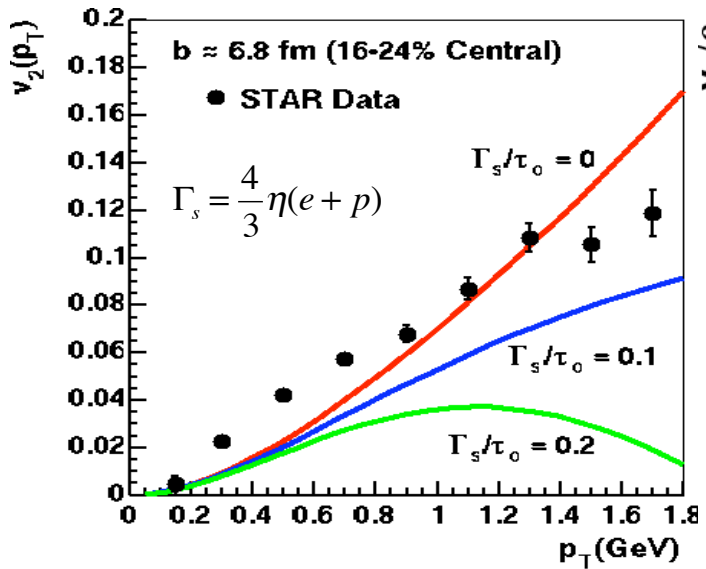


Flow Results and Hydrodynamic Limit

Aihong Tang for the STAR Collaboration

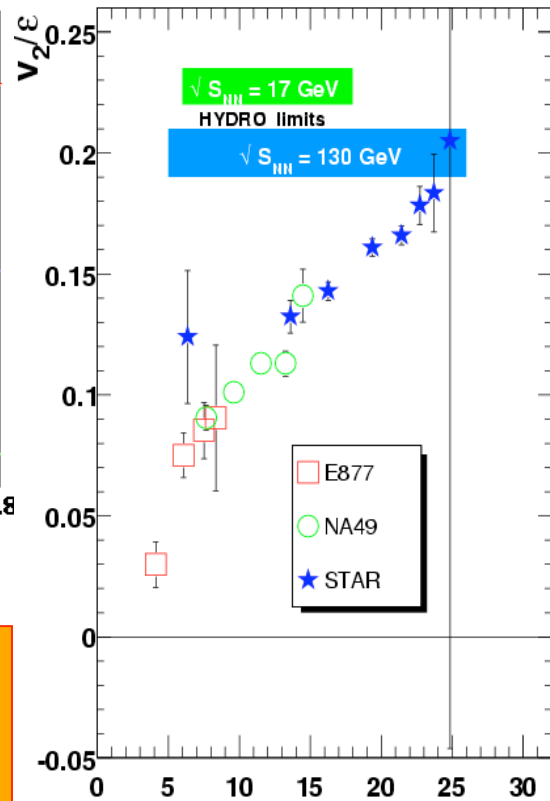


The story rewinded

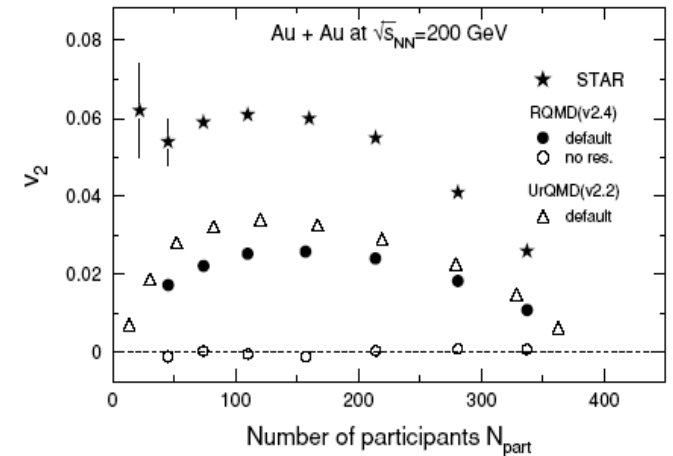


D. Teaney, PRC 68 034913 (2003)

Viscosity reduces v_2
Viscosity needs to be small
in order to explain data



STAR, PRC 66 034904 (2002)

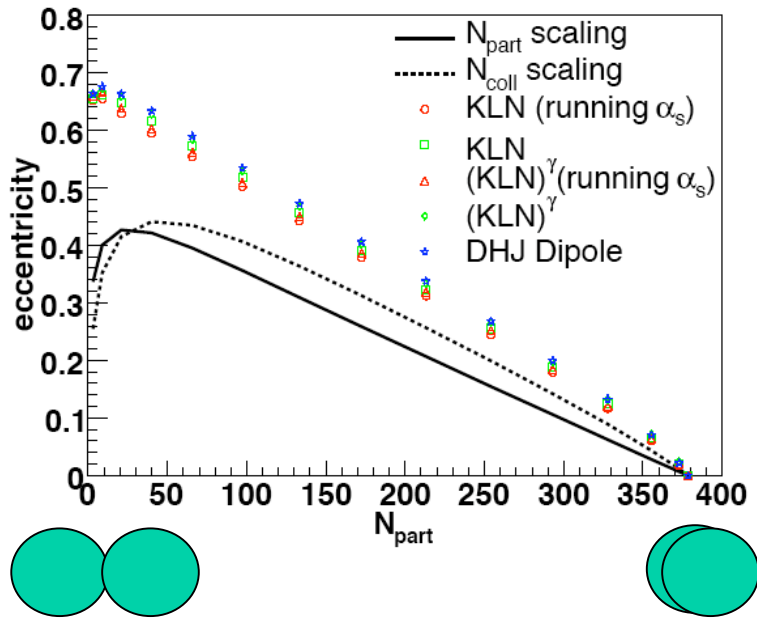


Y. Lu et al. Journal of Phys. G 32 1121 (2006)

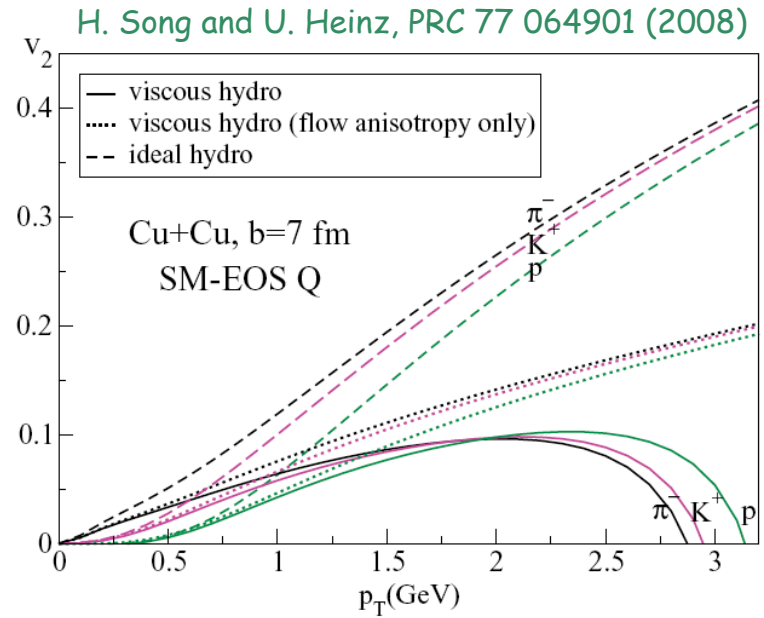
v_2/ϵ approaches the limit
of ideal hydrodynamics
Hadronic interaction
alone does not produce
enough v_2



