# Finite Density QCD with Infinite Coupling Constant Limit

S. Kim (Sejong U.)
Hyeong-Gyu Kim (Sejong U.)

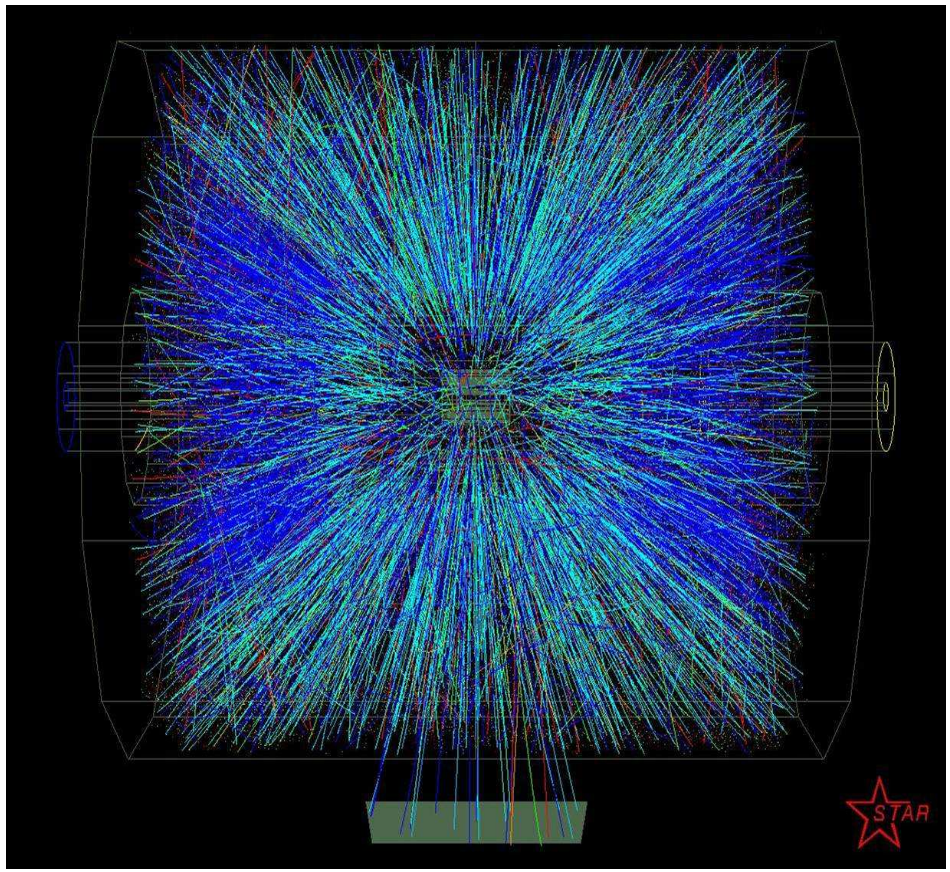
work under progress

# Plan of Talk

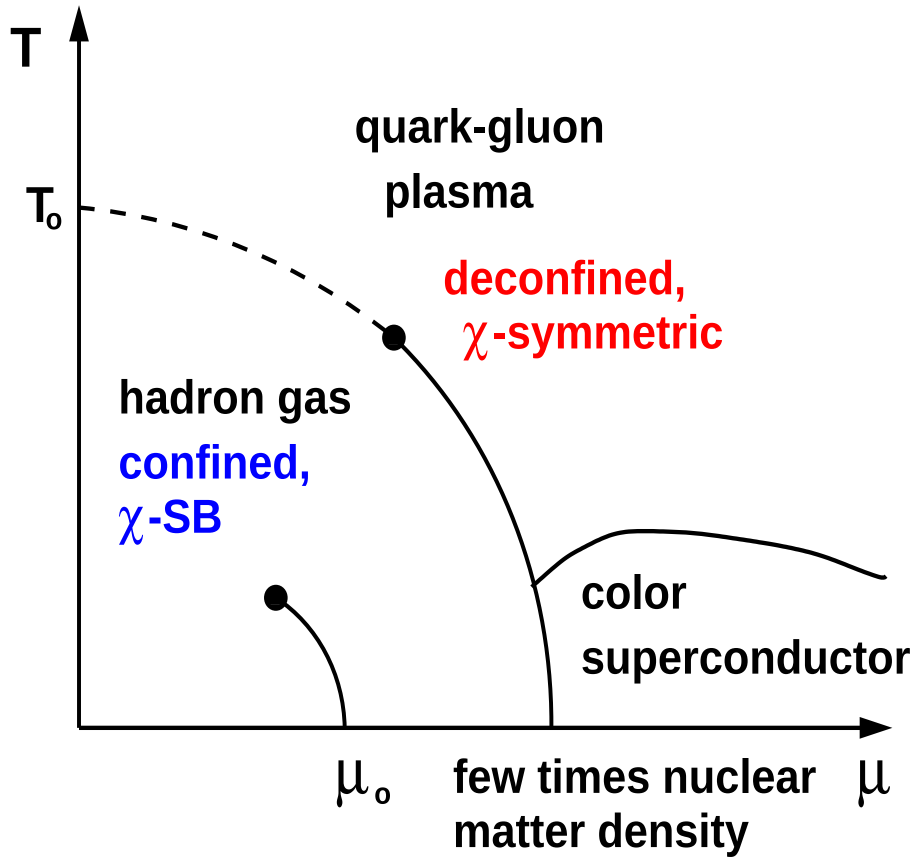
0. Motivation
