Viscous Hydrodynamics for Relativistic Heavy Ion Collisions

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in collaboration with Ulrich Heinz

References:

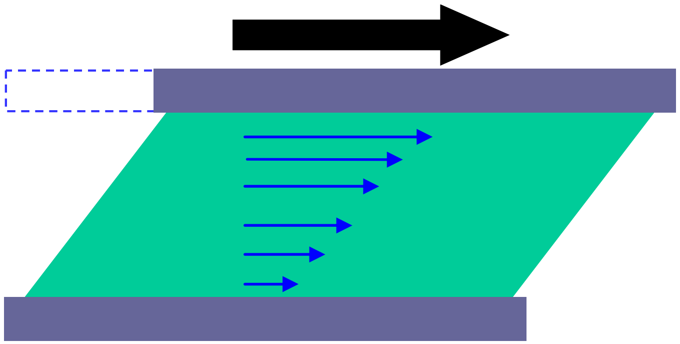
H. Song and U. Heinz, Phys.Rev.C78:024902 (2008). arXiv:0805.1756 [nucl-th].

H. Song and U. Heinz, Phys.Rev.C77:064901 (2008).

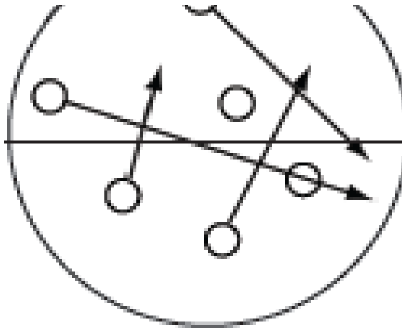
H. Song and U. Heinz, Phys. Lett. B658, 279 (2008).

What is viscosity

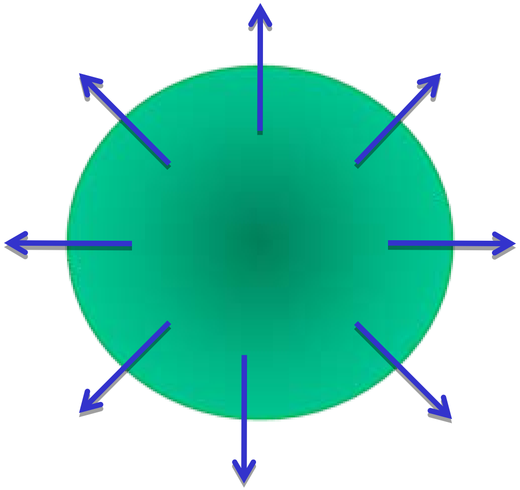
Shear viscosity –measures the resistance to flow



the ability of momentum transfer



Bulk viscosity –measure the resistance to expansion



-volume viscosity

Determines the dynamics of compressible fluid

The QGP viscosity

Kubo formulas:
- shear viscosity: $\eta = \frac{1}{20} \lim_{\omega\to 0} \int d^4x e^{i\omega t} < [T^{ij}(x) T^{ij}(0)] > \theta(t)$
- bulk viscosity: $\zeta = \frac{1}{18} \lim_{\omega\to 0} \int d^4x e^{i\omega t} < [T_i^i(x) T_i^i(0)] > \theta(t)$

Shear viscosity: uncertainty principle requires a lower limit for η / s

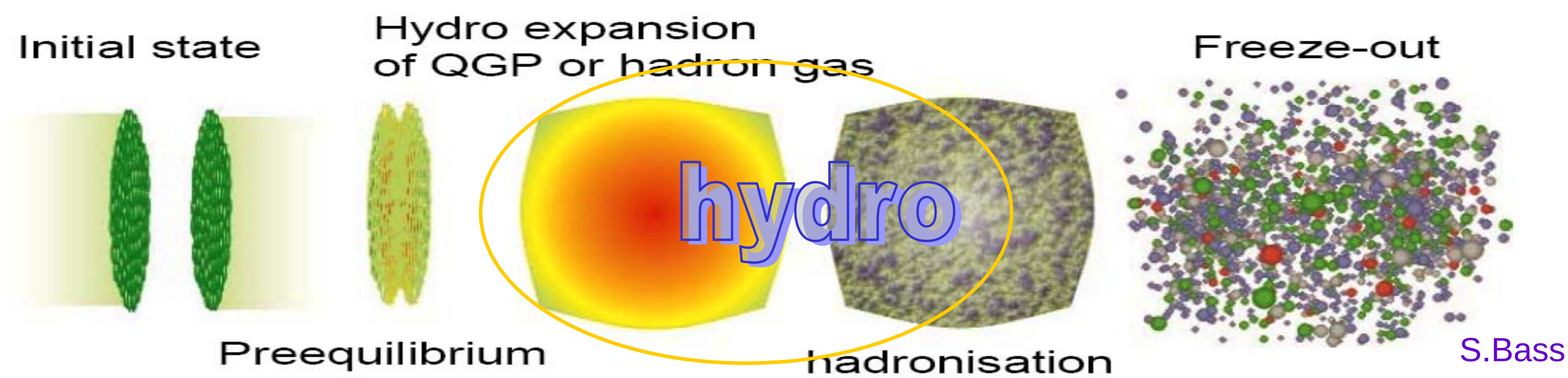
- -weakly coupled QCD: $\eta / s \sim 1$ — Arnold, Moore & Yaffe, 00,03
- -lattice SU(3) gluon dynamics : $\eta / s < 1$ — Meyer, PRD 07
- -strongly coupled AdS/CFT prediction : $\eta / s \geq 1/4\pi \sim 0.08$ — D.T. Son et al. '01,'05

Bulk viscosity: zero for classical massless particles, ζ / s reaches a peak near T_c

- -weakly coupled QCD prediction: $\zeta / s << 1$ — Arnold, Dogan & Moore, PRD06
- -lattice SU(3) gluon dynamics : $\zeta / s|_{\sim T_c} = 0.73 \Big|_{0.5}^{2.0}$ — Meyer, PRL08
- -LET+ assum. of spectral fun. + Lattice data: $\zeta / s|_{\sim T_c} \sim 0.8$ — Kharzeev, et al. 07-08
- -strongly coupled AdS/CFT prediction: $\zeta / s > 2\eta / s(1/3 - c_s^2)$ — Buchel, 07
  $\zeta / s|_{\sim T_c} \sim 0.05$ — Gubser, et al . 0806 ..

To extract the QGP viscosity from experimental data, we need viscous hydrodynamics

Viscous hydrodynamics



Conservation laws:

$$\partial_\mu T^{\mu\nu}(x) = 0 \qquad T^{\mu\nu} = (e + p + \Pi)u^\mu u^\nu - (p + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

Evolution equations for shear pressure tensor $\pi^{\mu\nu}$:

$$\tau_\pi \Delta^{\alpha\mu}\Delta^{\beta\nu}\dot{\pi}_{\alpha\beta} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta(\partial \cdot u)$$

-simplified Israel-Stewart eqn.

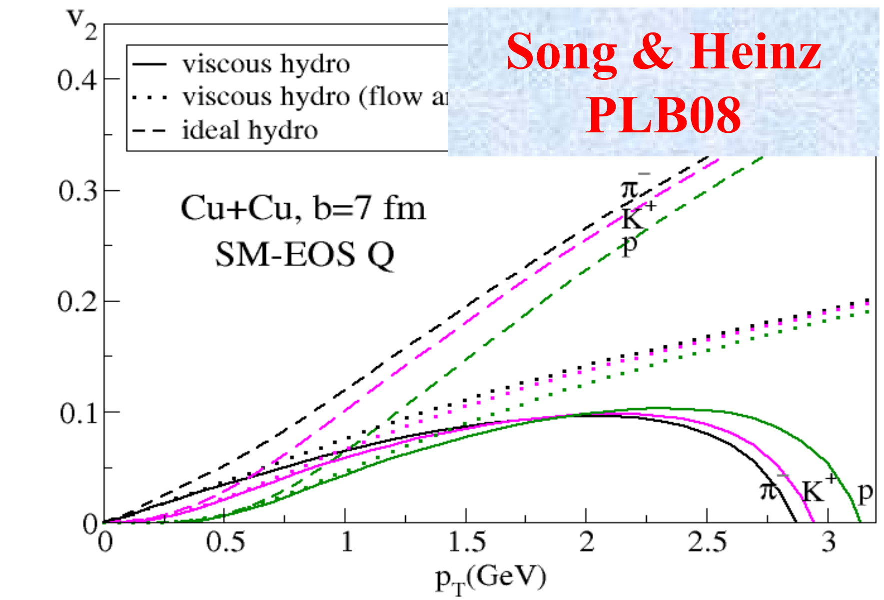
$$\partial_\mu S^\mu \geq 0$$

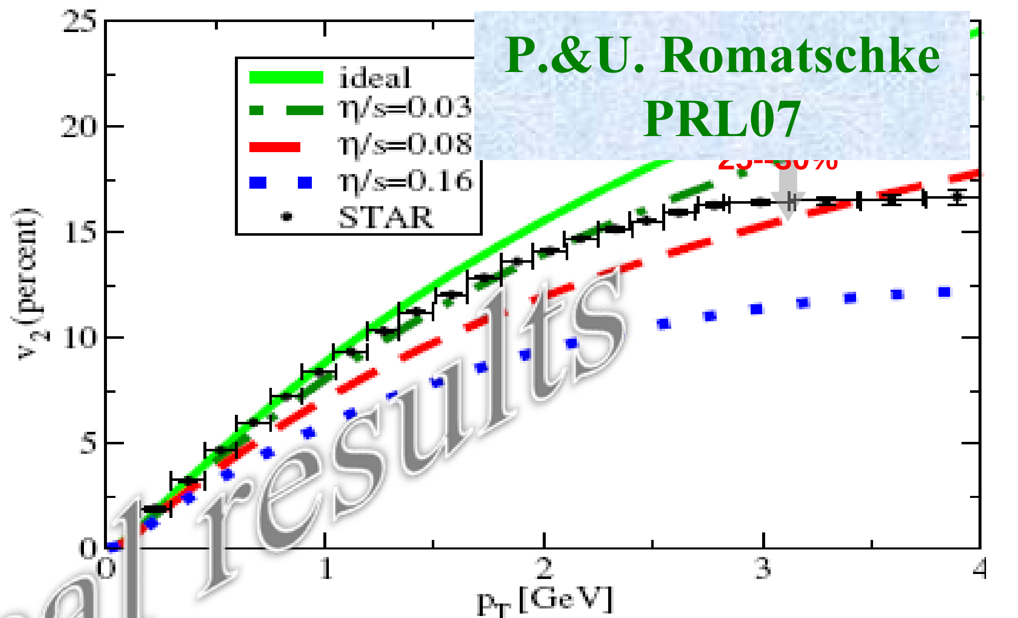
Input: "EOS" $\varepsilon = \varepsilon(p, n)$ initial conditions and final conditions

