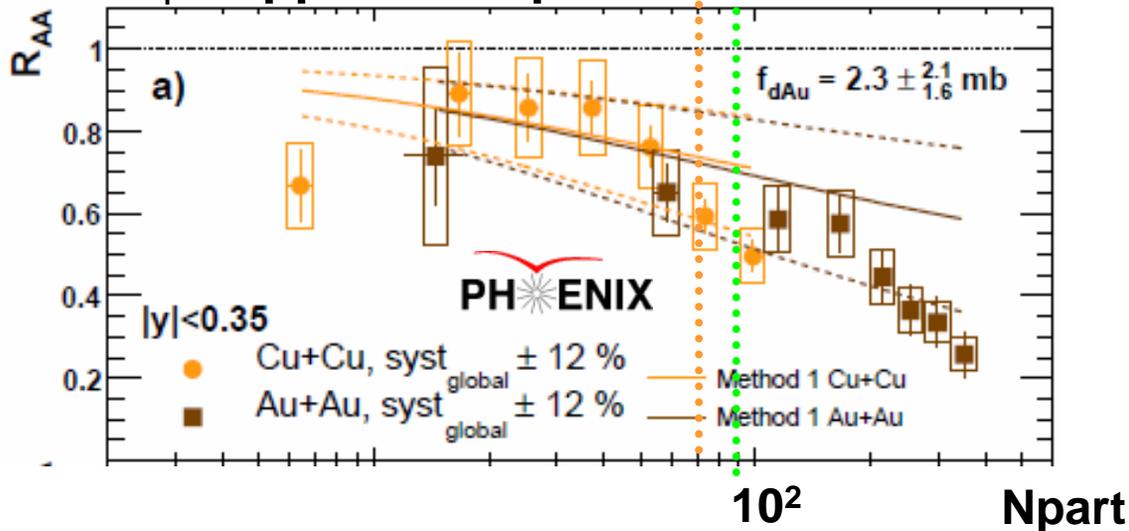
How about $\langle c\bar{c} \rangle$ suppression?

PHENIX Preliminary



$N_{\text{part}} \sim 90$ in
 AuAu@200GeV
 $\varepsilon_{\text{BJT}} \sim 2.4 \text{ GeV}/\text{fm}^2/c$

J/ ψ suppression pattern



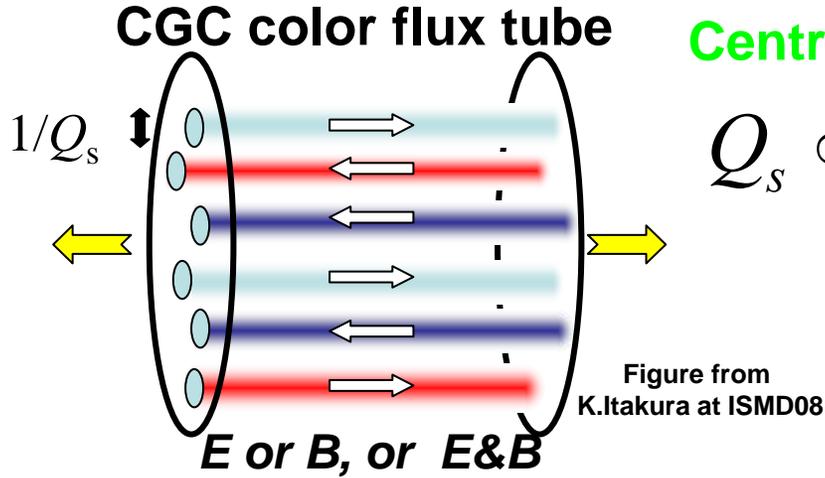
[arXiv:0801.0220v1 \[nucl-ex\]](https://arxiv.org/abs/0801.0220v1)

Summary

- 1. RHIC created strongly coupled high temperature & opaque state with partonic d.o.f. This is the very beginning of the scientific program on quantitative understanding of the QCD phase structure.**
- 2. 3D imaging of source shape from HBT type correlation suggests small but finite duration of pion emission time. There seems to be no strong indication of 1st order P.T. at 200GeV, but worth measuring at lower energy (in higher baryon density).**
- 3. PHENIX and STAR see very similar rapid transition of the two particle correlation in longitudinal direction at the similar centrality range in Au+Au@200 and 62GeV. However, a caution is necessary, because the rapidity window size is totally different: PHENIX is limited within 0.7 and perhaps STAR misses information at short rapidity window by the brute force subtraction process. Nevertheless, it is interesting to foresee the common reasoning.**

Open issue

- Is the rapid transition related with creation of CGC flux tube?
- Is the color electric field in tubes related with bag pressure (confinement)?
- Where is the threshold of the creation of the thermal system ?



Centrality can scan A

$$Q_s \propto A^{1/6} \sqrt{s}^{\sim 0.3/2}$$

A slide from S.H.Lee's talk on 14 Oct