1. QCD with infinite coupling constant
2. MDP Algorithm
3. Discussion

# 0. Motivation

- Heavy Ion Collision



- Finte T/$\mu$QCD phase diagram



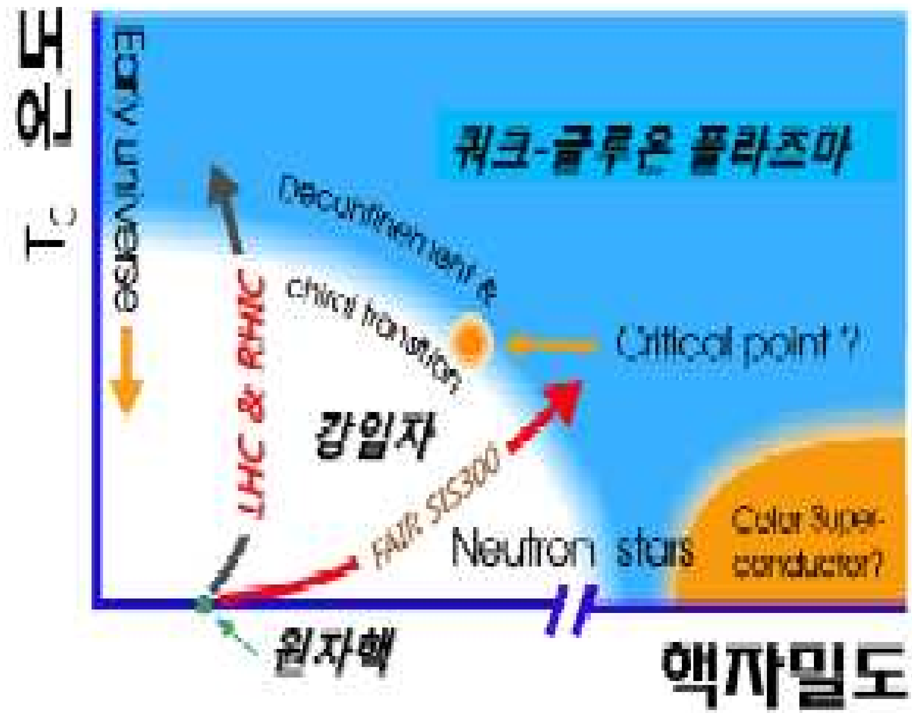



그림 2. 온도와 핵자밀도의 함수로 나타낸 핵 물질의 다양한 상(phase).