With $\eta, \zeta \to 0$ **viscous hydrodynamics** reduces to **ideal hydrodynamics**

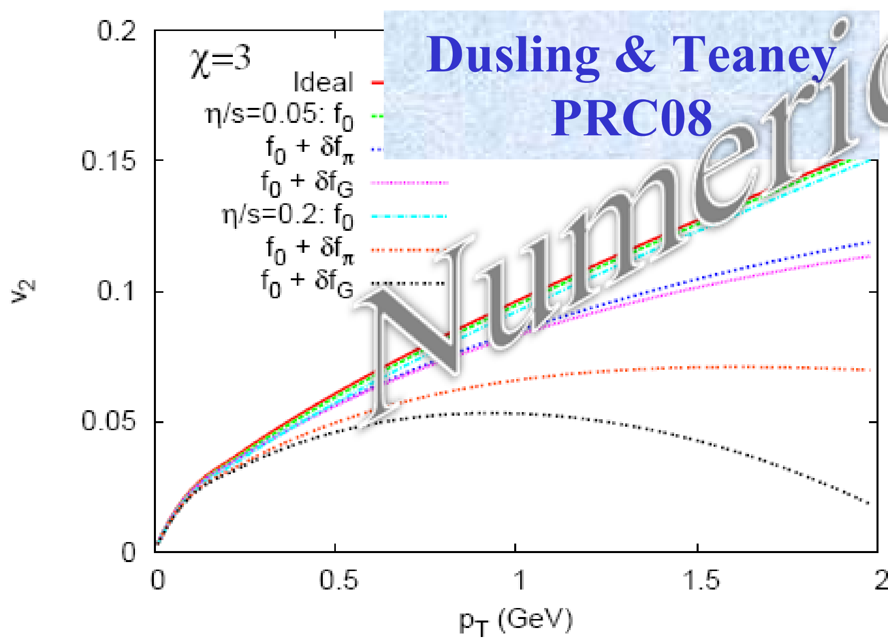
Bjorken appro. : $v_z = z/t$ reduces (3+1)-d hydro to (2+1)-d hydro (τ, x, y, η)

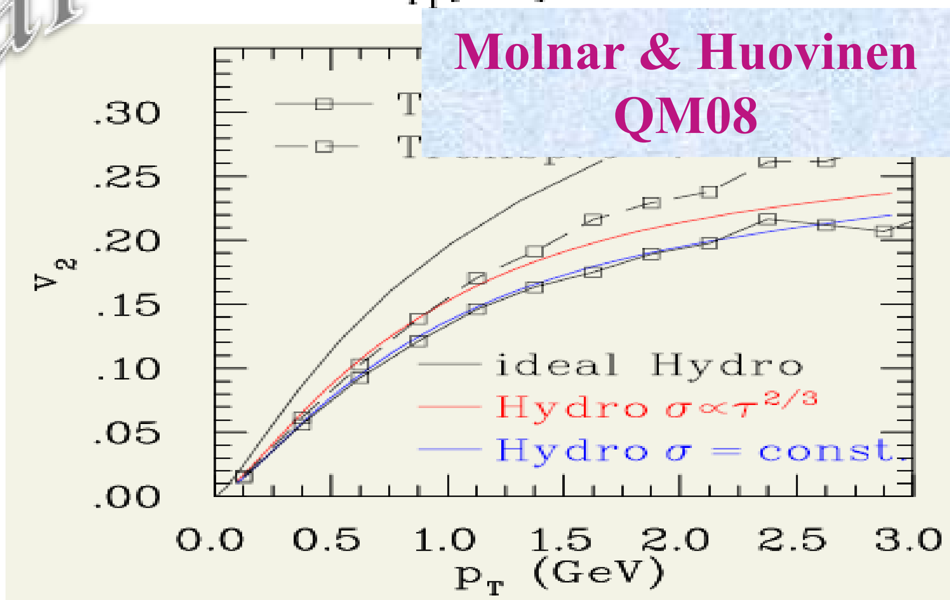
(2+1)-d viscous hydrodynamics





Numerical results





(2+1)-d viscous hydrodynamics

-Romatschke & Romatschke: full I-S eqn. EOS I EOS L*

PRL'07 **Au+Au,** $T_{dec} = 150 MeV$ (EOS L* here is the quasi-particle one based on lattice QCD)

-Song & Heinz: simplified I-S eqn. & full I-S eqn. EOS I SM-EOS Q EOS L

PLB'08 & PRC08 **Cu+Cu,** simplified I-S eqn., $T_{dec} = 130 MeV$

PRC08 (**Au+Au,** **Cu+Cu**, system size effects, full I-S eqn. vs. simplified I-S eqn., EOS L etc)

-Dusling & Teaney: Öttinger-Grmela (O-G) eqn. EOS I

PRC'08 **Au+Au,** decoupling by scattering rate, Nucl. Phys. A08(dilepton production)

-Huovinen & Molnar: full I-S eqn. EOS I

QM08 talk: comparing the results from viscous hydro and from transport model

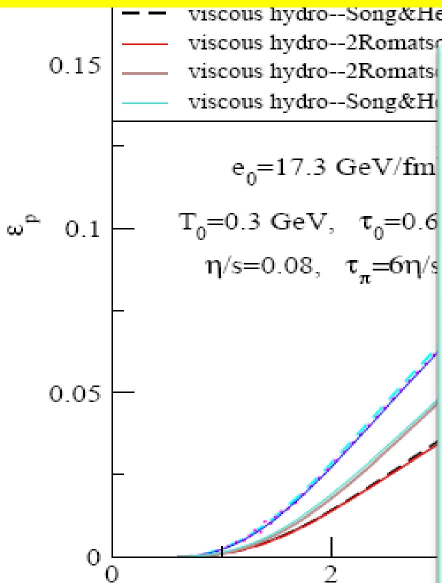
-Chaudhuri: simplified I-S eqn. EOS I EOS Q

arXiv:0708.1252 [nucl-th], arXiv:0801.3180 [nucl-th], arXiv:0803.0643 [nucl-th] **Au+Au**

Code Checking between Song & Heinz VISH2+1 and Romatschke Code (Nov.2007)

dynamics

EOS L *



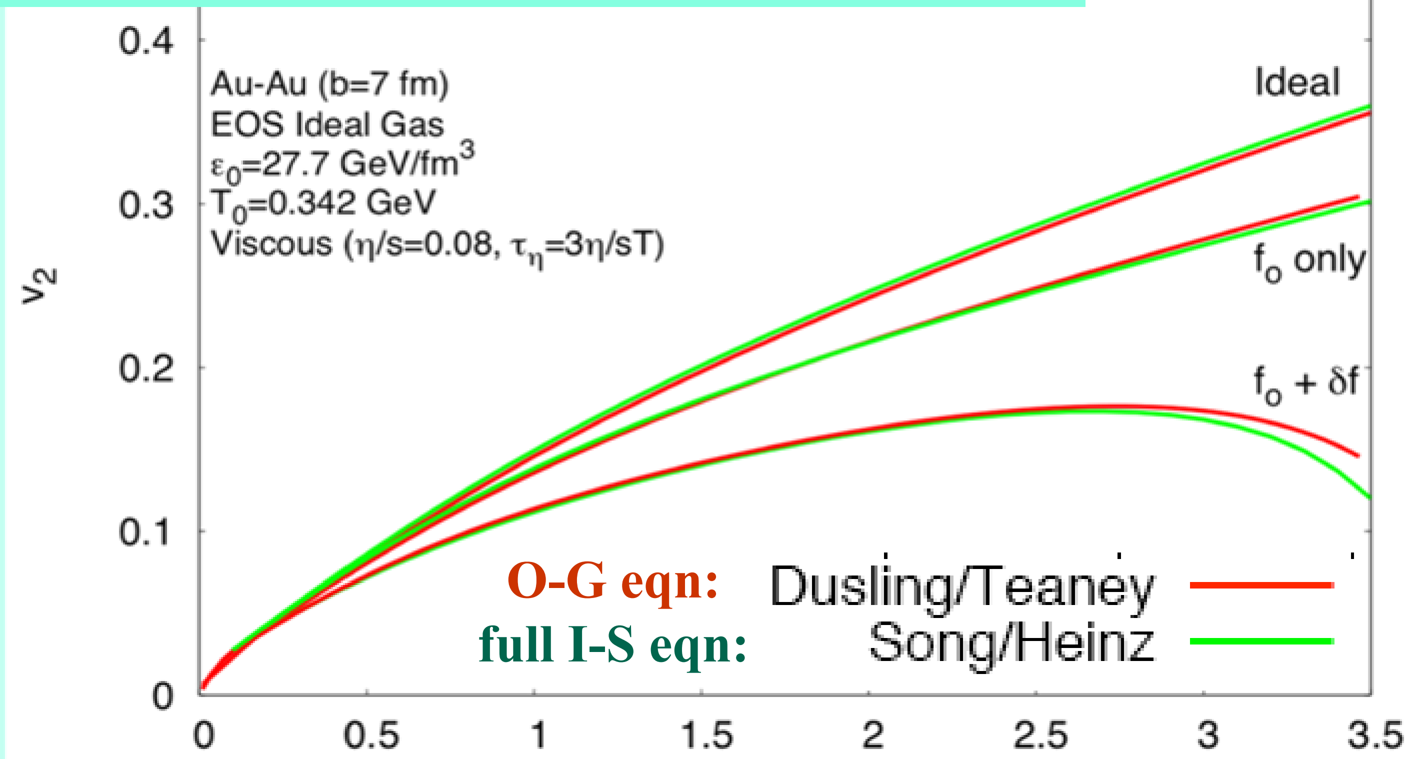
-Huovinen & Mo

QM08 talk: comp

-Chaudhuri: sir

arXiv:0708.1252 [n

Code Checking within TECHQM Collaboration (started on May, 2008)



https://wiki.bnl.gov/TECHQM/

Issues:

