Perturbative Shear Viscosity in $\mathcal{N}$=4 SYM

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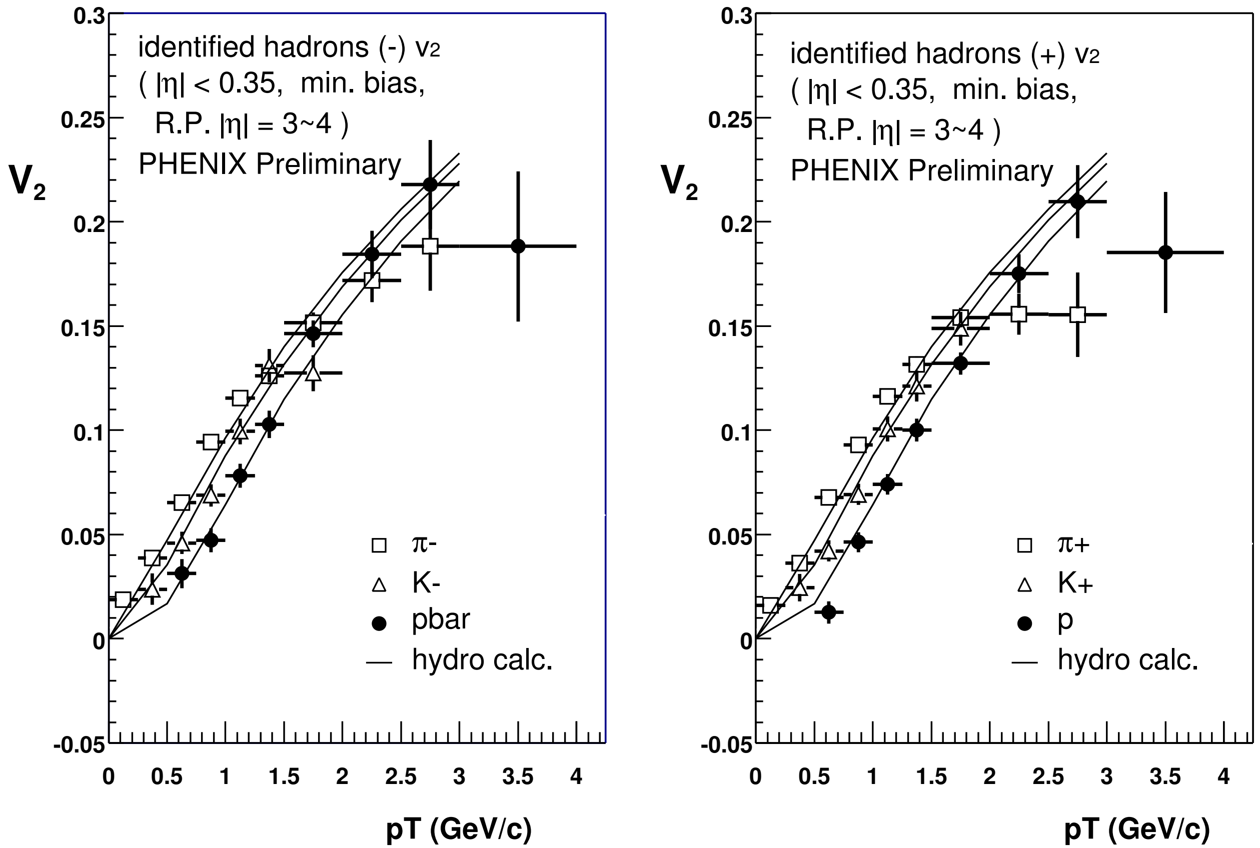
Part I

Story so far ...

We all love this



Heinz and Kolb, Ideal Hydro calculation

Let me remind you ...

- Ideal Hydrodynamics has 3 parts:
  - ∗ Energy-momentum conservation: $\partial_\mu \langle T^{\mu\nu} \rangle = 0$
  - ∗ Non-dissipative system $\langle T^{\mu\nu} \rangle_{\text{ideal}} = (\epsilon + \mathcal{P}) u^\mu u^\nu + \mathcal{P} g^{\mu\nu}$
  - ∗ Equation of state $\mathcal{P} = f(\epsilon)$, e.g. $\mathcal{P} = \frac{1}{3}\epsilon$

Dissipative system

- The stress-energy tensor gets disspative part

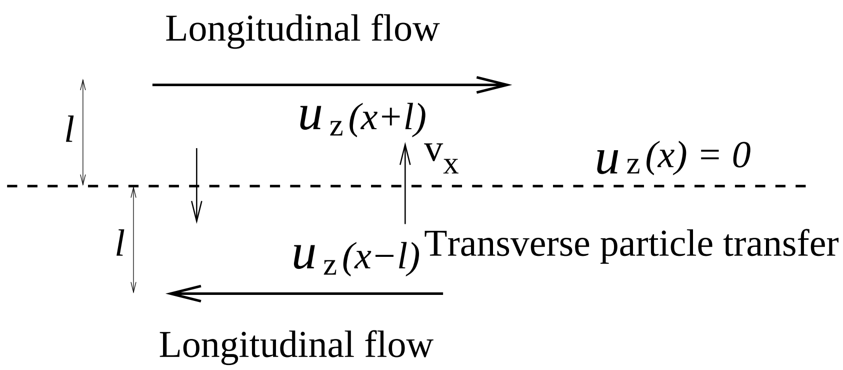
$$\langle T^{\mu\nu} \rangle = \left\langle T^{\mu\nu}_{\text{ideal}} \right\rangle + \pi^{\mu\nu}$$

- In the fluid rest frame,

$$\begin{aligned} \pi_{ij} \;&=\; -\frac{\eta}{\langle \epsilon + \mathcal{P} \rangle} \left(\partial_i \left\langle T_{0j} \right\rangle + \partial_j \langle T_{0i} \rangle - \frac{2}{3} \delta_{ij} \partial_k \langle T_{0k} \rangle \right) \\ &\quad - \frac{\zeta}{\langle \epsilon + \mathcal{P} \rangle} \delta_{ij} \partial_k \langle T_{0k} \rangle \end{aligned}$$

- Positive η $\Longrightarrow$ Entropy increases by mixing
- Positive ζ $\Longrightarrow$ Entropy increase by redistributing energy (including particle production)

Shear Viscosity



u_z: Flow velocity

v_x: Average speed of microscopic particles

- Rough estimate (fluid rest frame, or $u_z(x) = 0$ and $u_0(x) = 1$)

  $*$ The off-diagonal element T_{xz}: Current in the x-dir for the conserved momentum density $T_{0z} = (\epsilon + \mathcal{P})u_0 u_z$

$$\langle \epsilon + \mathcal{P} \rangle \, v_x u_0 (u_z(x-l) - u_z(x+l))$$
$$\approx -2 \, \langle \epsilon + \mathcal{P} \rangle \, v_x \, l \partial_x u_z(x) \sim -\eta \partial_x u_z$$