되었으나 이
특성을 정밀
리면 명백히
을 정밀측정
적시키고 동
데, 핵물리학
있다. LHC
지밀도가 초
입자의 수가
의 상태방정
를 결정할
히 LHC에서
배나 커서

- unlike finite $T$ domain of QCD phase diagram, it is difficult to study finite $\mu$ domain systematically

- Euclidean lattice QCD action becomes complex $\longrightarrow$ "sign problem"

- Monte Carlo simulation of lattice QCD action is unstable at best

- actually, our knowledge about finite $\mu$ domain of QCD phase diagram is NOT on firm ground

- $\longrightarrow$ model study

# 1. QCD with infinite coupling constant

- QCD lagrangian

$$\mathcal{L} = \bar{\psi}(\gamma_\mu D_\mu + m)\psi - \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \qquad (1)$$

- $D_\mu = \partial_\mu + igA^a_\mu T^a$
- $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + igf^{abc} A^b_\mu A^c_\nu$
- usually interested in weak coupling limit

  $g \to 0$ limit gives a free theory

- strong coupling limit: $g \longrightarrow \infty$

- rescaling $\tilde{A}^a_\mu = g A^a_\mu$

- $D_\mu = \partial_\mu + i\tilde{A}^a_\mu T^a$

- $F^a_{\mu\nu} = \frac{1}{g}[\partial_\mu \tilde{A}^a_\nu - \partial_\nu \tilde{A}^a_\mu + i f^{abc} \tilde{A}^b_\mu \tilde{A}^c_\nu]$

- QCD lagrangian in strong coupling limit (drop tilde)

$$\mathcal{L} = \bar{\psi}(\gamma_\mu D_\mu + m)\psi - \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} \tag{2}$$

- usually interested in strong coupling limit

  $g \longrightarrow \infty$ limit gives QCD string ground state

  (K.G. Wilson, Phys.Rev.D10 (1974) 2445)

• gauge kinetic term, $F^2$, drops in this limit

• gluon field becomes a random field

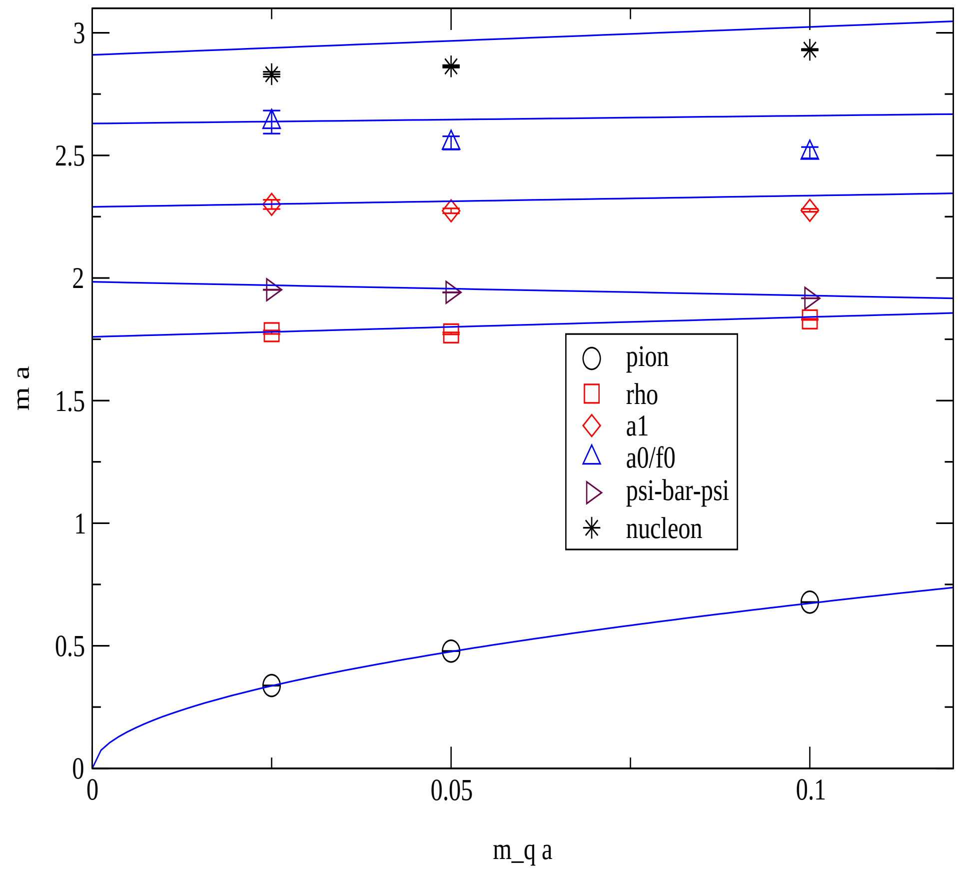
• only gauge singlet combination survives in the path integral