- verification of the codes individually developed by different groups
  - -VISH2+1 (Song & Heinz) vs. Romatschke code: (Nov. 2007)
  - -VISH2+1 (Song & Heinz) vs. Dusling & Teaney code: (May, 2008-)

(2+1)-d viscous hydrodynamics

-Romatschke & Romatschke: full I-S eqn. EOS I EOS L*

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Issues:

- verification of the codes individually developed by different groups
- **effects from different 2nd order formalisms**

  simplified I-S eqn. vs. full I-S eqn., I-S eqn. vs. O-G eqn.
- **effects from different EoS, systems sizes and freeze-out procedures**

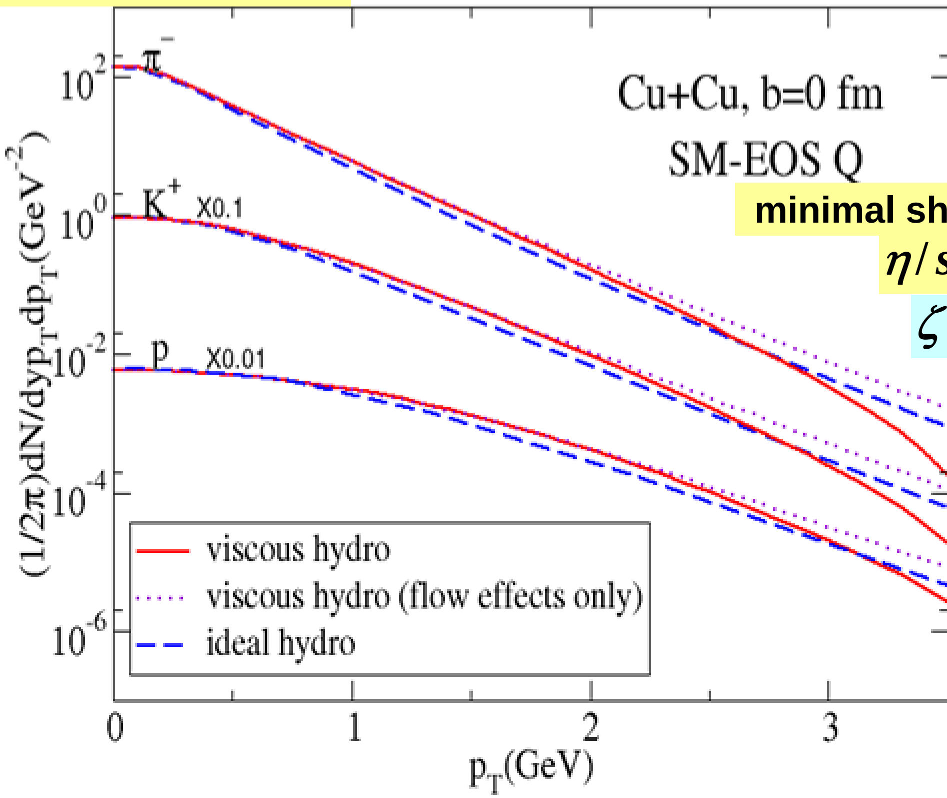
Shear viscosity effects:

Ideal hydro vs. viscous hydro

Viscous vs. ideal hydro – spectra & elliptic flow

Simplified I-S eqn.