  Here l: Mean free path

  $*$ Recall thermo. id.: $\langle \epsilon + \mathcal{P} \rangle = sT$

$$\eta \sim \langle \epsilon + \mathcal{P} \rangle \; l_{\text{mfp}} \; \langle v_x \rangle \sim s \, T \, l_{\text{mfp}} \; \langle v_x \rangle$$

Perturbative estimate

High Temperature limit

- $\eta/\mathrm{s} \approx T l_{\mathrm{mfp}} \approx \dfrac{T}{n\sigma} \sim \dfrac{1}{T^2\sigma}$

- The only energy scale: T

$$\sigma \sim \frac{(\text{coupling constant})^{\#}}{s} \sim \frac{(\text{coupling constant})^{\#}}{T^2}$$

Hence

$$\frac{\eta}{\mathrm{s}} \sim \frac{1}{(\text{coupling constant})^{\#}} \gg 1$$

- Perturbative QCD partonic 2-2 cross-sections

$$\sigma \propto \frac{\alpha_s^2}{s} f(t/s, u/s), \qquad s \sim T^2$$

Naively expect

$$\eta/\mathsf{s} \sim \frac{1}{\alpha_s^2}$$

Coulomb enhancement (cut-off by m_D) leads to

$$\eta/\mathsf{s} \sim \frac{1}{\alpha_s^2 \ln(1/\alpha_s)} \quad : \text{ Not an ideal hydro}$$

Ideal Hydro is non-perturbative

- Ideal hydro $\Longrightarrow$ $\eta/\mathrm{s} \ll 1$
- Questions:

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  - * How small can the QCD η/s be?

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- Questions:
  - ∗ When can we have this limit?
    - – Strong coupling.
  - ∗ Is there a lower bound to η/s?
    - – Maybe.
  - ∗ How small can the QCD η/s be?
    - – This talk.

Limits on gas η

[Danielewicz and Gyulassy, 1985]

- Recall: $\eta \sim \langle \epsilon + \mathcal{P} \rangle \, v_x l_{\mathrm{mfp}}$ also $\langle \epsilon + \mathcal{P} \rangle \, v_x \sim \langle p_x \rangle \, n$

  $\Longrightarrow \eta \sim \langle p_x \rangle \, n l_{\mathrm{mfp}}$

- Gas means $\Delta p_x \geq 1/l_{\mathrm{mfp}} \Longrightarrow \dfrac{\eta}{n} \gtrsim 1$

- Gas means $l_{\mathrm{mfp}} > n^{-1/3} \Longrightarrow \dfrac{\eta}{n} \gtrsim \langle p_x \rangle \, n^{-1/3}$

Part II

Shear viscosity in $\mathcal{N}=4$ SYM

Son, Starinets, Policastro, Kovtun, Buchel, Liu, ...

- Strong coupling limit, 4 ingredients
  - Kubo formula
  $$\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, d^3x\, e^{i\omega t}\, \langle [T_{xy}(x), T_{xy}(0)] \rangle$$
  - Gauge-Gravity duality
  $$\sigma_{\text{abs}}(\omega) = \frac{8\pi G}{\omega} \int dt\, d^3x\, e^{i\omega t}\, \langle [T_{xy}(x), T_{xy}(0)] \rangle$$
  - $\lim_{\omega\to 0} \sigma_{\text{abs}}(\omega) = A_{\text{blackhole}}$
  - Entropy of the BH : $\mathsf{s} = A_{\text{blackhole}}/4G$

Therefore, (including the first order correction)

$$\frac{\eta}{\mathsf{s}} = \frac{1}{4\pi}\left(1 + \frac{7.12}{(g^2 N_c)^{3/2}}\right)$$

PR annoucements

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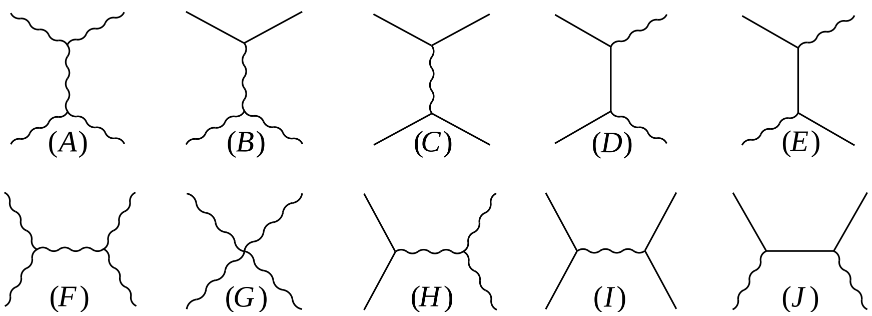
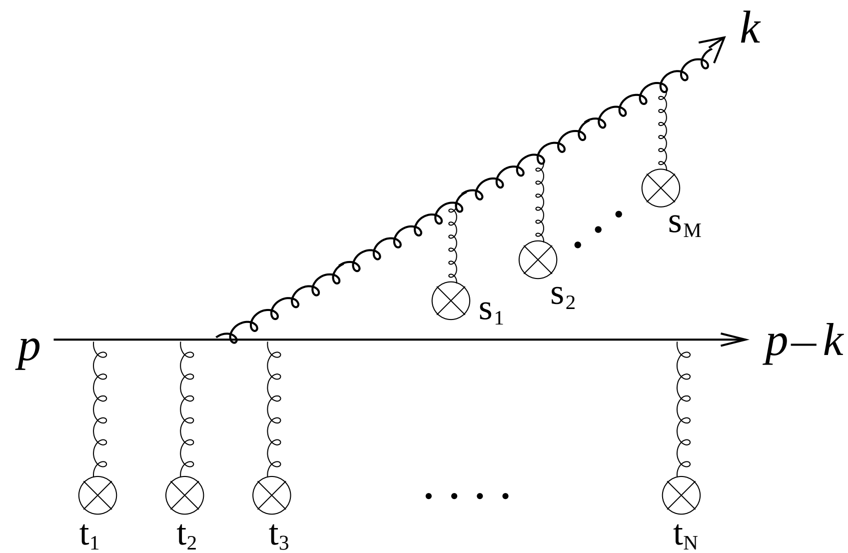
How close is the SYM to QCD?

Check the weak coupling limit! – This talk.