The story continues

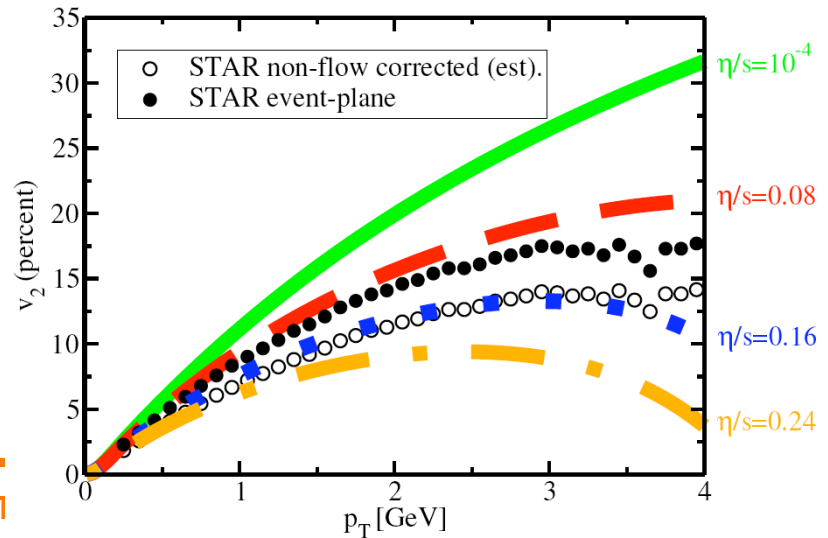
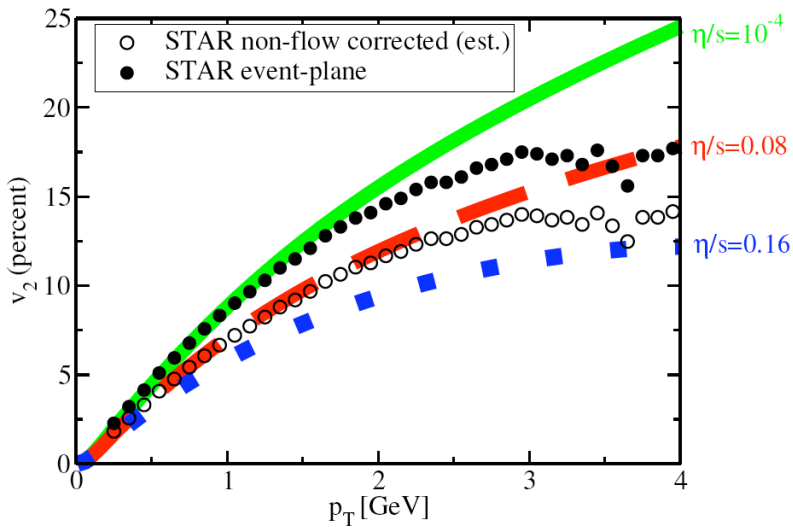


T. Hirano, RHIC & AGS Users Mtg 06



H. Song and U. Heinz, PRC 77 064901 (2008)

Glauber M. Luzum and P. Romatschke arXiv0804.4015 CGC



ing 1
and 0.16, 0.24



Are we saturated already ?

Are we still far away from there ?





How to view the hydro behavior better ? - Move away from it



- Ideal fluid and low viscosity \Leftrightarrow local equilibrium (small λ or large σ)

- **To study the local equilibrium, we have to move away from it,** say, check what if we relax the constraint of local equilibrium

- How to get a complete view? Study Boltzman equation for diluted system. It recovers Hydro when λ becomes small.

“To have a complete view of Lu Mountain, one has to move away from it.”

- Shi Su (1037~1101)



Transport Theory and Hydrodynamics

Transport Theory	Hydrodynamics
Microscopic	Macroscopic
Applicable out of equilibrium	Local equilibrium
Cannot describe phase transition	Can treat phase transition
$D \ll 1$	$K \ll 1$

D (Dilution parameter) =

$$\frac{\text{Typical distance between two particles}}{\text{Mean free path}}$$

K (Knudsen number) =

$$\frac{\text{Mean free path}}{\text{System size}}$$

Boltzmann Equation will be reduced to Hydrodynamics when both $D \ll 1$ and $K \ll 1$



Connecting Pieces

$$D \equiv \frac{n^{-1/3}}{\lambda} = \sigma n^{2/3}$$

n : particle density
 σ : parton cross section
 R : system size
 λ : mean free path

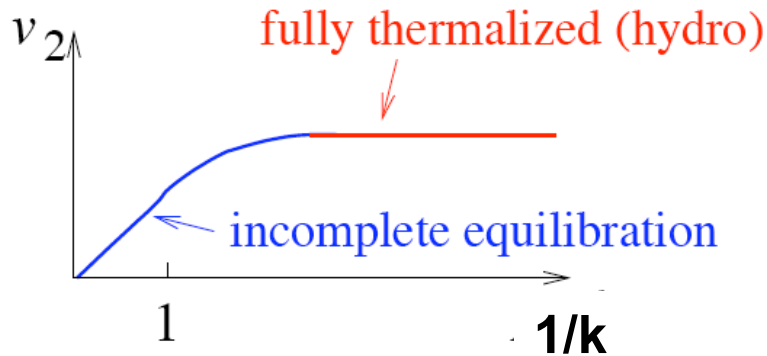
$$\frac{1}{K} \equiv \frac{R}{\lambda} \quad \leftarrow \text{Number of collisions.}$$