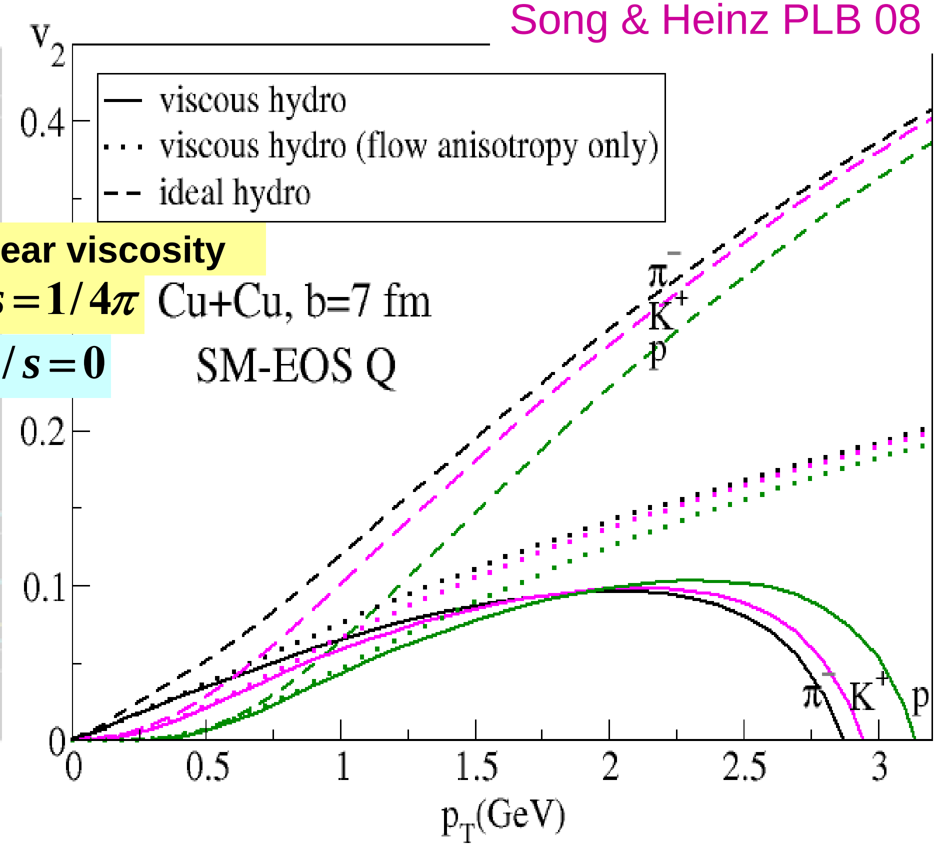
Song & Heinz PLB 08



minimal shear viscosity

$\eta/s = 1/4\pi$

$\zeta/s = 0$

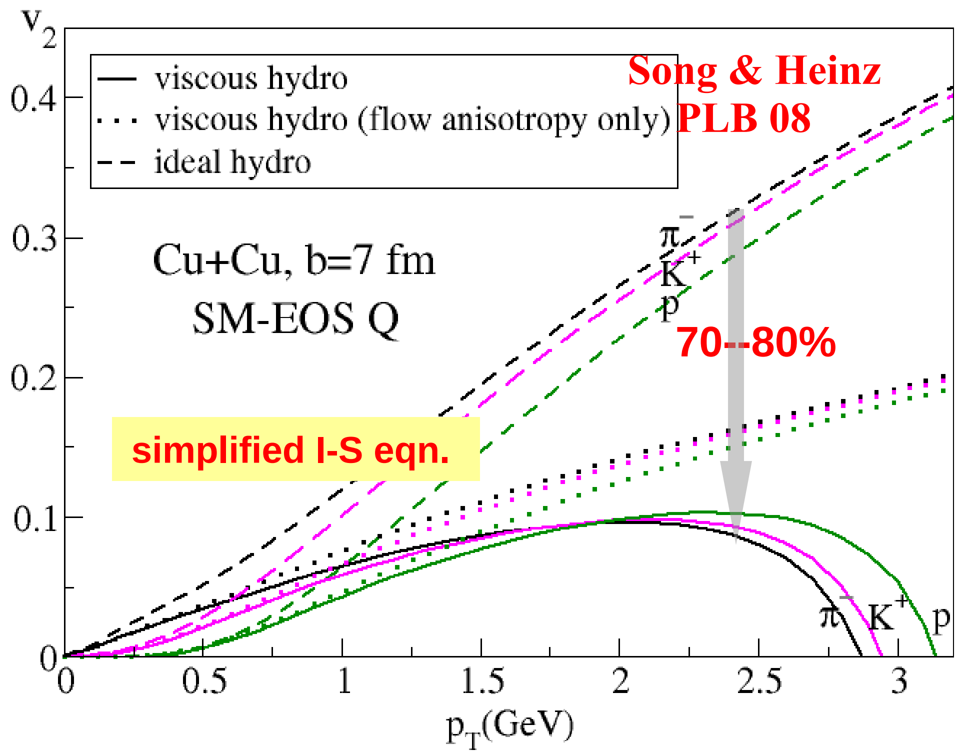


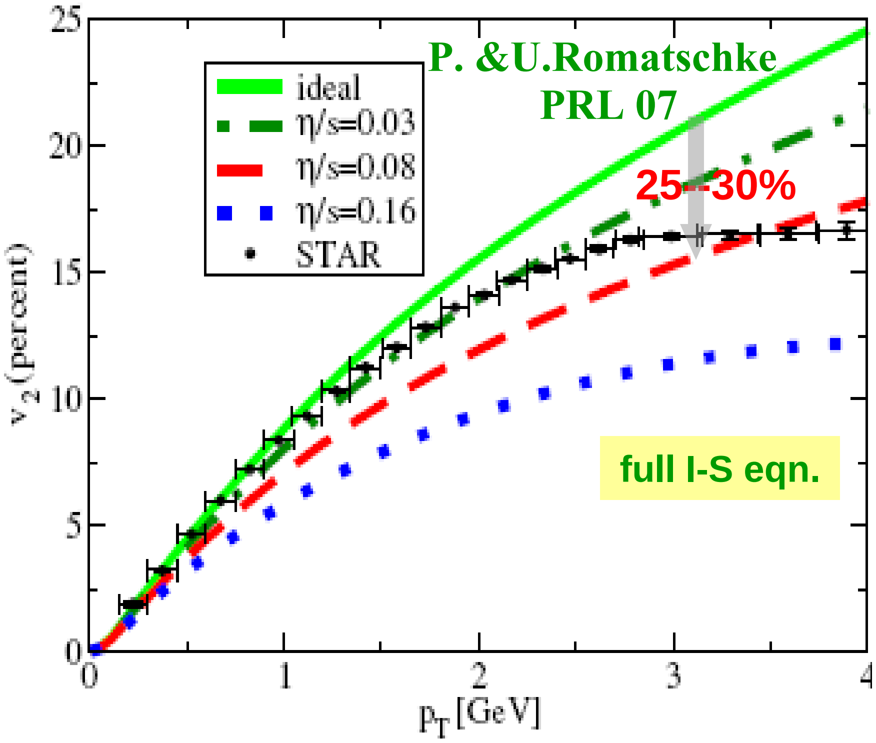
$$E\frac{dN}{d^3p} = \int_\Sigma \frac{p\cdot d^3\sigma(x)}{2\pi^3}\left[f_{eq}(x,p) + \delta f(x,p)\right] = \int_\Sigma \frac{p\cdot d^3\sigma(x)}{(2\pi)^3} f_{eq}(x,p)\left(1 + \frac{1}{2}\frac{p^\alpha p^\beta}{T^2(x)}\frac{\pi_{\alpha\beta}(x)}{(e+p)(x)}\right)$$

-More radial flow, flatter spectra;

-elliptic flow is very sensitive to shear viscosity

Comparison with Romatschke 07 results





- different systems & EOS: Cu+Cu, b=7, SM-EOS Q vs. Au+Au, min bias, EOS Lattice

- different Isreal-Stewart eqns. used: simplified I-S eqn. vs. full I-S eqn.

Effect of using different I-S eqns.?

Simplified I-S eqn. vs. full I-S eqn.:

simplified I-S eqn.:
$$\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right]$$

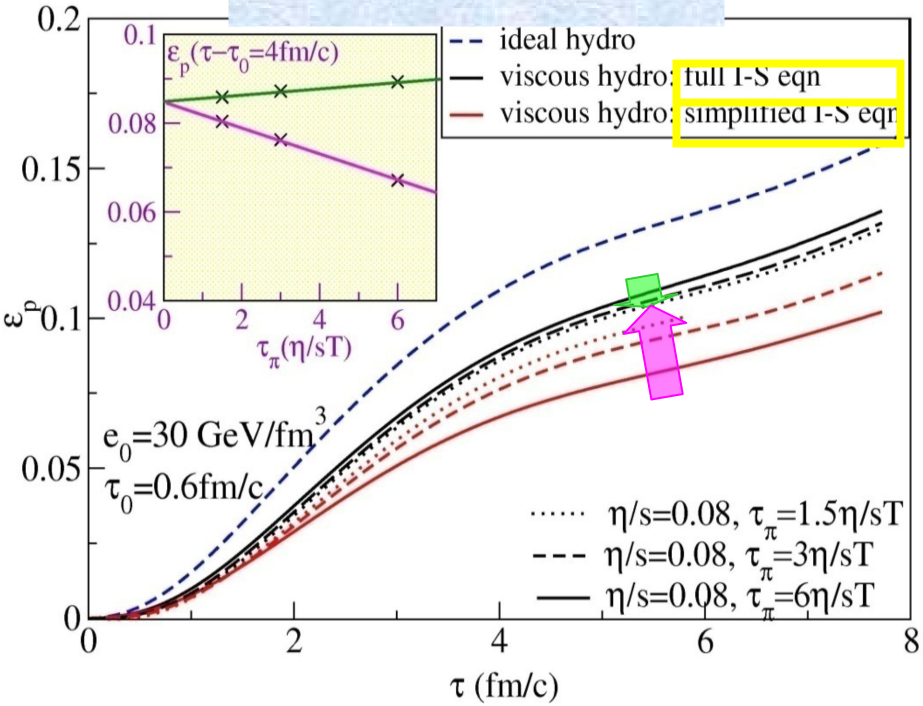
full I-S eqn.:
$$\Delta^{\mu\alpha}\Delta^{\nu\beta}D\pi_{\alpha\beta} = -\frac{1}{\tau_\pi}\left[\pi^{\mu\nu} - 2\eta\sigma^{\mu\nu}\right] + \frac{1}{2}\pi^{\mu\nu}[5D\ln T - \nabla_\alpha u^\alpha] - 2\pi^{\alpha(\mu}\omega_\alpha^{\nu)}$$

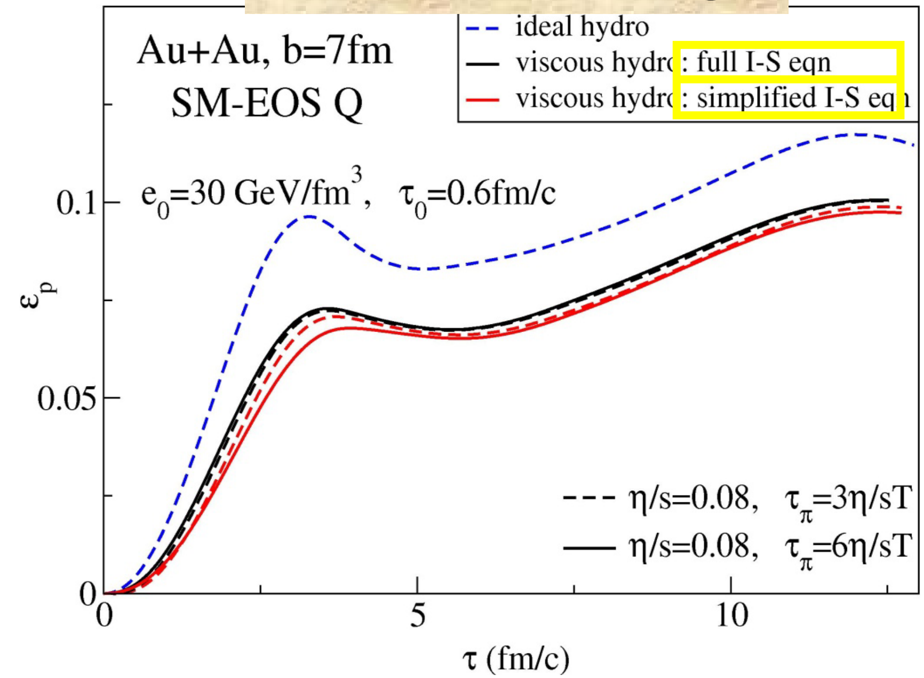
important for preserving the conformal symmetry

(Baier et al. '07)

simplified I-S eqn. vs. full I-S eqn.

EOS I Song & Heinz PRC 08 **SM-EOS Q**





-for EOS I, the difference between simplified I-S eqn and full I-S eqn could reach 30-50% for larger relaxation times.

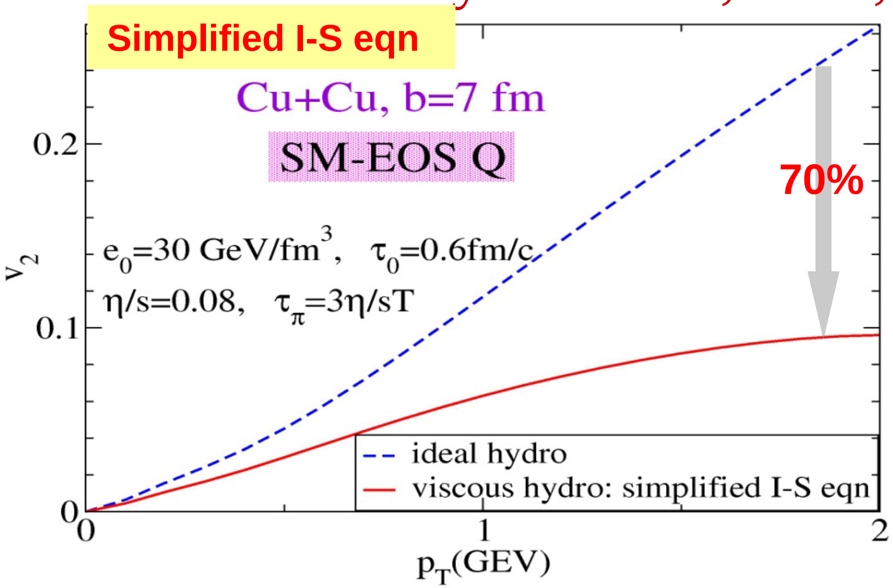
- for realistic EOS with a phase transition, the difference between simplified and full I-S for viscous v2 suppression are small

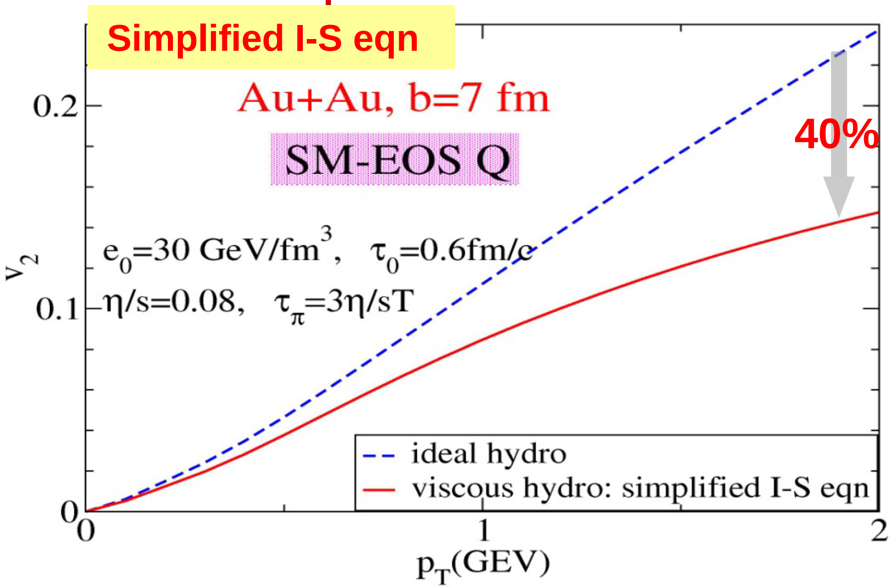
-numerical simulations also show that simplified I-S eqn. and full I-S eqn. **approach** the same Navier-Stokes limit as $\tau_\pi \rightarrow 0$