QCD η calc

Relevant processes



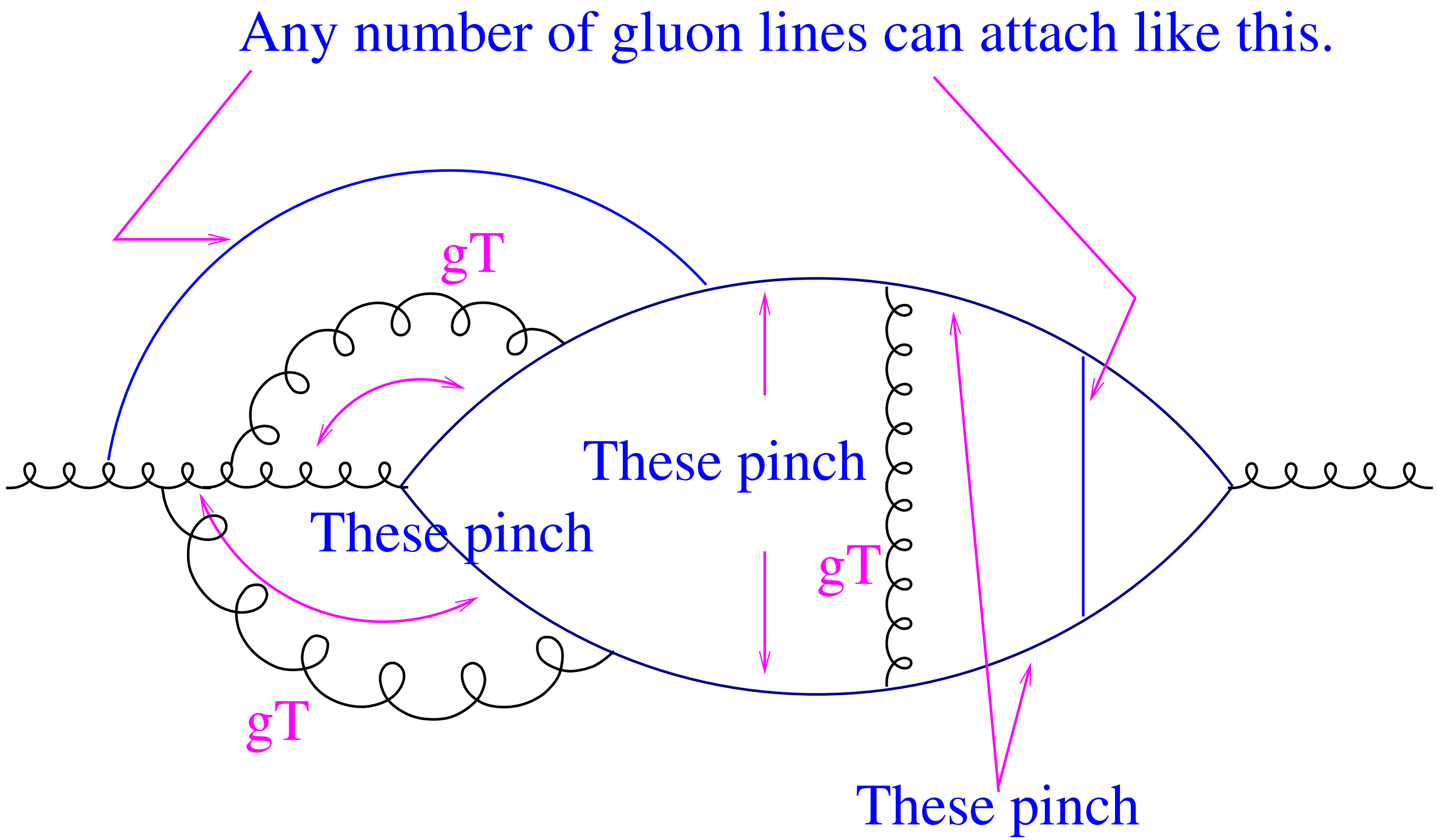


Use kinetic theory

$$\frac{df}{dt} = \mathcal{C}_{2\leftrightarrow 2} + \mathcal{C}_{1\leftrightarrow 2}$$

Complication: $1 \leftrightarrow 2$ process needs resummation (LPM effect)

QCD LPM diagrams



Adding one more rung = O(1).
Need to resum.

Procedure – Schematic

- Let $f = f_{\text{eq}} + f_{\text{eq}}[1 \pm f_{\text{eq}}]f_1$ with $f_{\text{eq}} = 1/(e^{p^\mu u_\mu(x)\beta(x)} \mp 1)$.

- LHS of the Boltzmann eq. in the fluid rest frame:

$$\begin{aligned} I &= \beta(x)\left(\left(\frac{1}{3} - v_s^2\right)\mathbf{k}^2 - v_s^2 m_{\text{phys}}^2\right)\nabla\cdot\mathbf{u}(x) \\ &\quad + \frac{\beta(x)}{2}\left(k_i k_j - \frac{\mathbf{k}^2}{3}\delta_{ij}\right)\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{u}\right) \end{aligned}$$

- RHS of the Boltzmann eq. in the fluid rest frame, e.g.:

$$\begin{aligned} \mathcal{C}_{2\leftrightarrow 2} &= \int |M|^2 \delta(p + k - p' - k') f_0(p) f_0(k)[1 \pm f_0(p')][1 \pm f_0(k')] \\ &\quad \times [f_1(p) + f_1(k) - f_1(p') - f_1(k')] \end{aligned}$$

with

$$f_1 = \beta A \nabla\cdot\mathbf{u} + \frac{\beta}{2}\left(\widehat{p}_i \widehat{p}_j - \frac{1}{3}\delta_{ij}\right) B(x,p)\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{u}\right)$$

Procedure − Cont.

- Now solve for $B(x,p)$ and get η from

$$\langle T^{\mu\nu}\rangle = T^{\mu\nu}_{\text{eq}} + \int f_{\text{eq}}[1 \pm f_{\text{eq}}] f_1 \left(k^\mu k^\nu + \frac{1}{4} g^{\mu\nu} \delta m^2 \right)$$

- Result:

$$\frac{\eta}{\text{s}} = \frac{A}{N_c^2 g^4 \ln(B/g\sqrt{N_c})} \quad A, B = \begin{cases} 34.8,\ 4.67,\ N_f = 0 \\ 46.1,\ 4.17,\ N_f = 3 \end{cases}$$

$\mathcal{N}$=4 SYM calculation

$$\mathcal{L} = -\frac{F^2}{4} - \frac{1}{2}\sum_{i<j} D_\mu\phi_{ij}\, D^\mu\phi^{ij} + i\lambda^\dagger D^\mu\sigma_\mu\lambda$$
$$- \frac{g^2}{4}\sum_{i<j,k<l}[\phi_{ij},\phi_{kl}][\phi^{ij},\phi^{kl}]$$
$$+ \sum_{i<j} g\sqrt{2} f^{abc}\left(\phi^{a,ij}\lambda_i^{b\,\dagger}\epsilon\lambda_j^c - \phi_{ij}^a\lambda^{i\,b\,\dagger}\epsilon\lambda^{j\,c}\right)$$

Customary to define the t'Hooft coupling $\lambda = g^2 N_c$

d.o.f. count: All fields in Adjoint representation.