$$\int dU U_i {U_j}^\dagger = \delta_{ij}$$

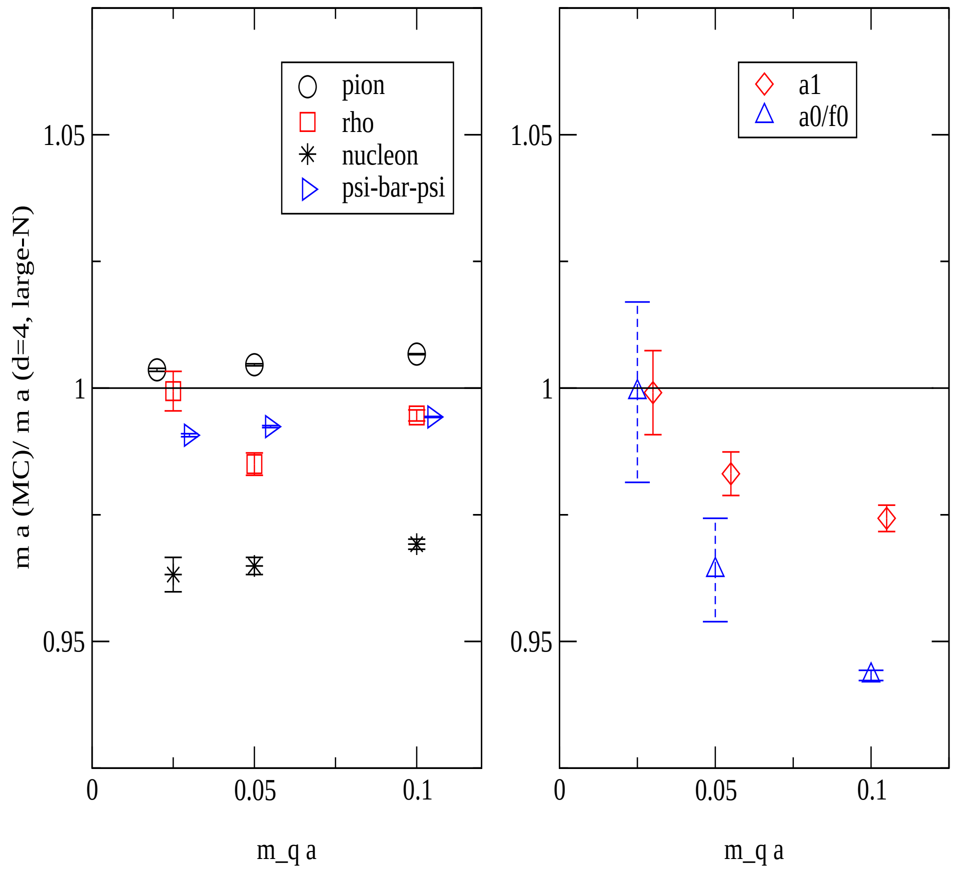
$$\int dU U_i U_j U_k = \frac{1}{3}\varepsilon_{ijk}$$

• can do analytic calculations (see e.g., N. Kawamoto and his collaborators' works)