-full I-S eqn. shows much weaker dependence to τ_π

System size effects to viscous v_2 suppression

system size, EOS, different I-S equations:

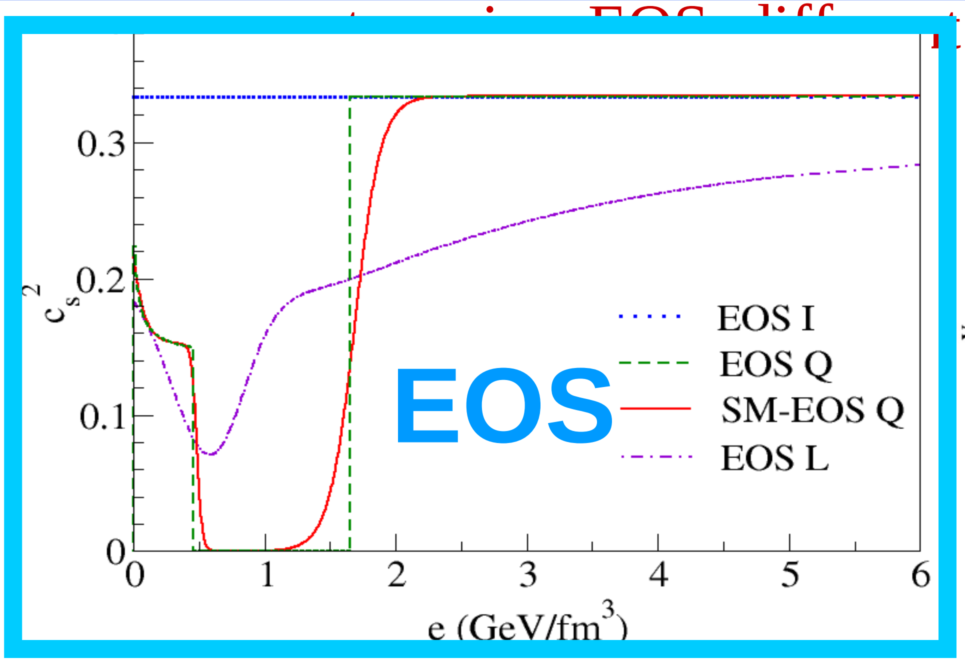


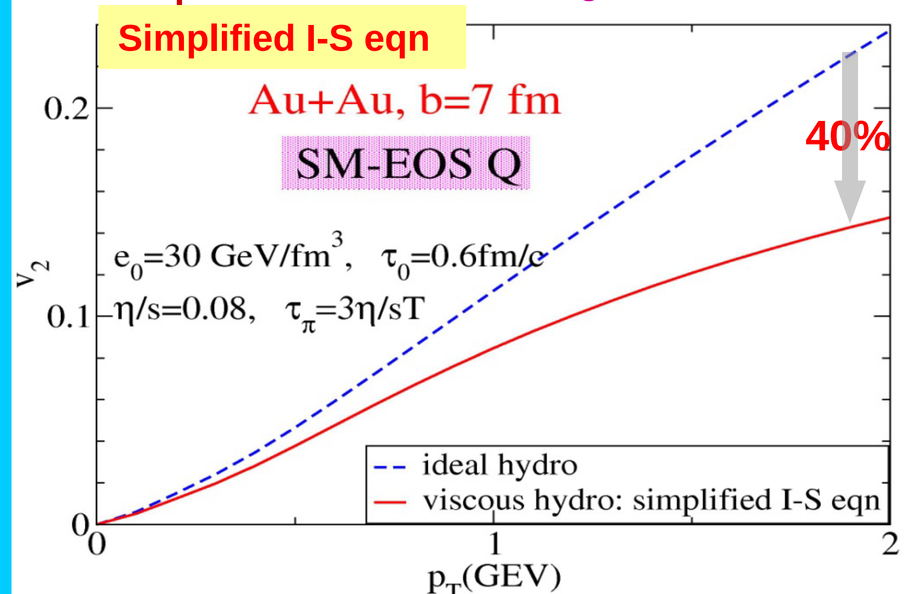


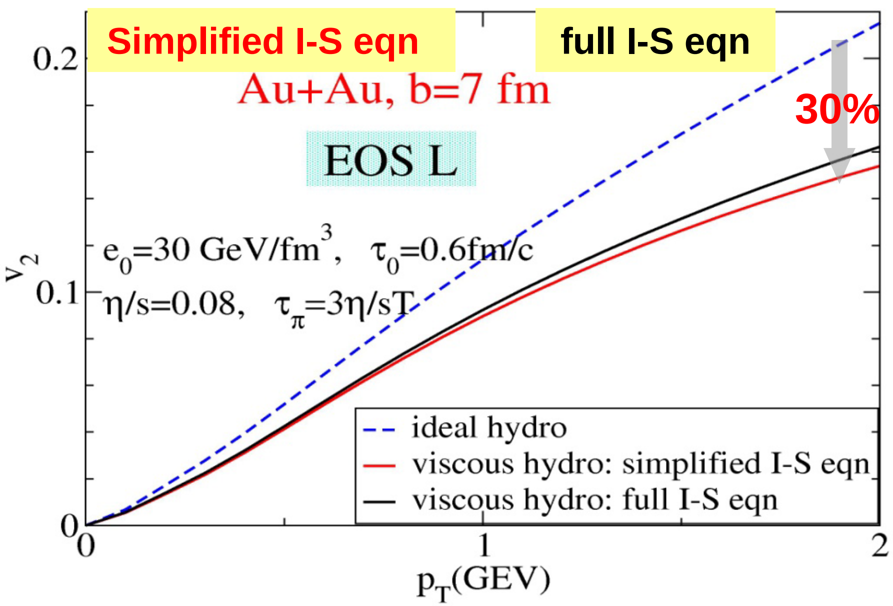
-system size: CuCu b=7fm vs. AuAu b=7fm:
~50-100% effect

EOS effects to viscous v_2 suppression

…t I-S equations: Song & Heinz PRC 08







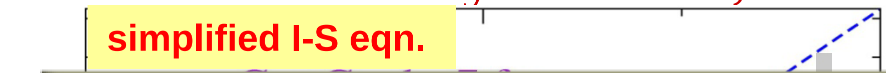
-system size: CuCu b=7fm vs. AuAu b=7fm: ~50-100% effect

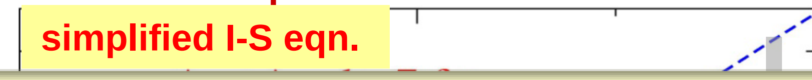
-EOS: SM-EOS Q vs. EOS L: ~25% effect

-different I-S eqns: simplified I-S eqn. vs. full I-S eqn.: ~5-10% effects (EOS Q and EOS L only)

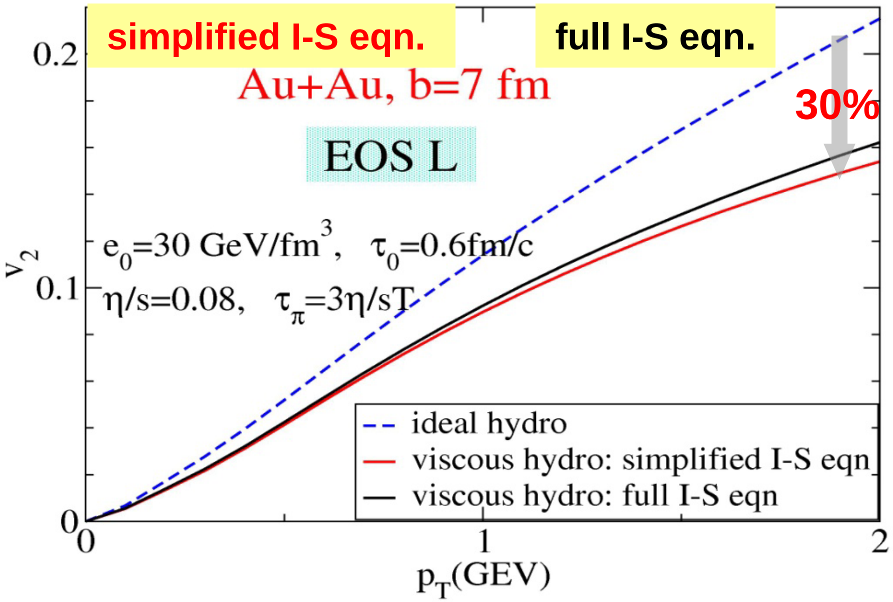
Different contributions to the suppression of v_2

System size, EOS, different I-S equations:





Considering all of these effects, the final suppression of v_2 for Au+Au with EOS L and the full I-S eqn., for minimal shear viscosity $\eta / s = \mathbf{0.08}$, is ~25%, approaching the results of P. & U. Romatschke (PRL 99, 172301 (2007)).