$$\lambda = \frac{1}{\sigma n}$$

$$n = \frac{1}{ct} \frac{1}{S} \frac{dN}{dy}$$

$$t \sim R / c_s$$

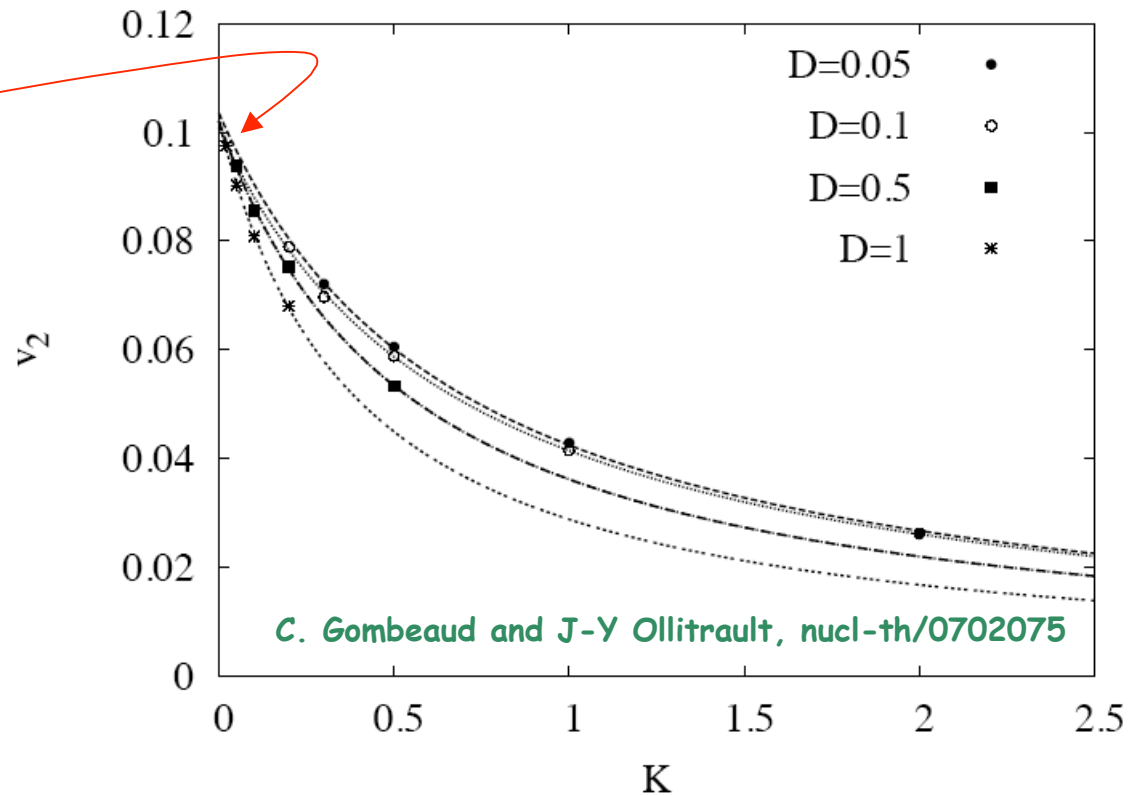
$$\frac{1}{K} = \sigma \left[\frac{1}{S} \frac{dN}{dy} \right] \frac{c_s}{c}$$





v_2 from Solving the Boltzmann Equation

Hydro limit is recovered when $D \ll 1$ and $K \ll 1$



$v_2 \propto 1/K$, and v_2 saturates eventually when the system reaches local equilibrium

$$\Rightarrow \frac{v_2}{\varepsilon} = \frac{v_2^{hydro}}{\varepsilon} \frac{1}{1 + K / K_0}$$

$K \rightarrow 0$ Hydro limit

$K \gg 1$ Low density limit,

$$v_2/\varepsilon \propto 1/S \, dN/dy$$



Choose the right $\{v_2, \varepsilon\}$ pairs

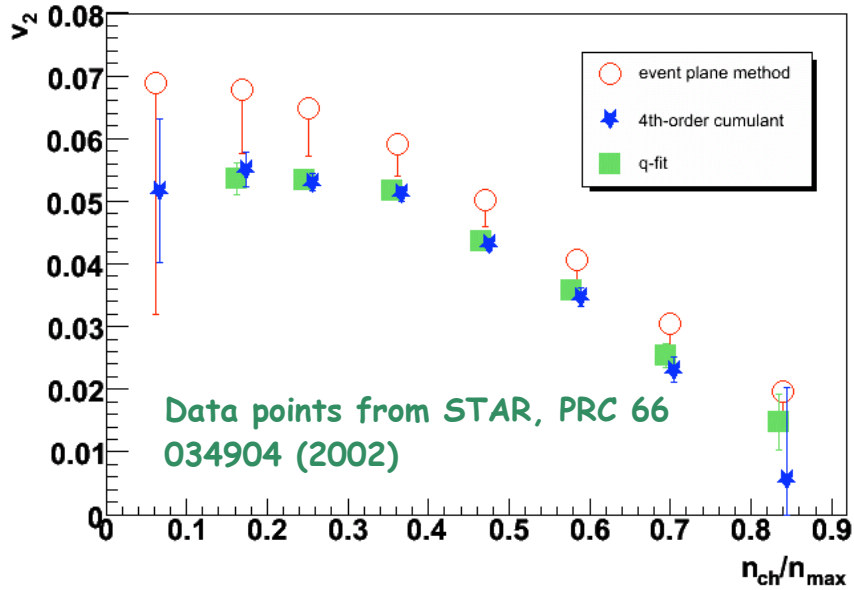
<p>v_2 that are sensitive to anisotropy w.r.t. the Reaction Plane v_2:</p> <p>$v_2\{4\}$, $v_2\{qDist\}$, $v_2\{qCumulant4\}$, $v_2\{ZDCSMD\}$</p>	<p>v_2 that are sensitive to anisotropy w.r.t. the Participant Plane :</p> <p>$v_2\{2\}$, $v_2\{EP\}$, $v_2\{uQ\}$ etc.</p>
<p>ε that are sensitive to anisotropy w.r.t. the Reaction Plane:</p> <p>$\varepsilon\{std\}$, $\varepsilon\{4\}$</p>	<p>ε That are sensitive to anisotropy w.r.t. the Participant Plane:</p> <p>$\varepsilon\{part\}$ $\varepsilon\{2\}$</p>

R.Bhalerao and J-Y. Ollitrault, Phys. Lett. B 614 (2006) 260

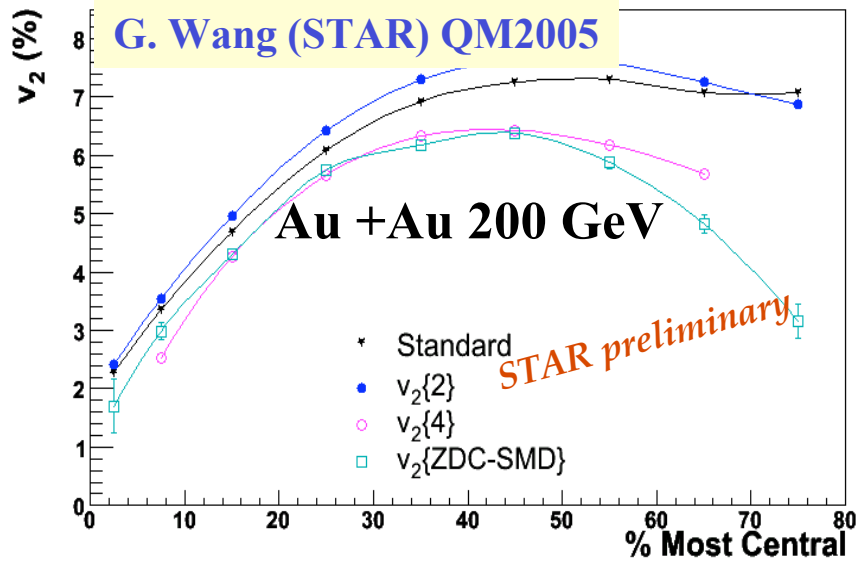
S.Voloshin, A.Poskanzer, A.Tang and G.Wang, Phys. Lett. B 659 (2008) 537