- Transverse gauge field A_μ^a
- 4 Weyl fermions λ_i^a with $i = 1-4$
- 6 real scalar $\phi_{ij}^a = -\phi_{ji}^a$
- Total: $2+8+6=16$ d.o.f. per color index a. 128 for $N_c = 3$.
- QCD with $N_c = 3$, $N_f = 3$: $2\times3\times2\times3+2\times8 = 52$

SYM viscosity calculation

- Follow the same procedure as the QCD case. Many differences:
  - ∗ DOF counting is quite different.
  - ∗ Thermal masses are all the *same*: $m^2_{\text{th}} = \lambda T^2$
  - ∗ Many scattering channels unavailable in QCD open up:
    - ∘ Common (although differently weighted)

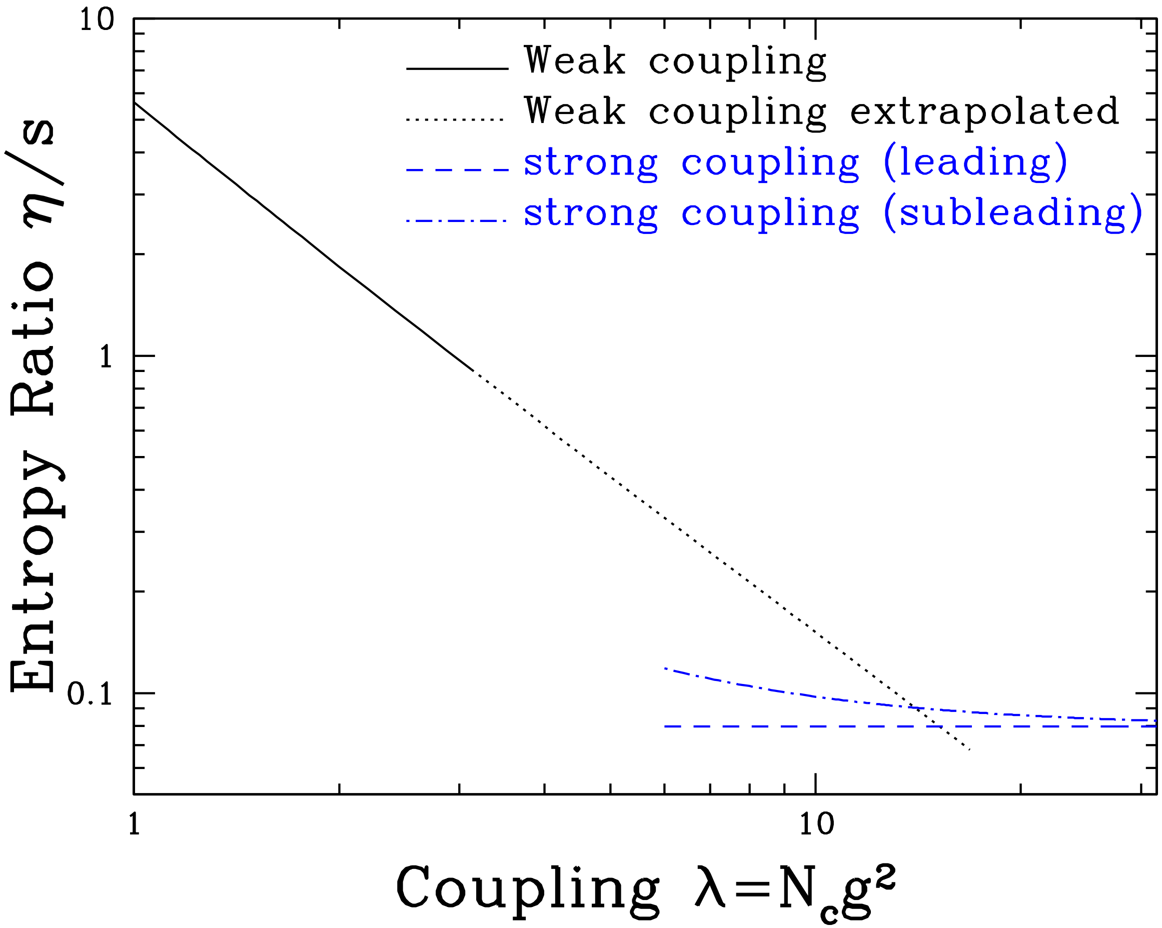
      $FF \leftrightarrow FF \quad FG \leftrightarrow FG$

      $GG \leftrightarrow GG$
    - ∘ New channels involving scalar

      $SS \leftrightarrow SS \quad SF \leftrightarrow SF$

      $SF \leftrightarrow GF \quad SG \leftrightarrow SG$
    - ∘ Also more $1 \leftrightarrow 2$ channels open up: SFF, GSS, GFF, GGG

SYM Result

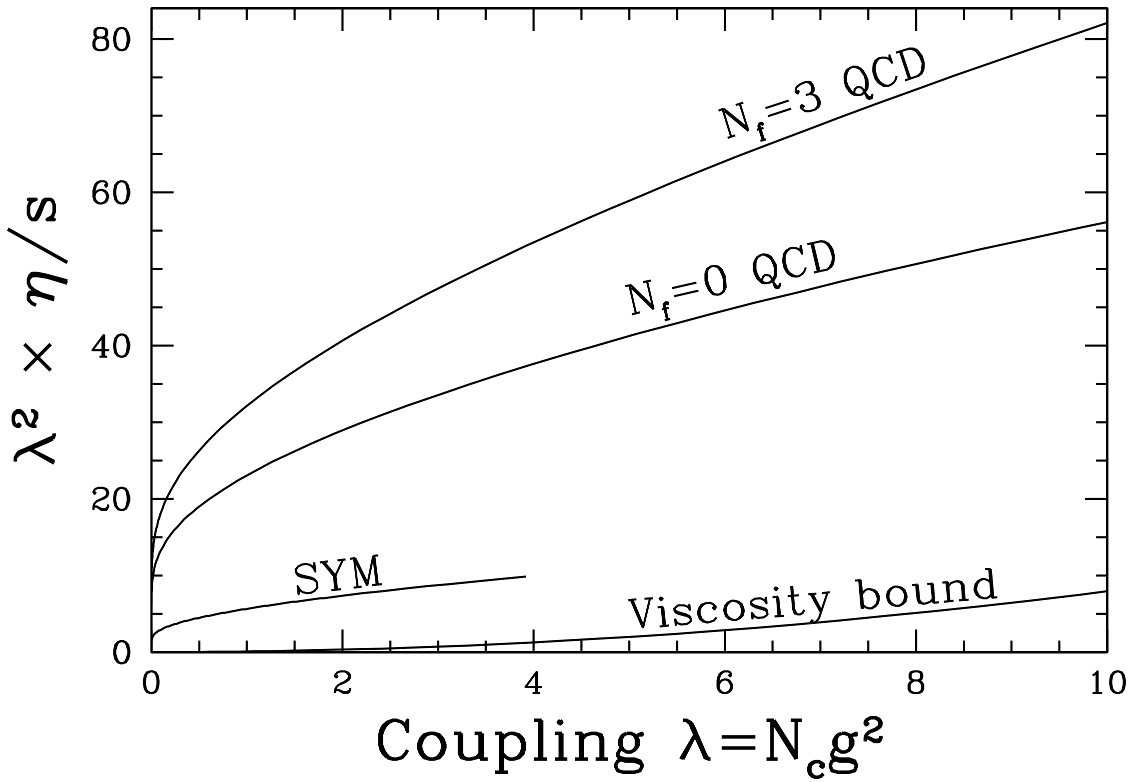


SYM:

$$\left(\frac{\eta}{s}\right)_{\mathrm{SYM}} \simeq \frac{6.174}{\lambda^2 \ln(2.36/\sqrt{\lambda})}$$

QCD: $\left(\frac{\eta}{s}\right)_{\mathrm{QCD}} \simeq \frac{46}{\lambda^2 \ln(4.2/\sqrt{\lambda})}$ How do you compare?