• Comparison between analytic works and Monte Carlo result




(S.K. and Ph. de Forcrand, Phys. Lett. B645(2007) 339 )


pion
rho
nucleon
psi-bar-psi
a1
a0/f0
1.05
1
0.95
0
0.05
0.1
m a (MC)/ m a (d=4, large-N)
m_q a
m_q a

# 2. Monomer-Dimer-Polymer (MDP) Algorithm

- F. Karsch, K.H. Mutter, Nucl. Phys. 313 (1989) 541
- partition function of lattice QCD in strong coupling limit

$$\mathcal{Z} = \int d\bar{\psi} d\psi \int dU e^{S_F} \tag{3}$$

- $S_F = \bar{\psi}(D_0\eta_0 + D_i\eta_i + m)\psi$

$$D_i\psi = \frac{1}{2}\{U_i(x)\psi(x+i) - U_i^\dagger(x-i)\psi(x-i)\}$$

$$D_0\psi = \frac{1}{2}\{U_0(x)e^{\mu}\psi(x+0) - U_0^\dagger(x-0)e^{-\mu}\psi(x-0)\}$$

• $\psi$ is staggered quark field (i.e., 1-component in spin, 3-component in color grassman field )

• each lattice site can have only $\bar{\psi}_3(x)\bar{\psi}_2(x)\bar{\psi}_1(x)\psi_1(x)\psi_2(x)\psi_3(x)$ combination due to grasmmann property of quark field

• With $M(x) = \sum_{a=1,2,3} \bar{\psi}_a(x)\psi_a(x)$,

$B(x) = \psi_1(x)\psi_2(x)\psi_3(x)$,

$\bar{B}(x) = \bar{\psi}_3(x)\bar{\psi}_2(x)\bar{\psi}_1(x)$

• only gauge singlet combination survives in the path integral

• with

$$F(x,y) = \int dU e^{\bar{\psi}(x)U(x,y)\psi(y) - \bar{\psi}(y)U^{\dagger}(y,x)\psi(x)}$$

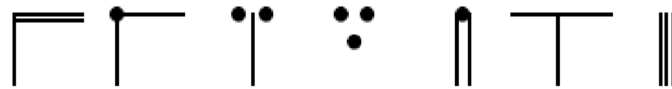
$$= 1 + \frac{1}{3}M(x)M(y) + \frac{1}{12}\{M(x)M(y)\}^2 + \frac{1}{36}\{M(x)M(y)\}^3$$
$$-\eta(x,y)^3\bar{B}(x)B(y) - \eta(y,x)^3\bar{B}(y)B(x) \quad (4)$$

$$\mathcal{Z} = \int d\bar{\psi}d\psi e^{m\sum_x M(x)} \prod_{x,y} F(x,y) \quad (5)$$

• each lattice site can have $n_M + n_D = 3$ ($n_M$ is the number of monomer, $n_D$ is the number of dimer)