v_2 methods



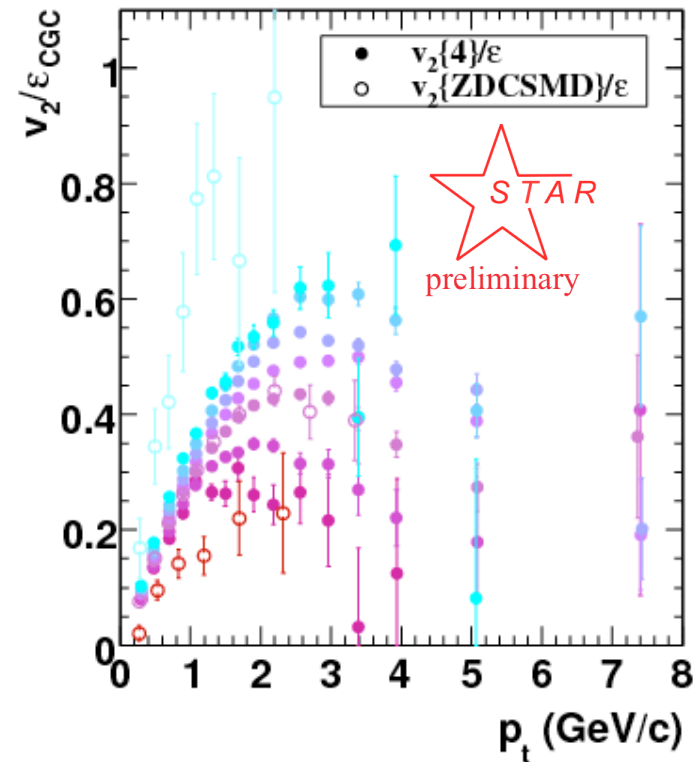
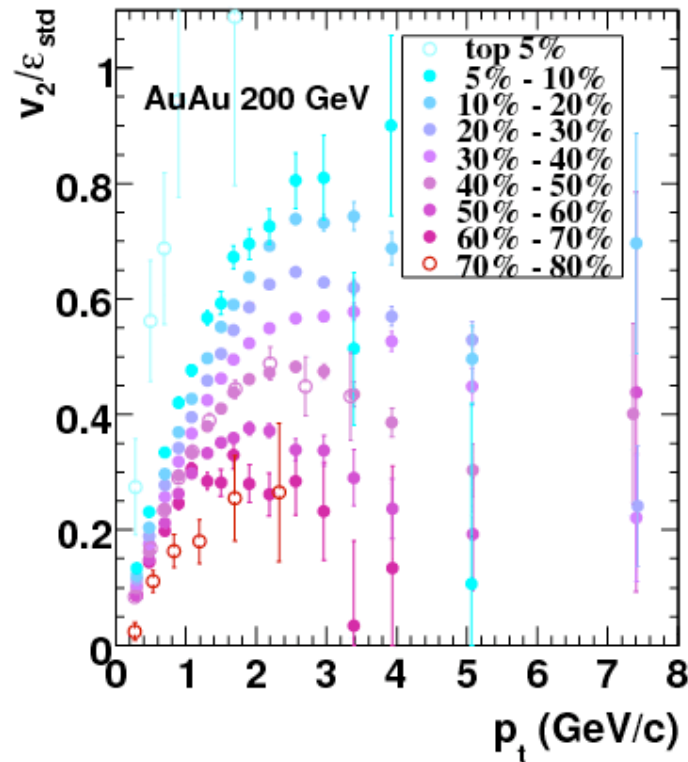
$$v_2\{4\}=v_2\{qDist\}$$



$$v_2\{4\}=v_2\{ZDC-SMD\}$$



Flow increases



The p_t where v_2/ϵ peaks increases with p_t - the applicable range
For hydrodynamics extends to large p_t in central collisions.
 v_2/ϵ for the CGC case sees hints of saturation.

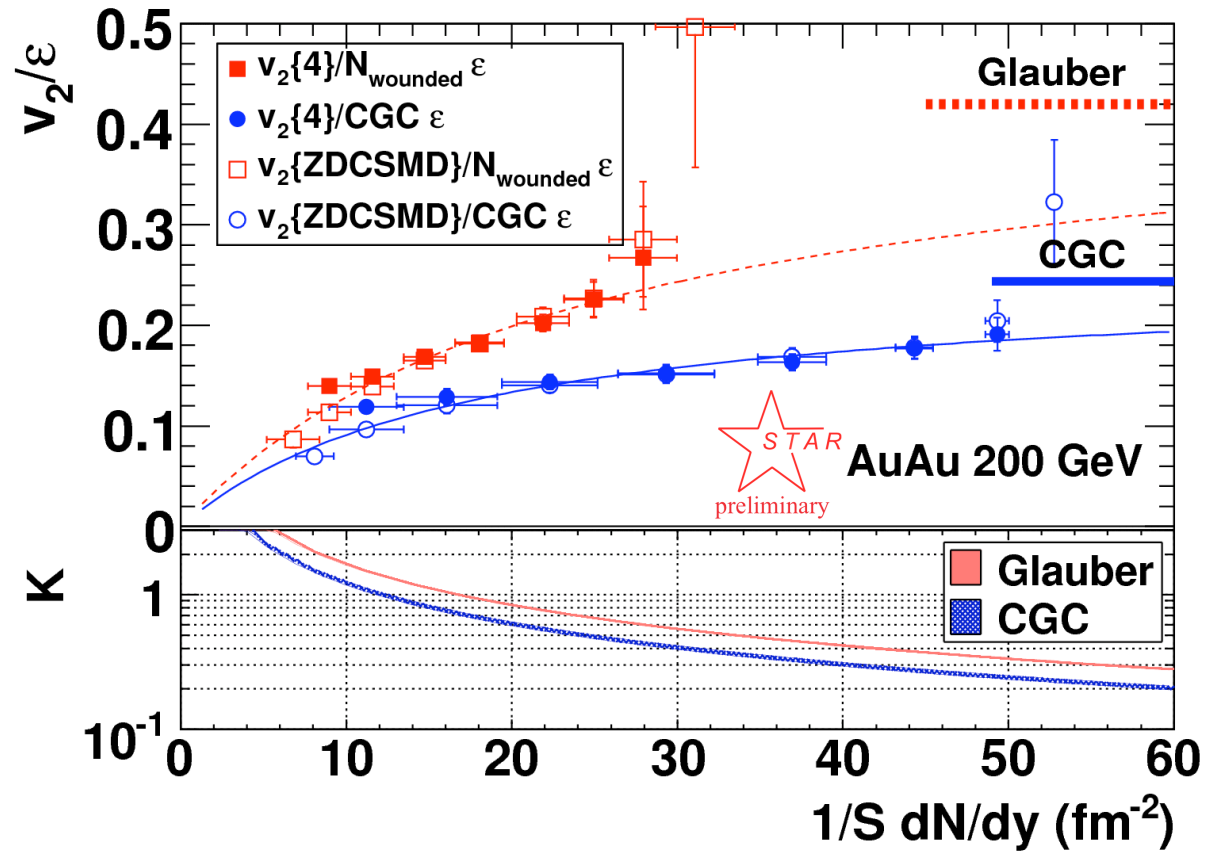


How much deviation from ideal hydro ?