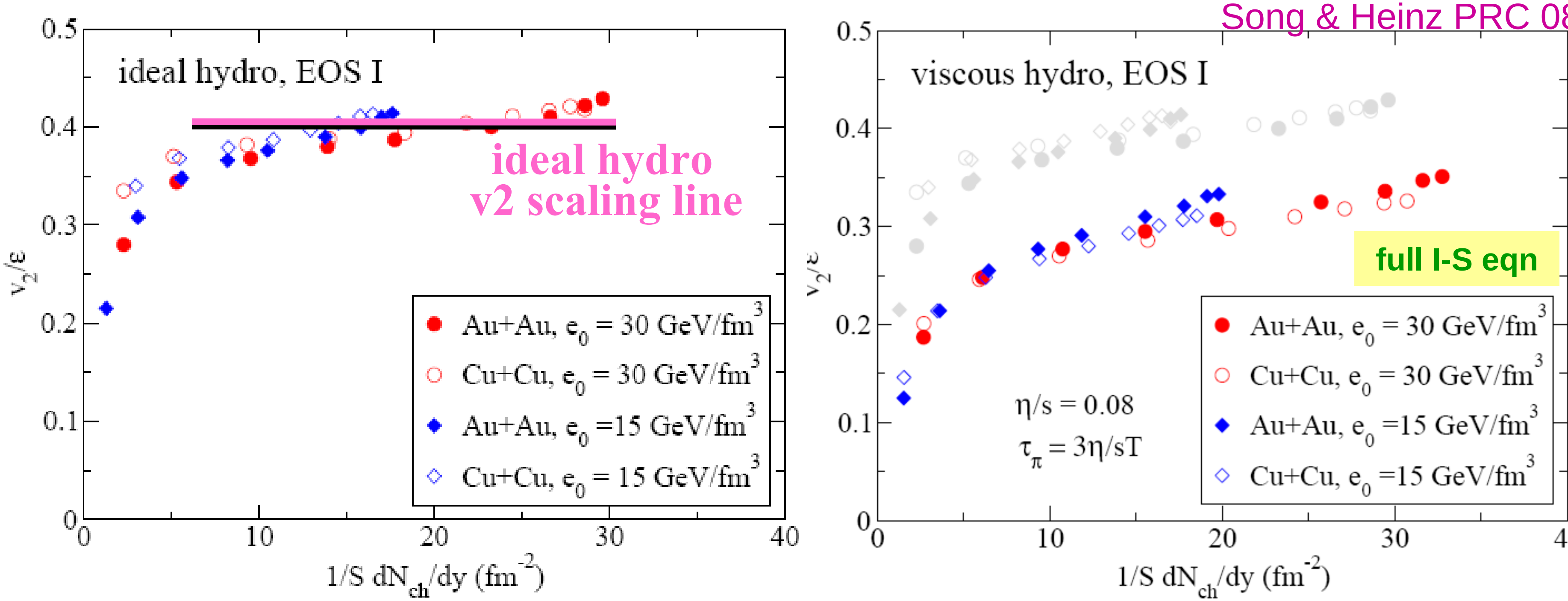
- system size: CuCu b=7fm vs. AuAu b=7fm: ~50-100% effect
- EoS: SM-EOS Q vs. EOS L: ~25% effect
- different I-S eqn.: simplified vs. full I-S eqn.: ~5-10% effect (EOS Q and EOS L only)

Comment: To extract QGP viscosity from exp. data by using viscous hydro, one needs a better description of EoS (Lattice EoS + chemical non-equil. HRG EoS)

System size effects

Multiplicity scaling of v_2/ε EOS I

Song & Heinz PRC 08



Ideal hydrodynamics: multiplicity scaling of v_2/ε is weakly broken:

- freeze-out condition introduces time scale, breaking scale invariance of id. hydro eqns.
- Initial profiles for Cu+Cu and Au+Au systems are not identical after a rescaling

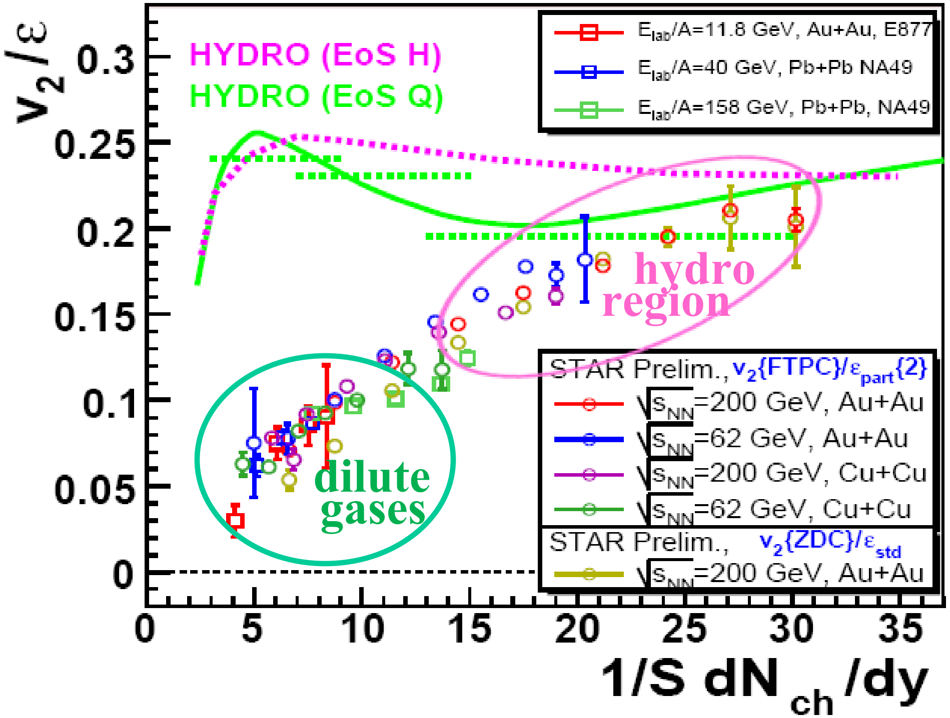
Viscous hydrodynamics: additional scale breaking by shear viscosity, resulting in fine structure of v_2/ε :

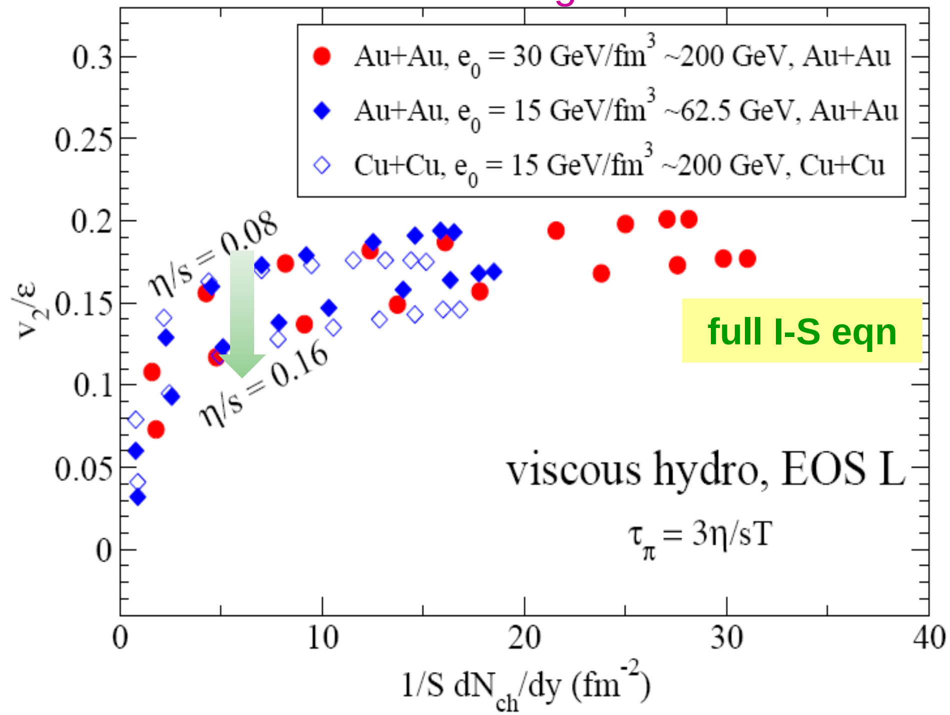
- for similar initial energy density, Cu+Cu curves are slightly below the Au+Au curves
- at fixed $\frac{1}{S}\frac{dN_{ch}}{dy}$, the $e_0 = 15\mathrm{GeV/fm}^3$ curves are slightly above the $e_0 = 30\mathrm{GeV/fm}^3$ ones

Viscous effects are larger for smaller systems and lower collision energies

Multiplicity scaling of v_2/ε EOS L

Song & Heinz PRC 08





- experimental data show qualitatively similar fine ordering as viscous hydro prediction

- to reproduce slope of v_2/ε vs. (1/S)dN/dy, a better description of the highly viscous hadronic stage is needed: *T*-dependent η/s, viscous hydro + hadron cascade

- the experimental v_2/ε vs. (1/S)dN/dy scaling (slope and fine structure) is another good candidate to constrain η/s (insensitive to Glauber-type vs. CGC initialization)