• or occupied by baryon loop

- allowed lattice site types

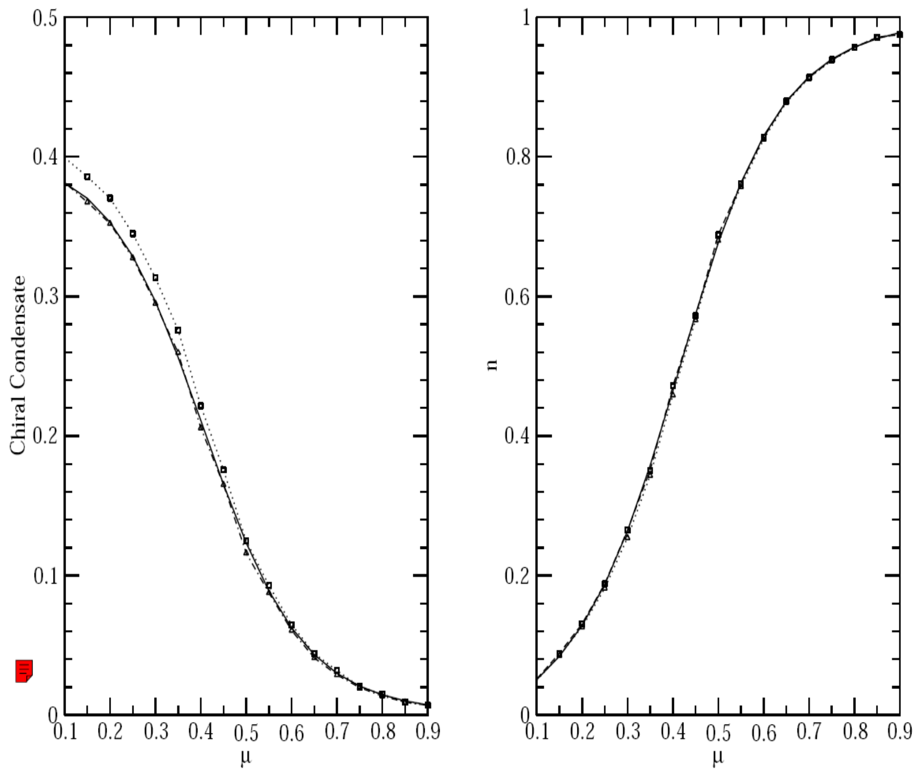
| [illegible] | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| |  | | | | | | |
| $n_{D_1}(x)$ | 1 | 2 | 1 | 0 | 0 | 3 | 0 |
| $n_{D_2}(x)$ | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| $n_{D_3}(x)$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| $w(x)$ | 3 | 6 | 3 | 1 | 3 | 6 | 1 |

- Metropolis algorithm

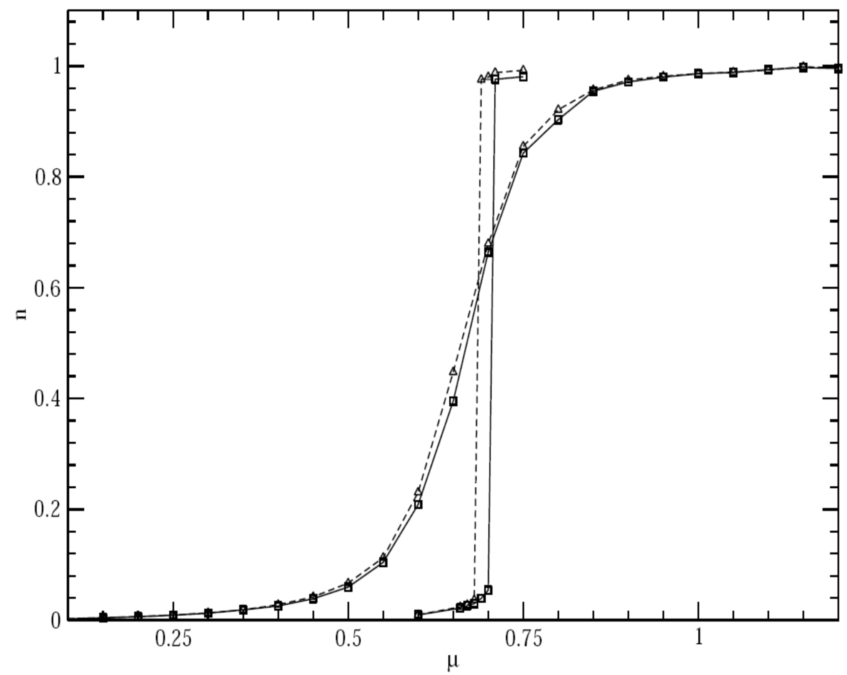
- either cut a dimer, which removes a link and add monomer at end sites