$$\frac{v_2}{\varepsilon} = \frac{v_2^{hydro}}{\varepsilon} \frac{1}{1 + K / K_0}$$

$$K = \lambda / R$$

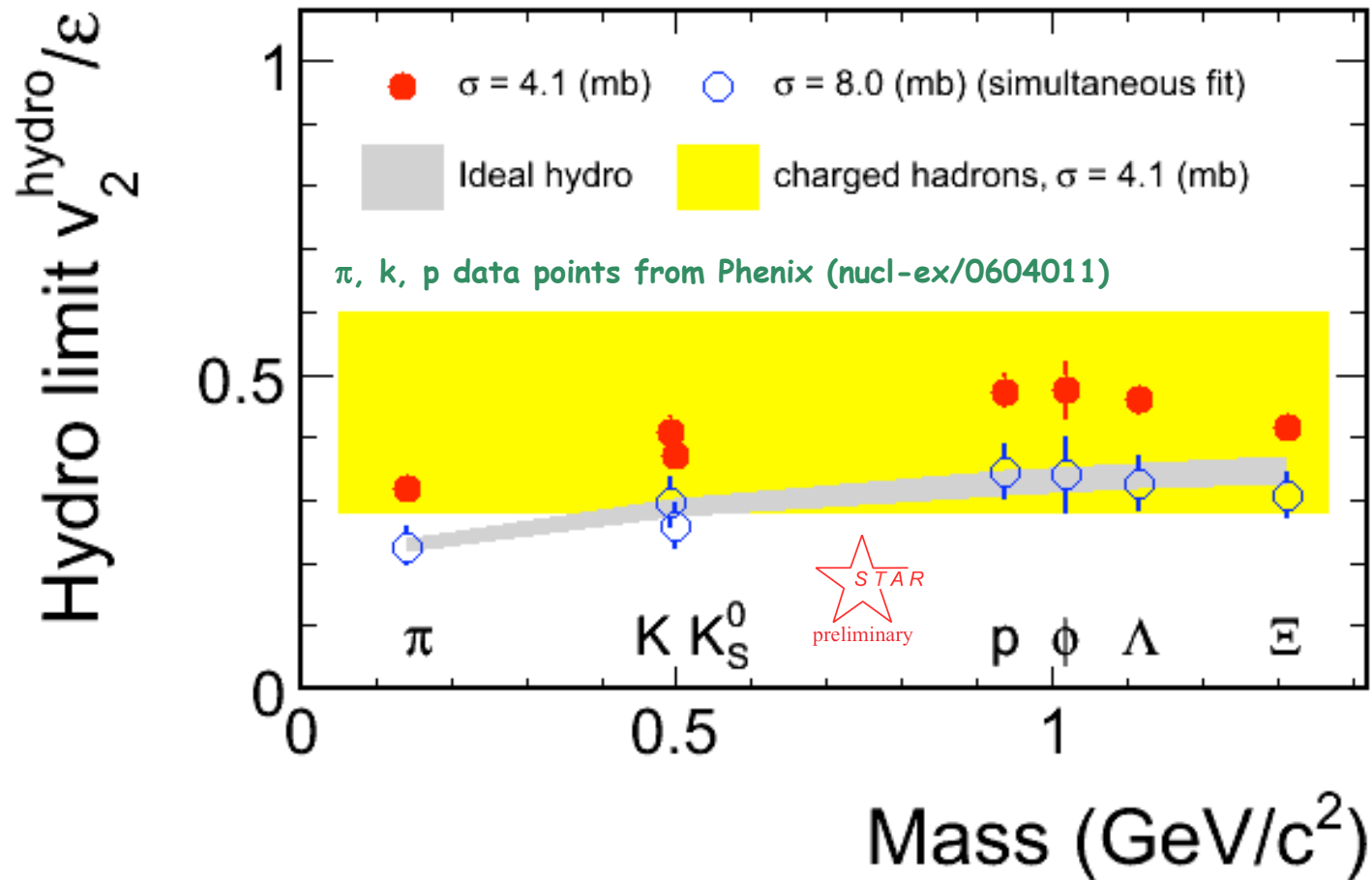
$$\frac{1}{K} = \frac{\sigma}{S} \frac{dN}{dy} C_s$$



Fitting function from Drescher, Dumitru, Gombeaud, J.Ollitrault, Phys. Rev. C76, 024905(2007)
 CGC ε obtained from A.Adil, H-J Drescher, A.Dumitru, A.Hayashigaki and Y.Nara, Phys. Rev. C 74 044905 (2006)



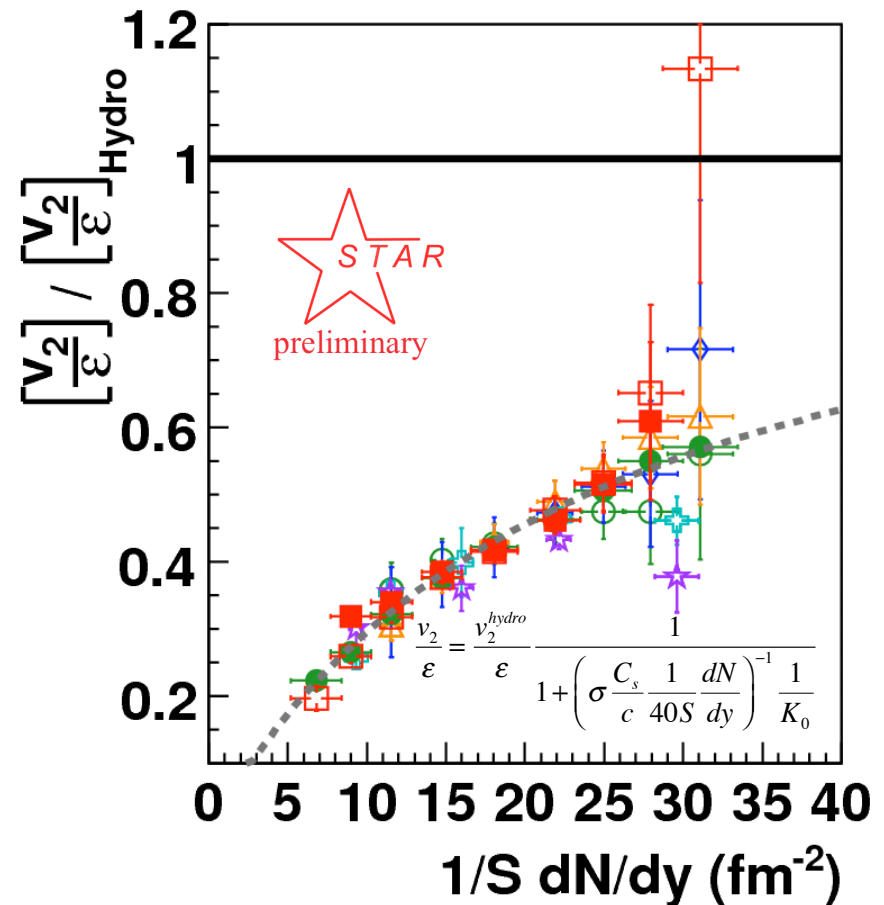
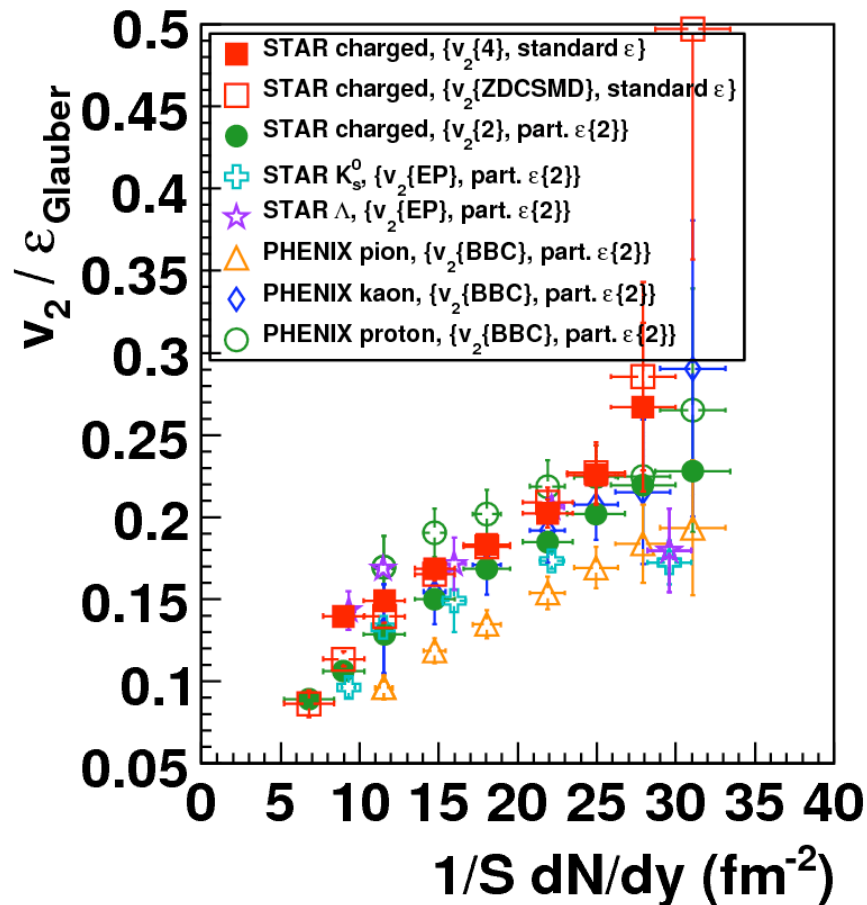
Hydro limit for ID'd particles