Comparison 1 – Same λ



- At the same $\lambda = g^2 N_c$, $(\eta/\mathrm{s})_{\mathrm{QCD}} \approx 7 \times (\eta/\mathrm{s})_{\mathrm{SYM}}$

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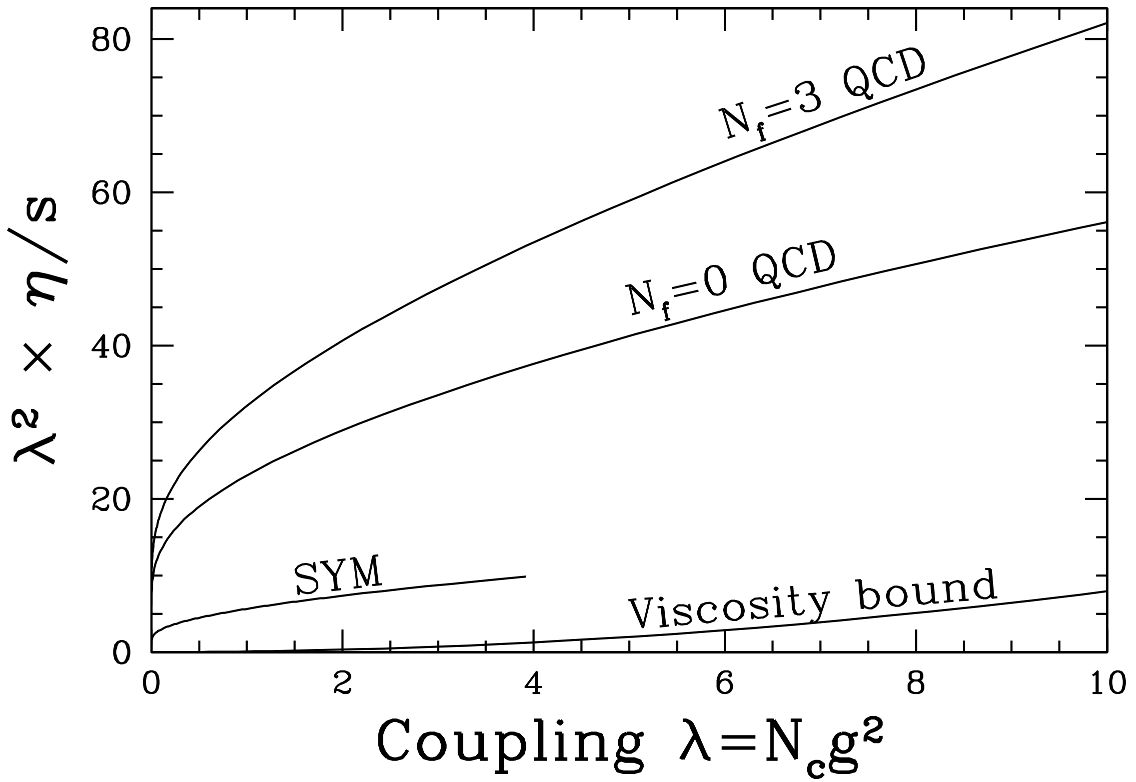

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- SYM seems to grossly underestimate $\eta/$s!

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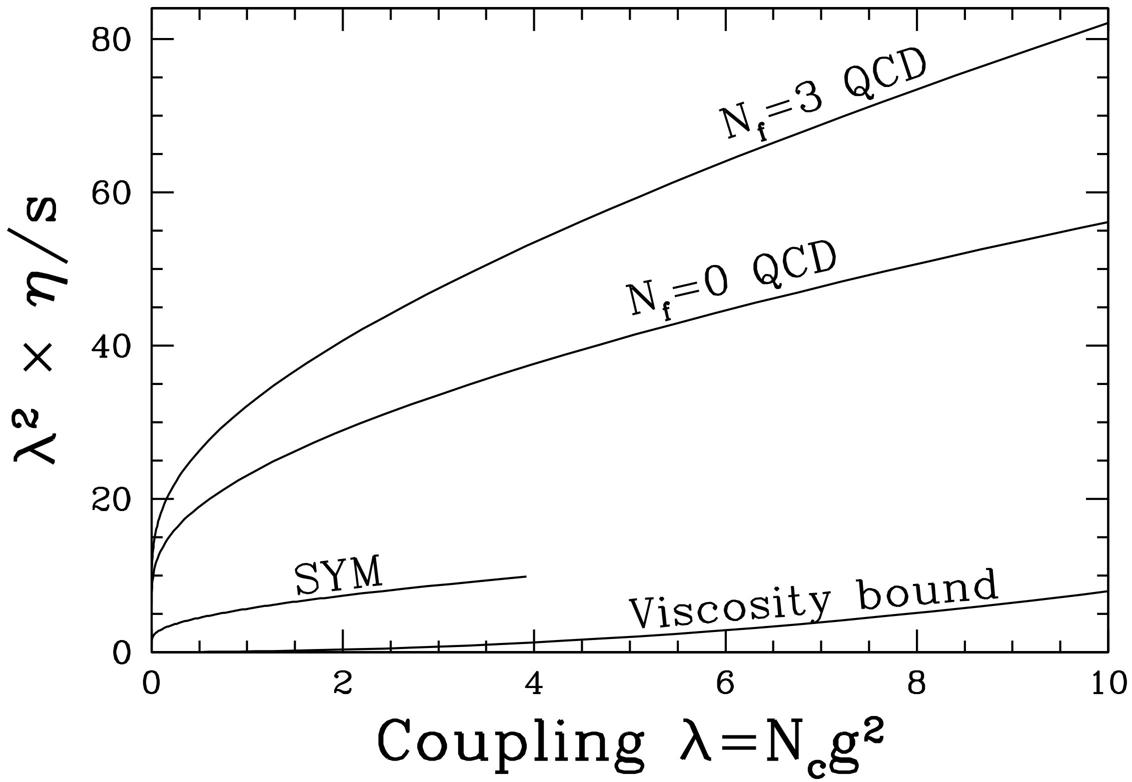
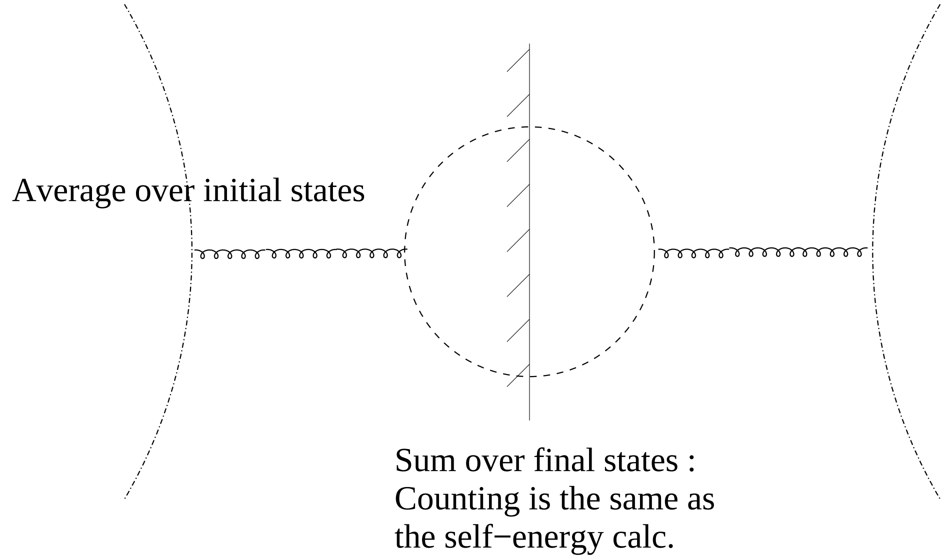