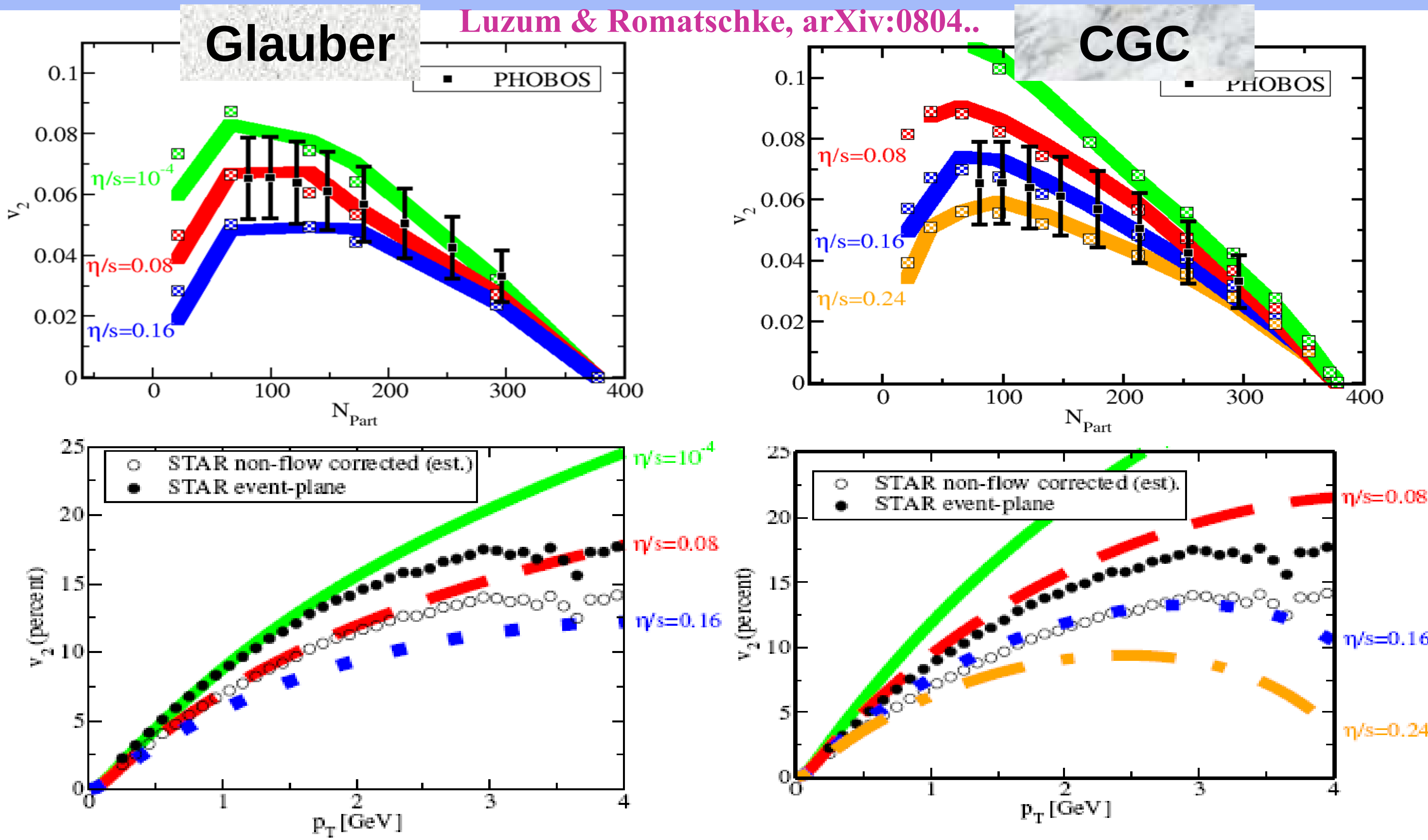
- this requires, however, experimental and theoretical improvements: reduced error bars, accounting for *T*-dependence of η/s, ζ/s near T_c, modeling hadronic phase with realistic cascade

A first attempt to extract η / s from experimental data

-Luzum & Romatschke,
arXiv:0804.4015 [nucl-th]

Extracting η / s from elliptical flow data

Luzum & Romatschke, arXiv:0804..

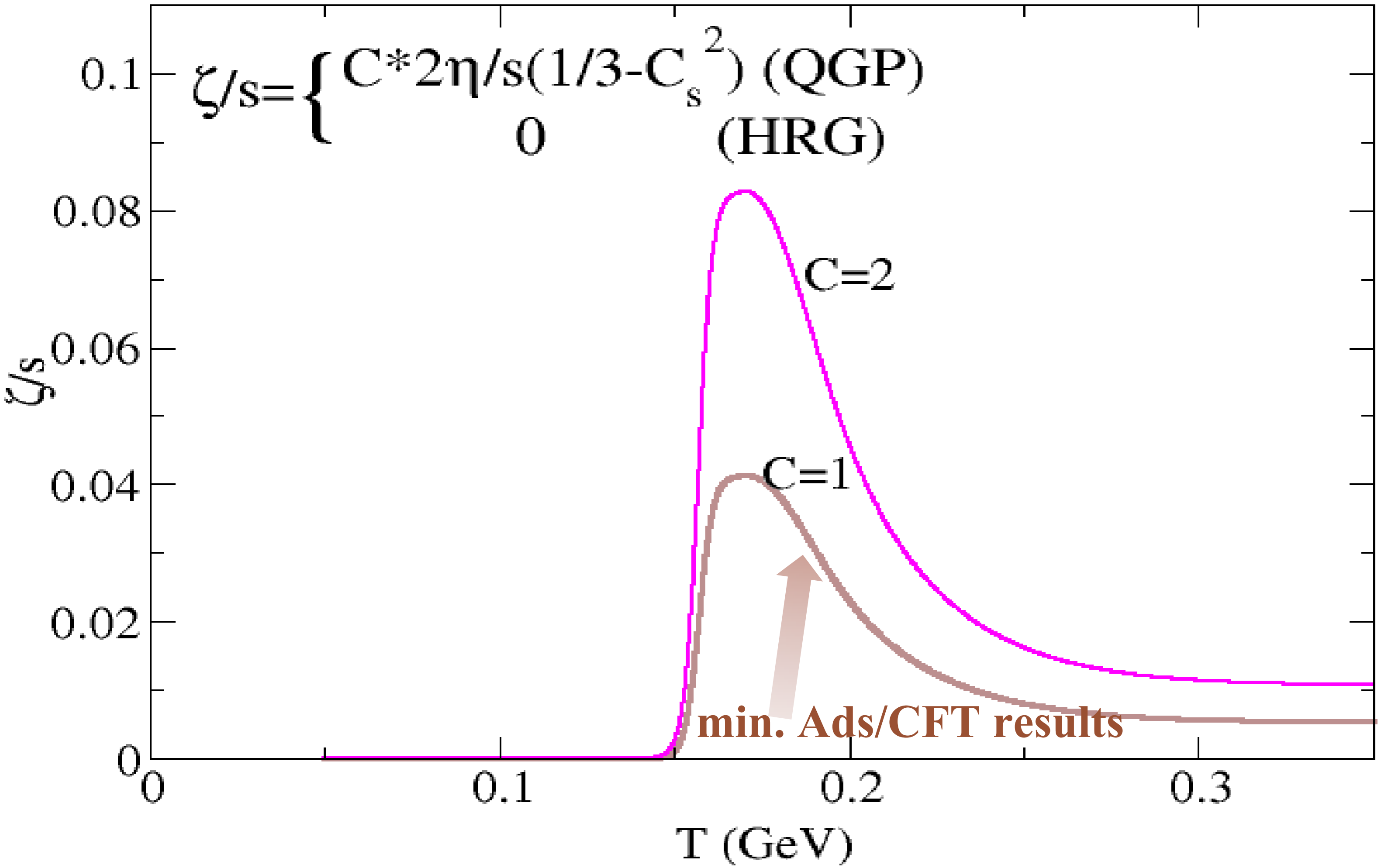


-Glauber vs.CGC ~100% effects on the extracted value of η / s, (highly viscous late hadronic stage, bulk viscosity , non-eq. chemistry in HG, etc are not included)

- $\eta / s \leq 5 \times (1/4\pi)$ (estimated from these data and results by other groups)