Fitted limit increases with mass



How much deviation from ideal hydro ?

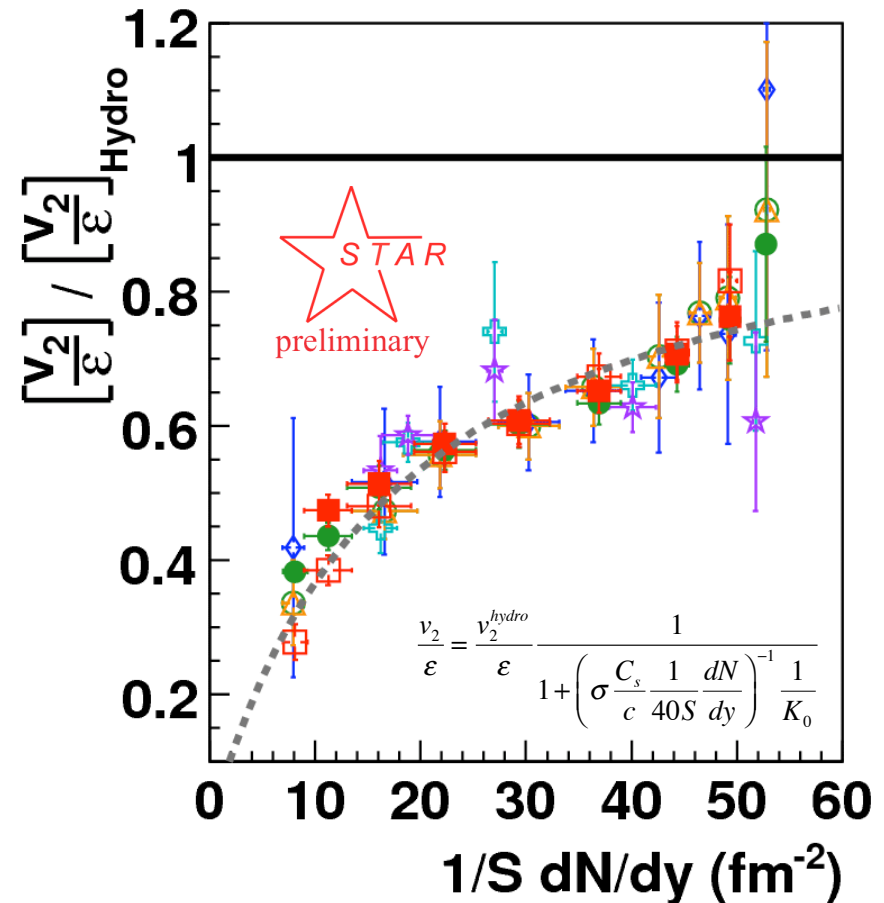
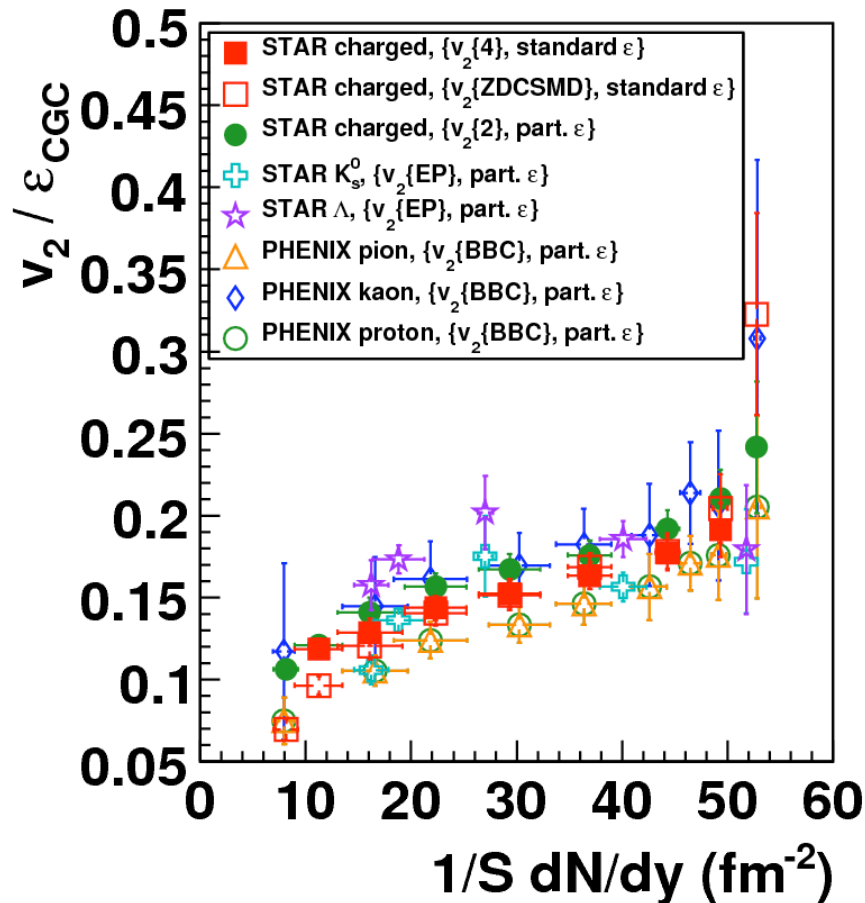


π, k, p data points from Phenix (nucl-ex/0604011)

With the assumption that parton cross section (σ) is same for all ID'd particles, a universal trend of approaching hydro limit is established.
 Anti-correlation found between σ and C_s .
 Good constraint on EoS



How much deviation from ideal hydro ?