- At the same $\lambda = g^2 N_c$, $(\eta/\text{s})_{\text{QCD}} \approx 7 \times (\eta/\text{s})_{\text{SYM}}$
- But is this a meaningful comparison? η/s scales with what?

Scaling of η/s

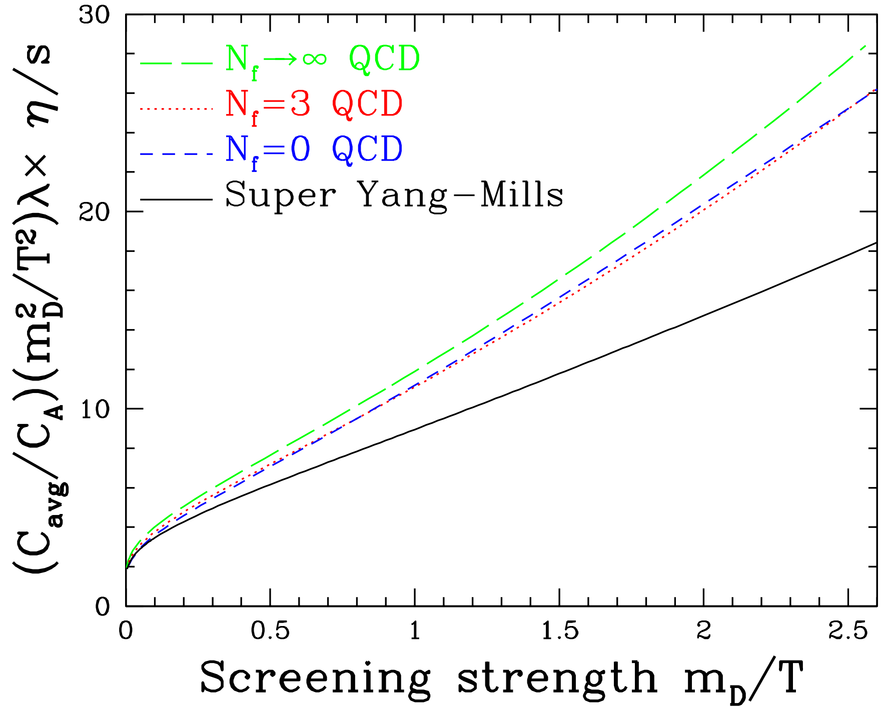
Recall: Perturbatively, $s/\eta \sim T^2\sigma$



- Coulomb σ:
  - Average over initial states: $\sim C_{\mathrm{avg}} g^2$
  - Sum over final states: $\sim m_D^2/T^2$

$$\sigma \sim \frac{C_{\mathrm{avg}}\, g^2\, m_D^2/T^2}{T^2} \sim \frac{C_{\mathrm{avg}}/C_A\, m_D^2/T^2\, \lambda}{T^2}$$

Comparison 2 – Same m_D/T



$$C_{\text{avg}}^{-1} = \frac{C_{\text{matt}}^{-1} g_{*\,\text{matt}} + C_A^{-1} g_{*\,\text{adj}}}{g_*}$$

1 fermionic d.o.f. = (7/8) bosonic d.o.f.

[Large N_f calc: G.Moore, JHEP 0105:039,2001]

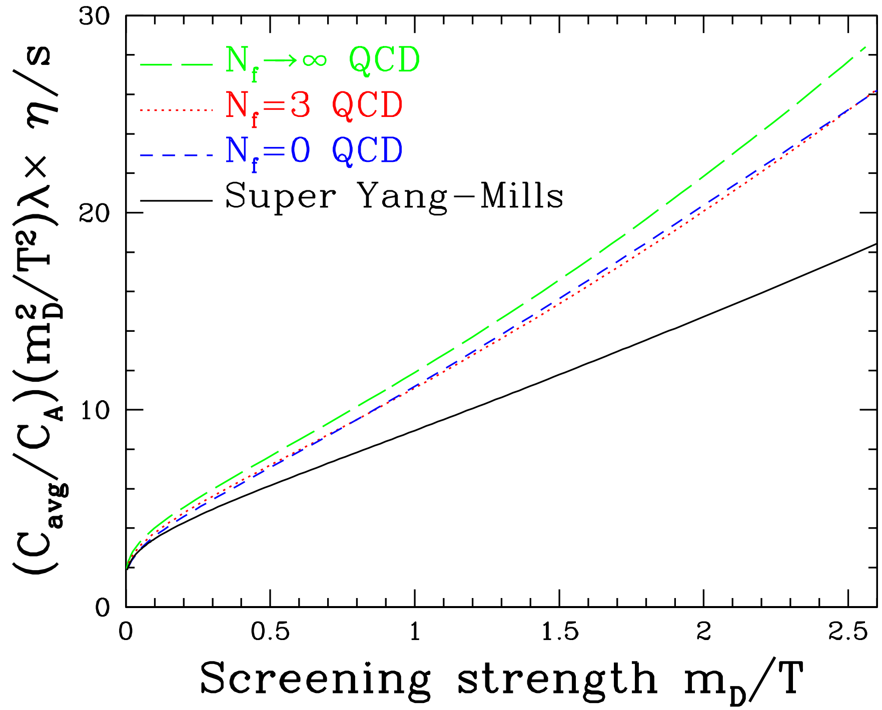
- Same m_D means $\lambda_{\text{QCD}} = 4\lambda_{\text{SYM}}$ with $N_c = N_f = 3$.