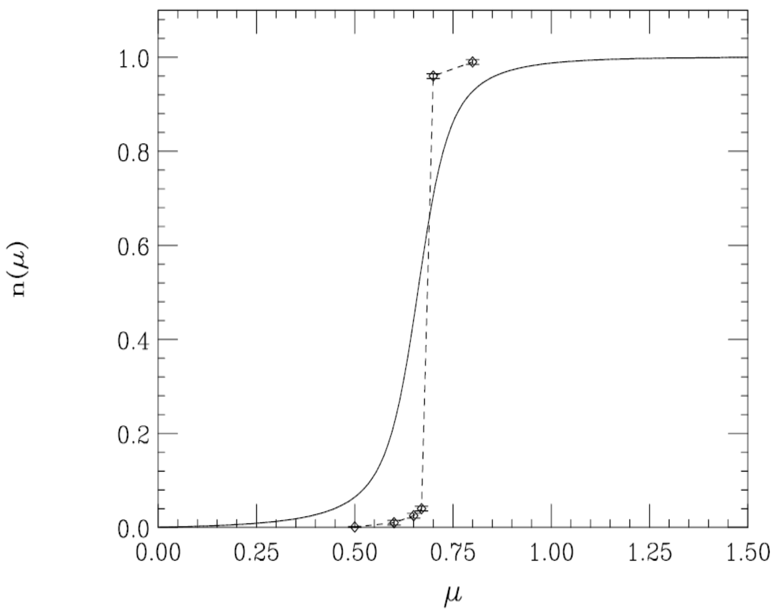
- or connect two monomers at the neighboring sites, which removes two monomers and add a link between the two sites

- throw dice to satisfy detailed balance



Chiral Condensate
μ
n
μ


n
μ
0
0.2
0.4
0.6
0.8
1
0.25
0.5
0.75
1



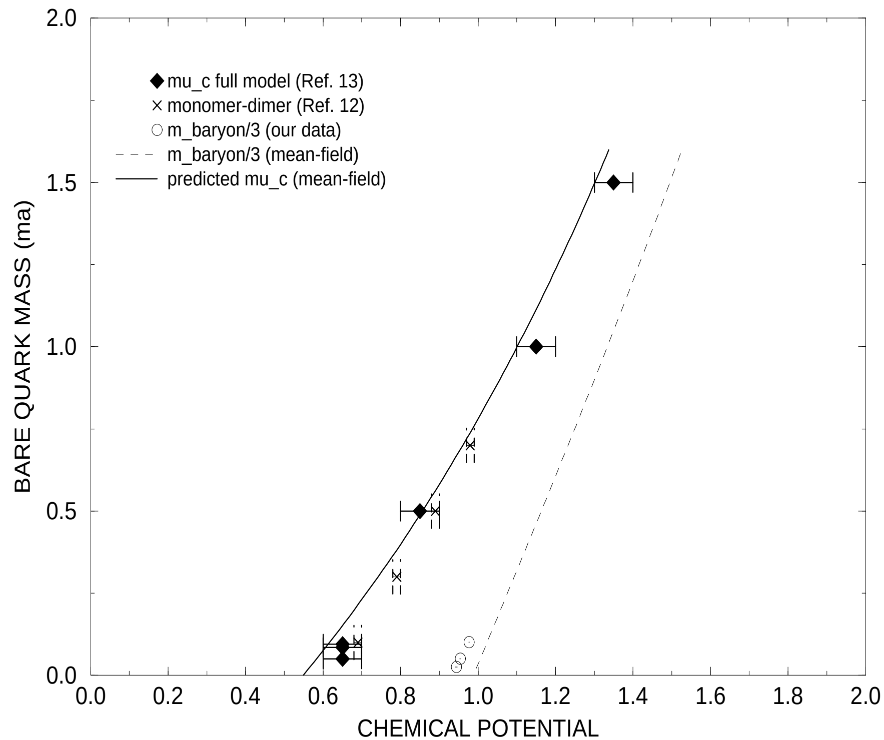

(R. Aloisio et al, Nucl. Phys. B564(2000) 489 )

# 3. Discussion

- We could reproduce F. Karsch, K.H. Mutter, Nucl. Phys. 313 (1989) 541

- MDP alogrithm has a PROBLEM with the chiral limit or with heavy quark mass

- further investigation under way




(S.K. and Ph. de Forcrand, Phys. Lett. B645(2007) 339 )