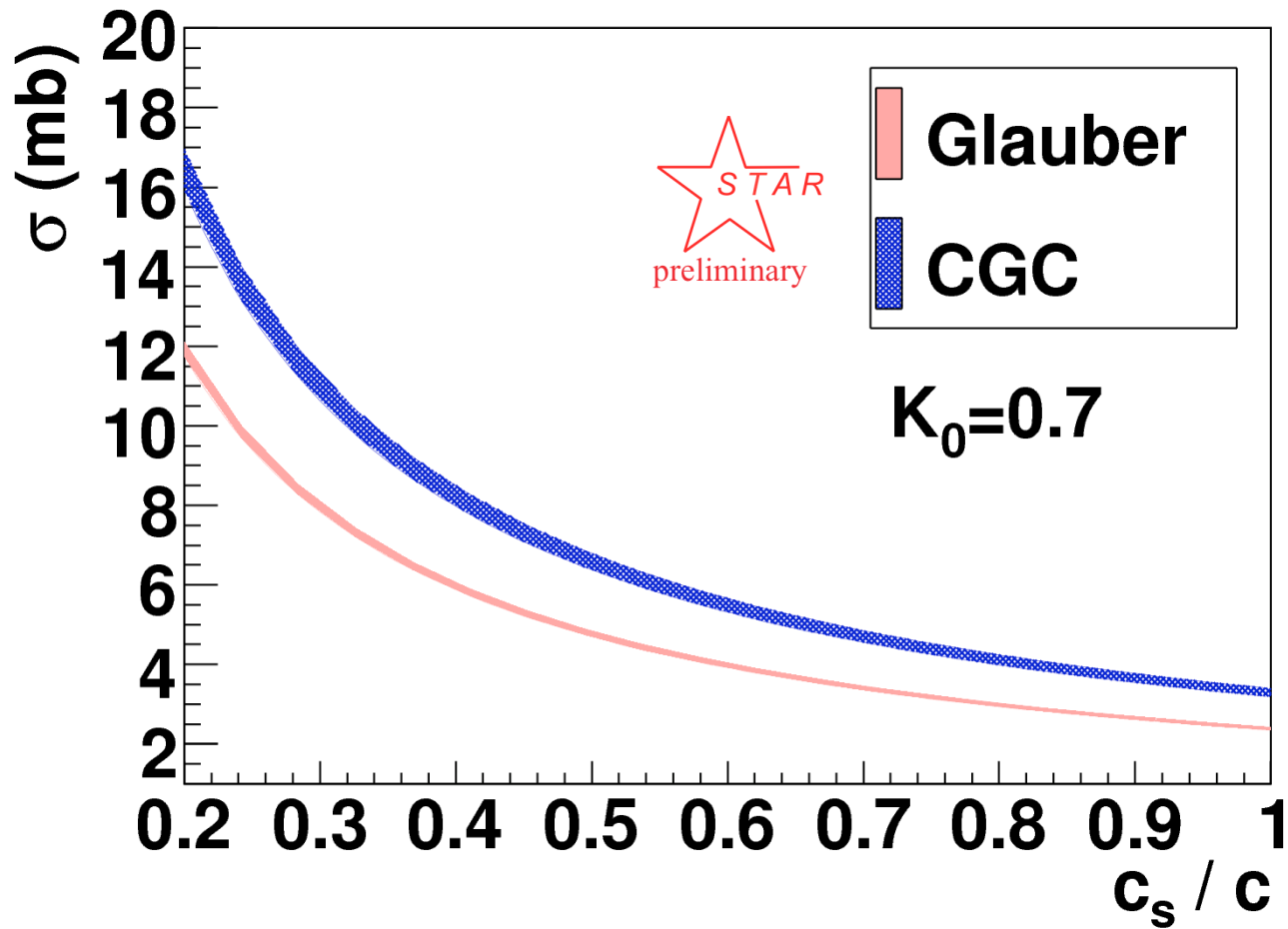


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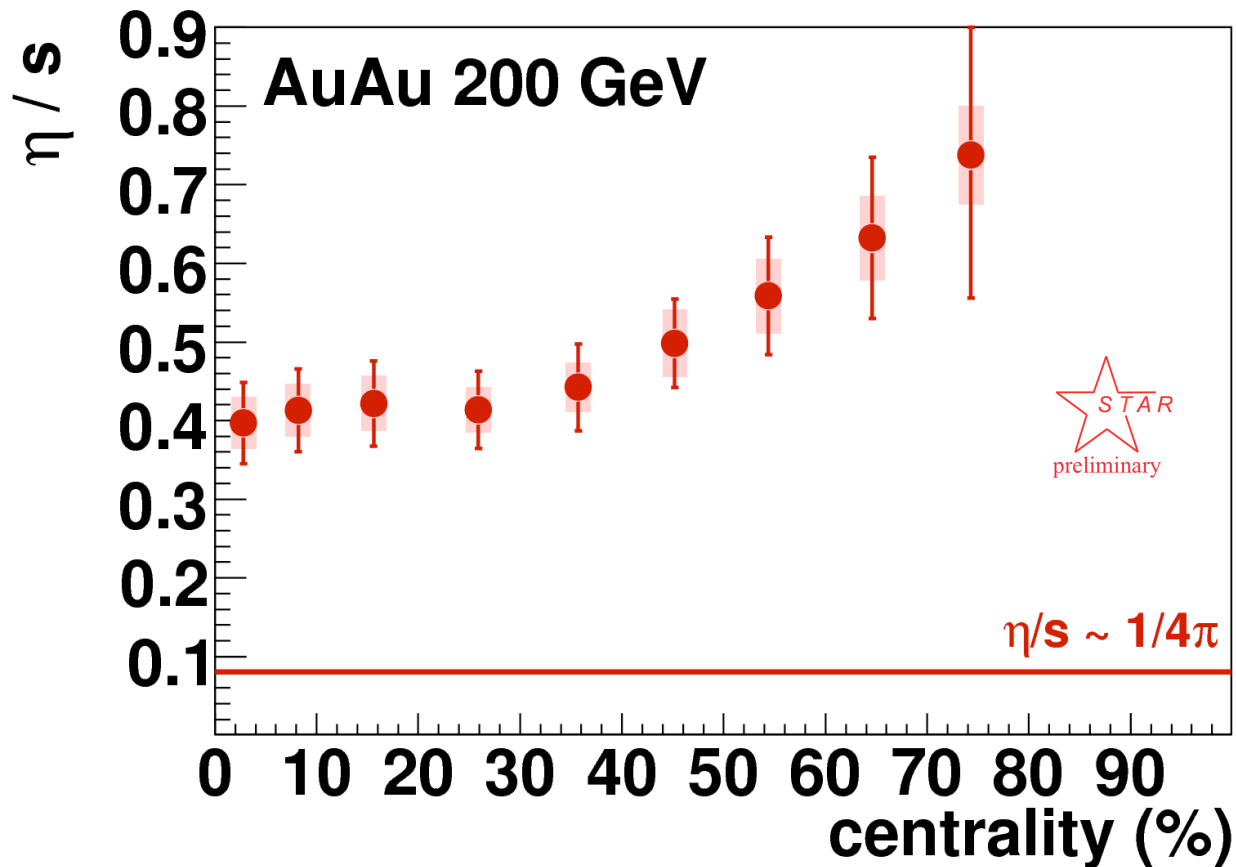
Constraint on EoS