  Or $\alpha_s = 0.5 \leftrightarrow \lambda_{\text{SYM}} = 4.7 \leftrightarrow (m_D/T) = 3.1$

$$\left(\frac{\eta}{s}\right)_{\text{QCD}} \sim \frac{30}{(C_{\text{avg}}/C_A)\,(4\pi N_c \alpha_s)\,(m_D/T)^2} = \frac{30}{(0.55)(19)(9.6)} = 0.3$$

$$\left(\frac{\eta}{s}\right)_{\text{SYM}} \sim \frac{20}{(C_{\text{avg}}/C_A)\,(\lambda)\,(m_D/T)^2} = \frac{20}{(1)(4.7)(9.6)} = 0.4$$

Comparison 2 – Same m_D/T



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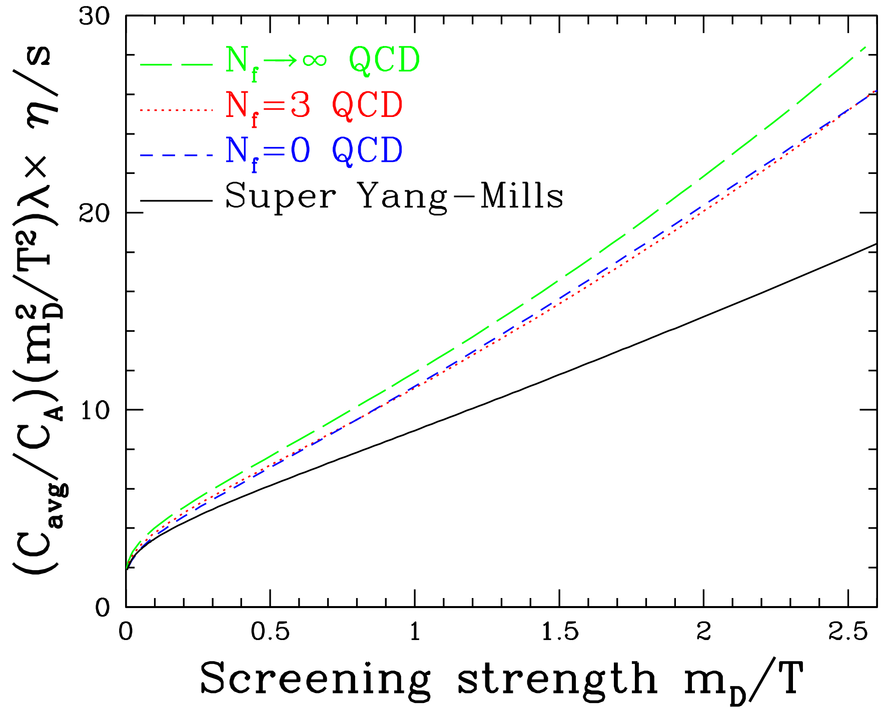

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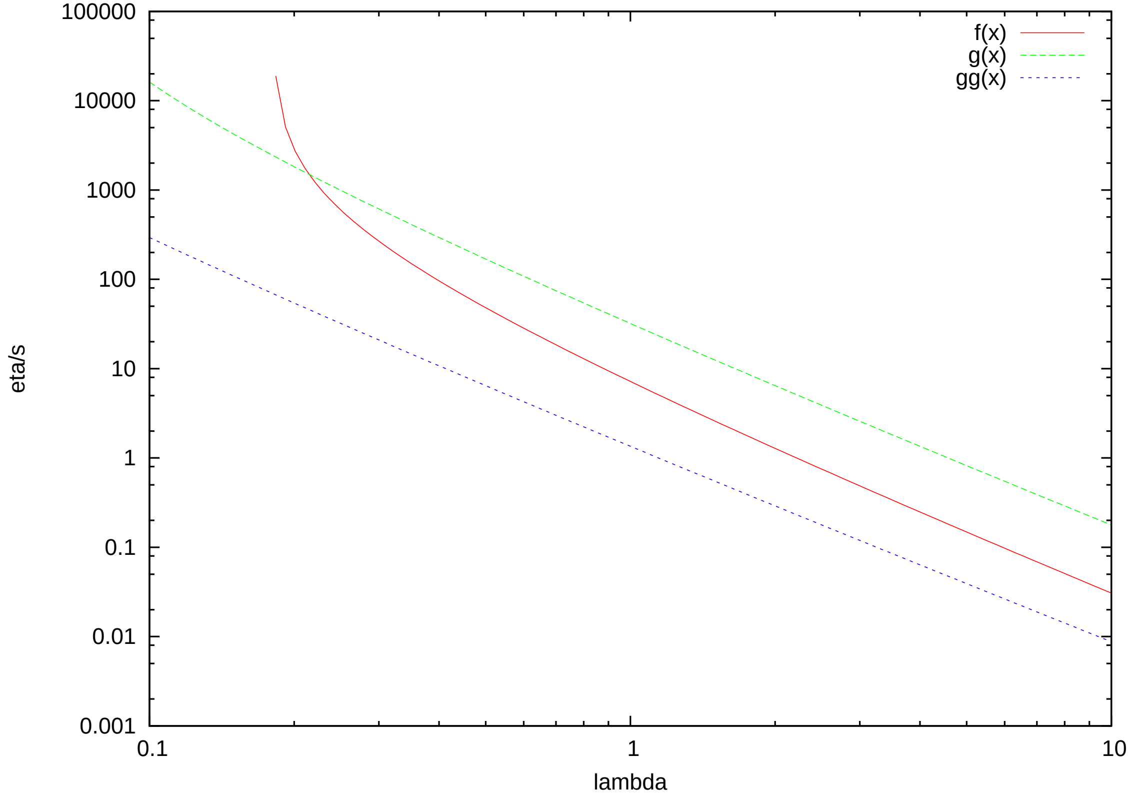
- They are actually comparable!



SYM

QCD at the same λ

QCD at the same m_D/T

Conclusions and Perspectives

Conclusions

- In the weak coupling limit (same λ),

$$\left(\frac{\eta}{\mathsf{s}}\right)_{\mathsf{QCD}} \approx (6 \sim 7) \times \left(\frac{\eta}{\mathsf{s}}\right)_{\mathsf{SYM}} \gg 1 \quad \text{Not the right way to compare!}$$

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- The right way to compare different η/s ratio: At the same m_D/T.

$$\frac{\mathsf{s}}{\eta} \sim (C_{\mathsf{avg}}/C_A)\,(g^2 N_c)\,(m_D^2/T^2)\,f(m_D/T)$$

With $N_c = 3$, $N_f = 3$,

$$\left(\frac{\eta}{\mathsf{s}}\right)_{\mathsf{QCD}} \approx \frac{1}{2} \times \frac{f_{\mathsf{SYM}}(m_D/T)}{f_{\mathsf{QCD}}(m_D/T)} \times \left(\frac{\eta}{\mathsf{s}}\right)_{\mathsf{SYM}} \lesssim \left(\frac{\eta}{\mathsf{s}}\right)_{\mathsf{SYM}}$$

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  * Teaney: [Phys.Rev.C68:034913,2003]

$$\frac{4}{3}\frac{\eta}{\langle \epsilon + \mathcal{P} \rangle \, \tau_0} < 0.1$$