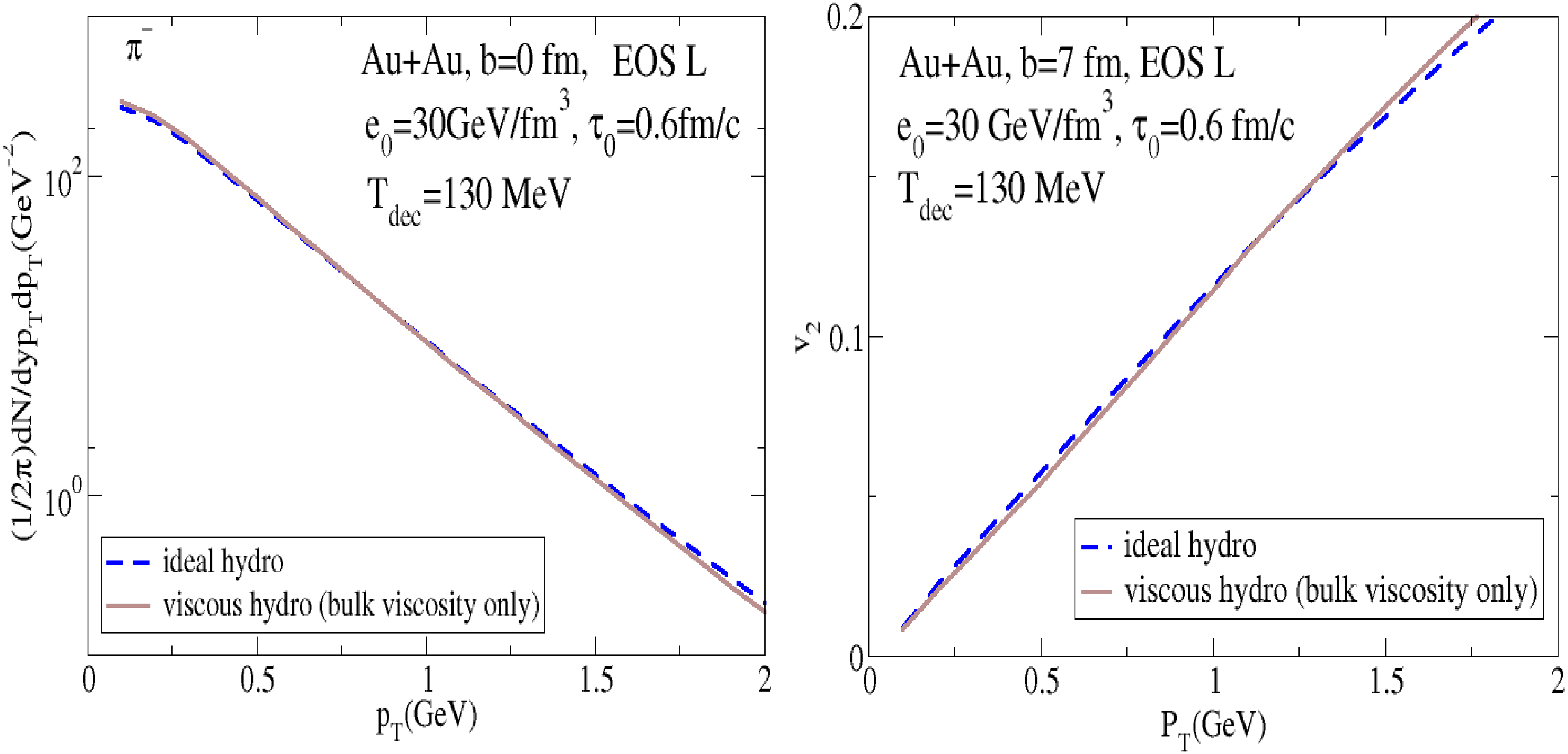
Effects from bulk viscosity

-Preliminary results



Ideal vs. Viscous hydro – spectra & elliptic flow

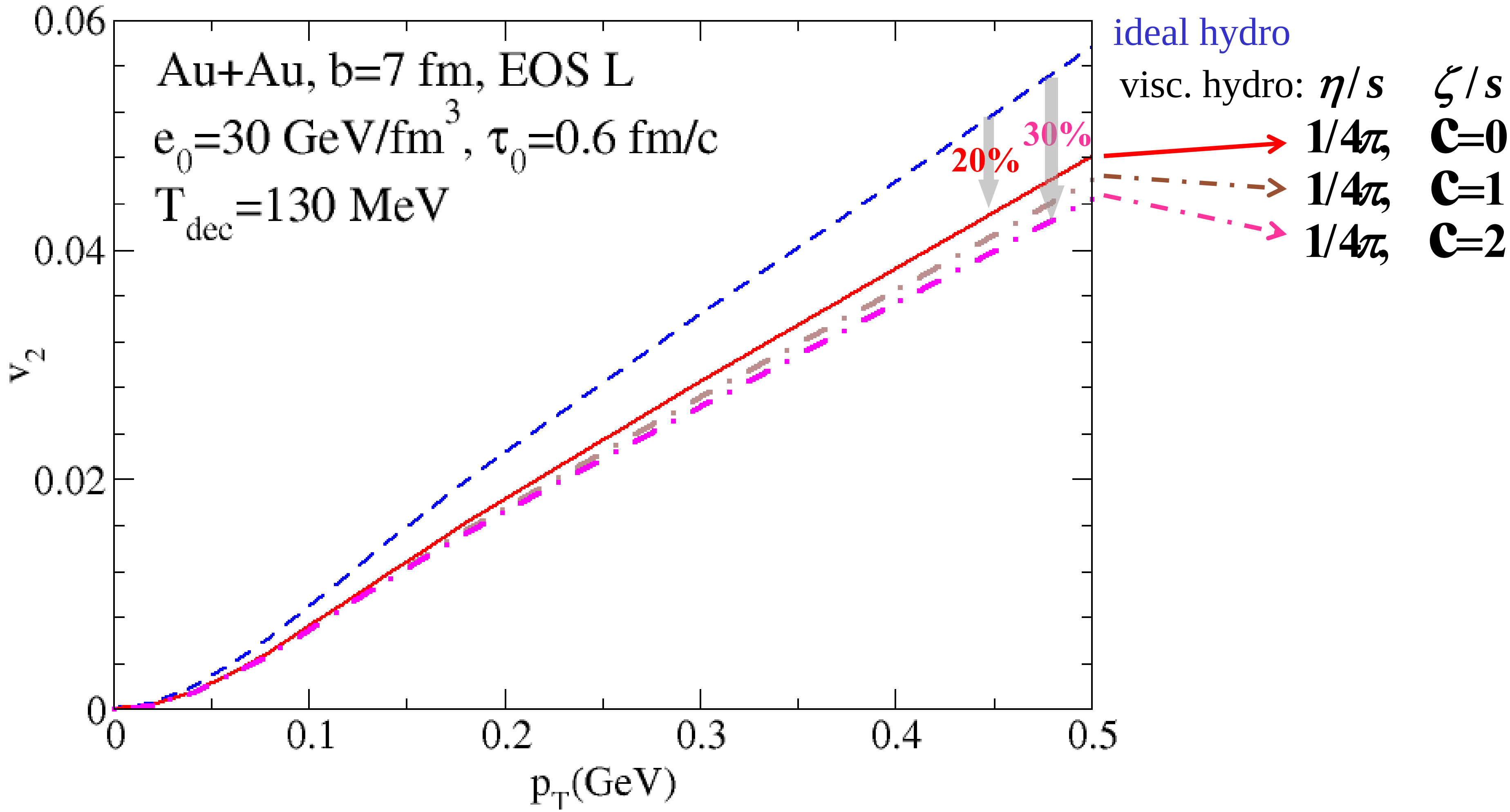
Viscous hydro: **bulk viscosity only,** using C=1 here



-negative bulk pressure could effectively soften the EoS near phase transition, which prohibits the development of radial flow and momentum anisotropy.

-spectra becomes steeper, v_2 is suppressed at lower p_T region

Viscous v2 suppression: **shear** and **bulk** viscosity



-2 x min bulk viscosity could result in **~50%** additional v_2 suppression

-when extracting the η/s from exp. data, bulk viscous effects cannot be neglected

Summary and discussion

- v_2 is sensitive to η/s
  - use full I-S to minimize sensitivity to τ_π
- multiplicity scaling of v_2/ε is a good candidate to extract the QGP viscosity:
  - larger viscous effects in smaller systems and at lower collision energies
  - multiplicity scaling of v_2/ε is insensitive to Glauber model vs. CGC initialization.

A first attempt to constrain η/s from RHIC data indicates $\eta/s \leq 5\times(1/4\pi)$,

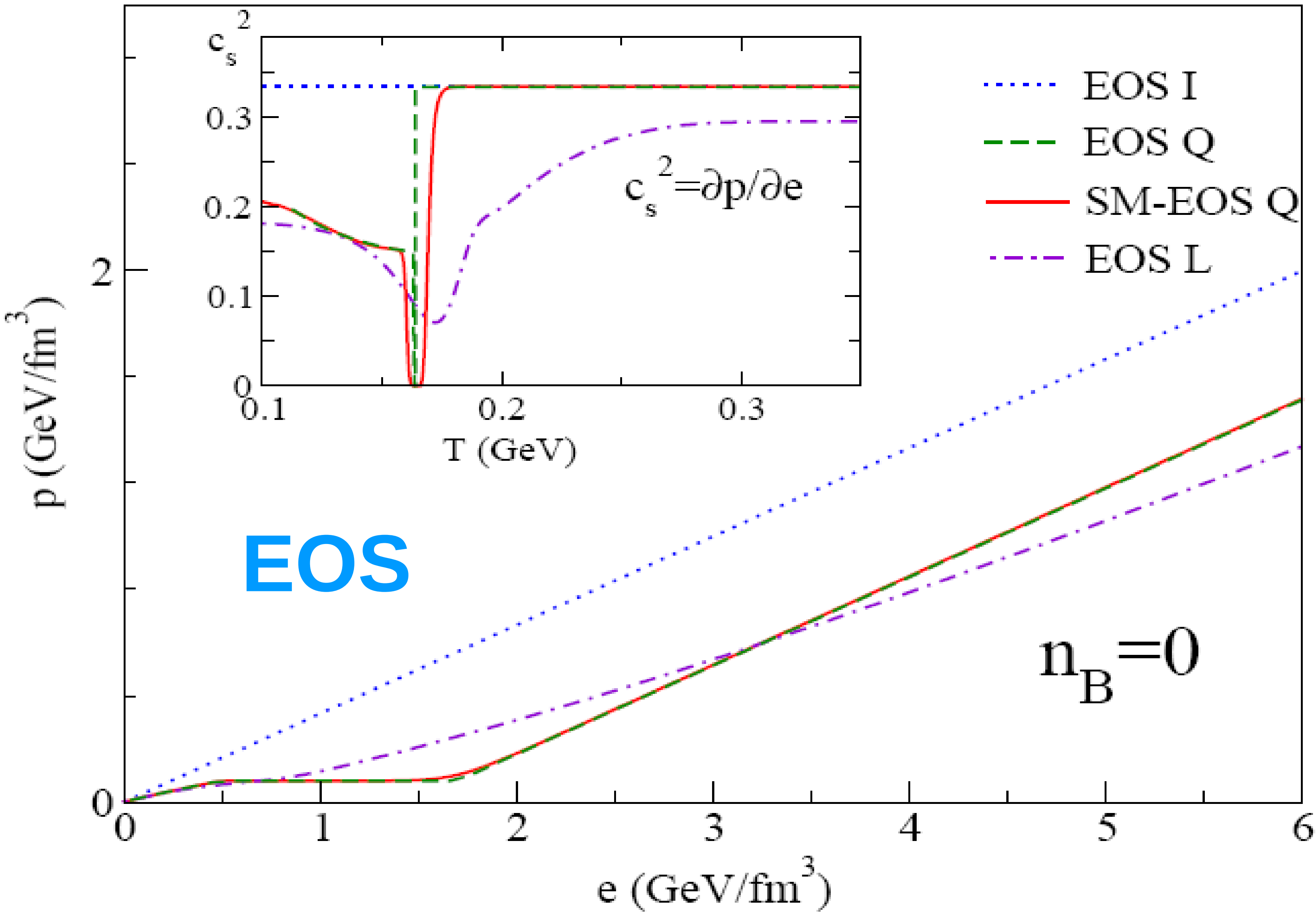
BUT:
to extract QGP viscosity, one must consider (at least) all the following aspects:

- a realistic EOS: EOS L vs. SM-EOS Q ~25% (for v_2 and v_2/ε)
- initial conditions: CGC initialization vs. Glauber initialization ~15-30% (for v_2)
- bulk viscosity: with vs. without bulk viscosity ~?%
- hadronic stage : viscous hydro+ hadron cascade in the furthure ?

… … … …

Thank You

EOS



c_s^2
0.3
0.2
0.1
0
0.1
0.2
0.3
T (GeV)
$c_s^2 = \partial p / \partial e$
EOS I
EOS Q
SM-EOS Q
EOS L
p (GeV/fm^3)
2
0
EOS
$n_B = 0$
0
1
2
3
4
5
6
e (GeV/fm^3)

EOS