$$\sigma \left(\frac{c_s}{c} \right) k_0 = \text{const.}$$



Constraint on η/s



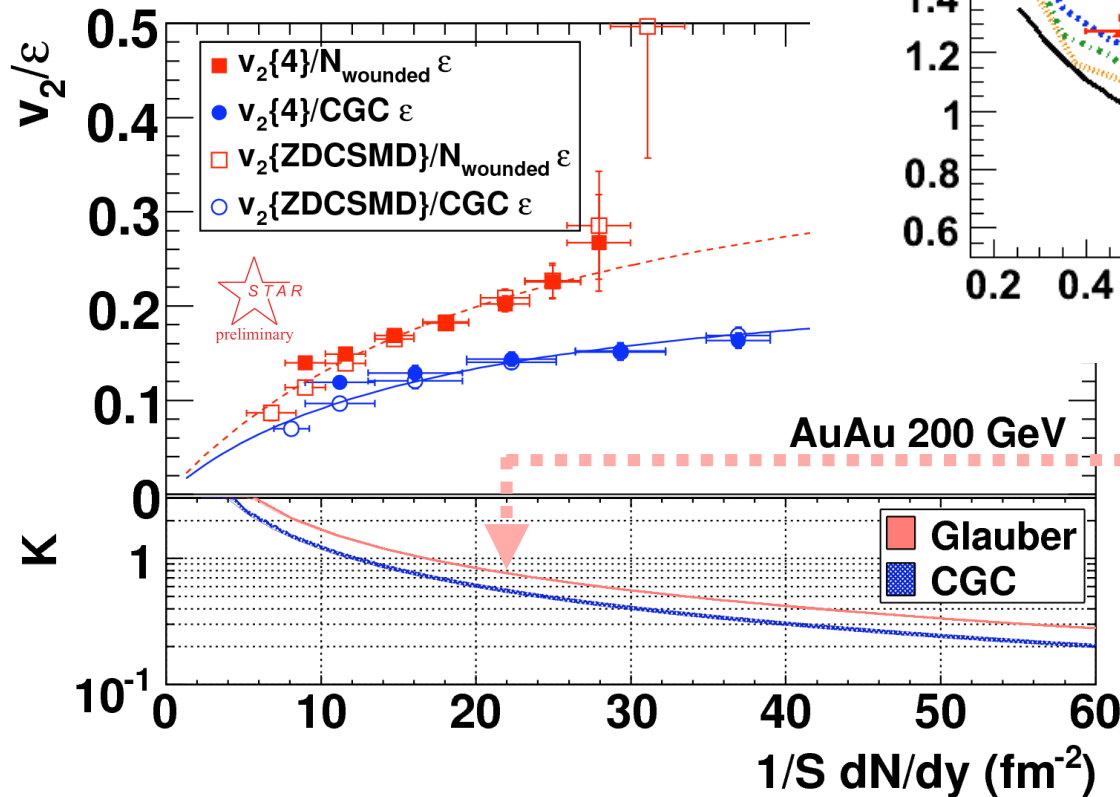
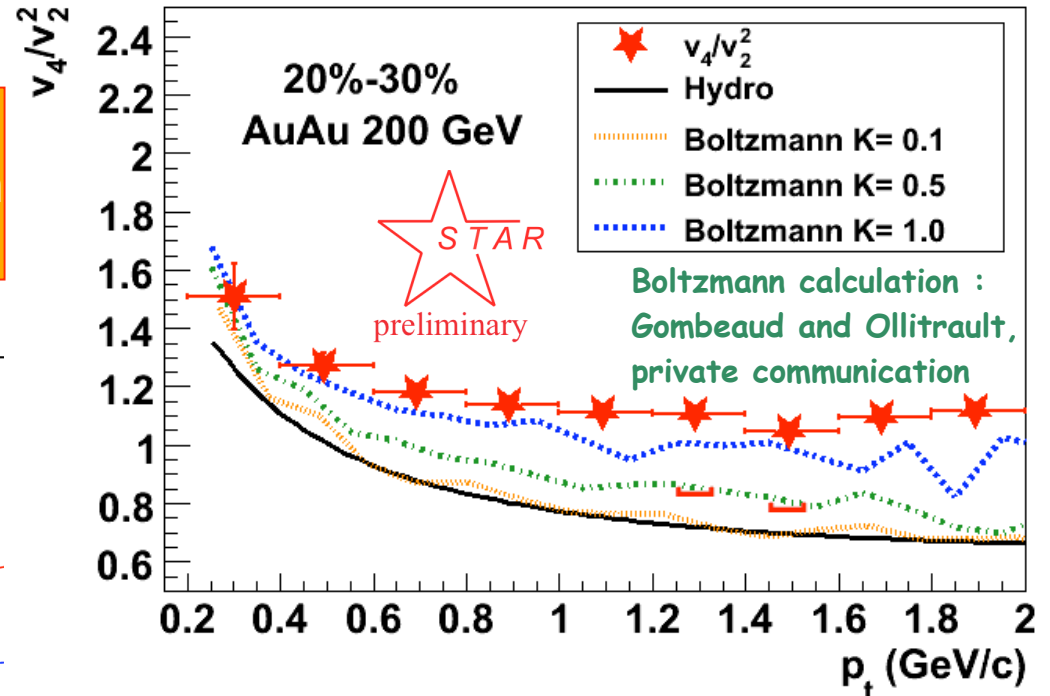
$\eta/s \sim 4 \times$ quantum limit.

Discrepancies between different approaches need to be understood.



A slightly different approach on Knudsen number

For centrality 20-30%, Knudsen number needs to be greater than 0.5 in order to explain data



$$K = \frac{\lambda}{R}$$

K=0 : Ideal Hydro
K>>1 : Free Streaming



Summary

- v_2/ε , if examined with transport motivated formula with certain assumptions, approaches Hydro limit in a similar way for different particle species.
- In central AuAu collisions, v_2/ε is considerably away from Hydro limit. This conclusion is independent of PID, initial conditions, and choice of $\{v_2, \varepsilon\}$ pairs.
- Knudsen number extracted from fitting v_2/ε vs. $1/S$ dN/dy , as well as that extracted from v_4/v_2^2 , stays finite for central collisions - not as zero as required by ideal hydrodynamics.
- Good constraint on EoS obtained.
- For the first time the centrality dependence of η/s is presented.