  With $\langle \epsilon + \mathcal{P} \rangle = T\mathrm{s}$ this is equivalent to

$$\frac{\eta}{\mathrm{s}} < 0.1 \times T\,\tau_0$$

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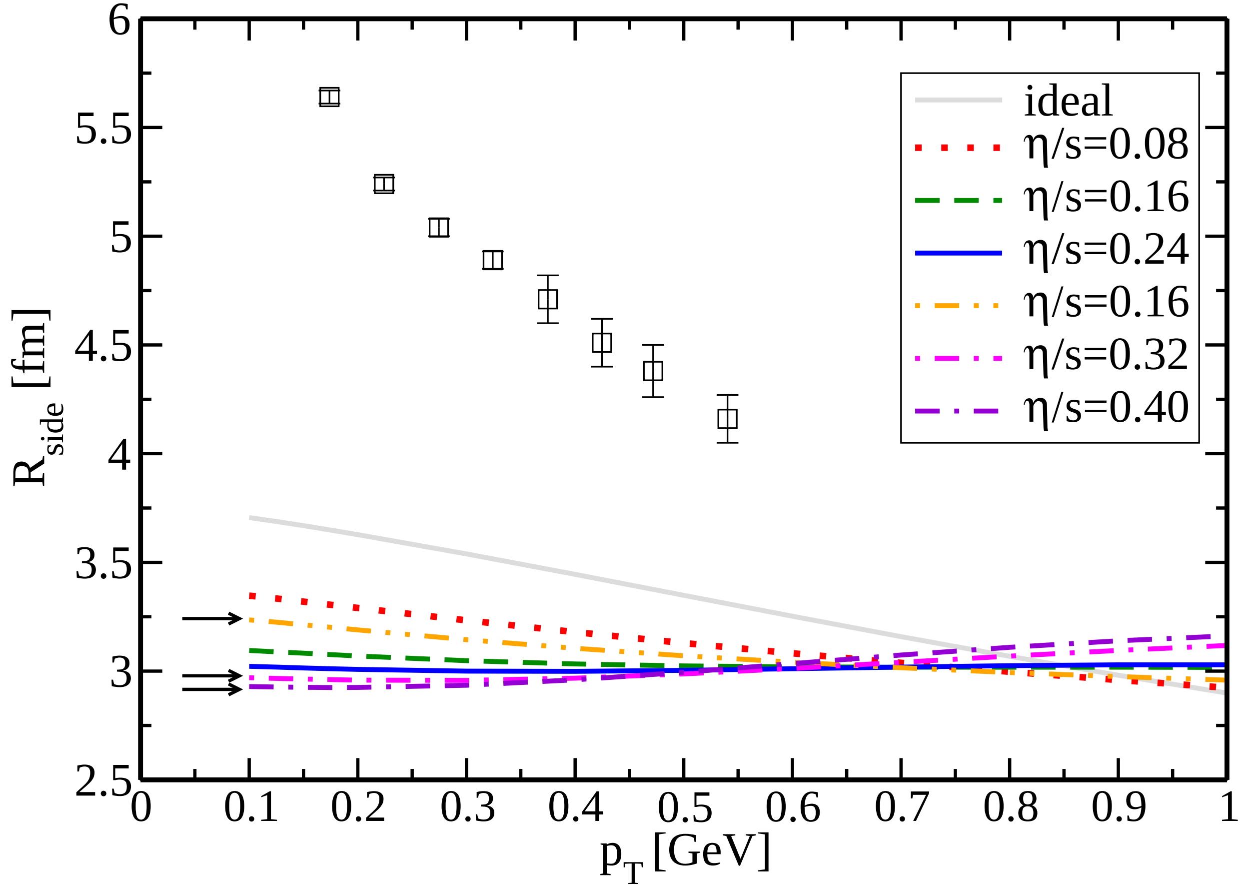
- Caveats: This is an extrapolation!

Perspective



P.Romatschke, nucl-th/0701032

No matter what you do with η/s, you are never going to get this right!

Backups

Numerology

Debye masses

$$\begin{aligned} m_D^{\mathrm{SYM}}/T &= \sqrt{2\,\lambda_{\mathrm{SYM}}} \\ m_D^{\mathrm{QCD}}/T &= \frac{1}{\sqrt{3}}\sqrt{C_A g^2 + N_f g^2/2} \\ &= \frac{1}{\sqrt{3}}\sqrt{N_c g^2 + N_f g^2/2} \\ &= \sqrt{3/2}\,g \end{aligned}$$

with $N_c = 3$, $N_f = 3$. Hence $m_D^{\mathrm{SYM}} = m_D^{\mathrm{QCD}}$ means

$$g = \sqrt{4\lambda_{\mathrm{SYM}}/3}$$

or

$$\alpha_s = \frac{g^2}{4\pi} = \frac{\lambda_{\mathrm{SYM}}}{3\pi}$$

and

$$\lambda_{\mathsf{QCD}} = g^2 N_c = 4\lambda_{\mathsf{SYM}}$$

If $\lambda_{\mathsf{SYM}} = 0.1$, $\alpha_s = 0.011$, $\lambda_{\mathsf{QCD}} = 0.4$, $m_D/T = \sqrt{2\lambda_{\mathsf{SYM}}} = 0.45$.

$$\left(\frac{\eta}{\mathsf{s}}\right)_{\mathsf{QCD}} \sim \frac{7}{(C_{\mathsf{avg}}/C_A)\,(\lambda_{\mathsf{QCD}})\,(m_D/T)^2} = \frac{7}{(0.55)(0.4)(0.2)} = 160$$

$$\left(\frac{\eta}{\mathsf{s}}\right)_{\mathsf{SYM}} \sim \frac{5}{(C_{\mathsf{avg}}/C_A)\,(\lambda)\,(m_D/T)^2} = \frac{5}{(1)(0.1)(0.2)} = 250$$

$$\frac{(\eta/\mathsf{s})_{\mathsf{QCD}}}{(\eta/\mathsf{s})_{\mathsf{SYM}}} = \frac{1}{0.55 \times 4} \times \frac{\bar{\eta}_{\mathsf{QCD}}}{\bar{\eta}_{\mathsf{SYM}}} = \frac{1}{2.2} \times \frac{\bar{\eta}_{\mathsf{QCD}}}{\bar{\eta}_{\mathsf{SYM}}}$$

SYM

$$C_{\text{ave}}^{\text{SYM}} = C_A = 3$$

QCD

$$\begin{aligned} C_{\text{ave}}^{\text{QCD}} &= \frac{g_*}{C_{\text{matt}}^{-1} g_{*\,\text{matt}} + C_A^{-1} g_{*\,\text{adj}}} \\ &= \frac{(7/8) * 36 + 16}{(3/4) * (7/8) * 36 + (1/3) * 16} \\ &= 1.64 \end{aligned}$$

$$1.64/3 = 0.55$$