Hydrodynamics in Heavy Ion Collisions

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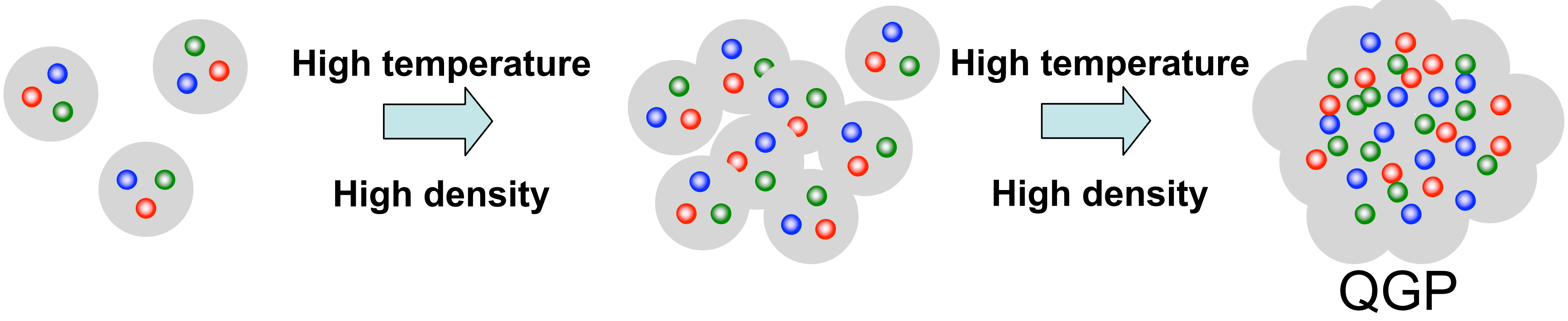
- Introduction
- Ideal Hydrodynamics
  - Success of hydrodynamics at RHIC
  - Limitation of hydrodynamics
  - Current status of ideal hydrodynamic models

    initial conditions, equation of states, freezeout process
- Viscous Hydrodynamics
- Applications
  - Jets in medium
  - LHC
- Summary

Quark-Gluon Plasma

- QCD Phase Transition



High temperature

High density

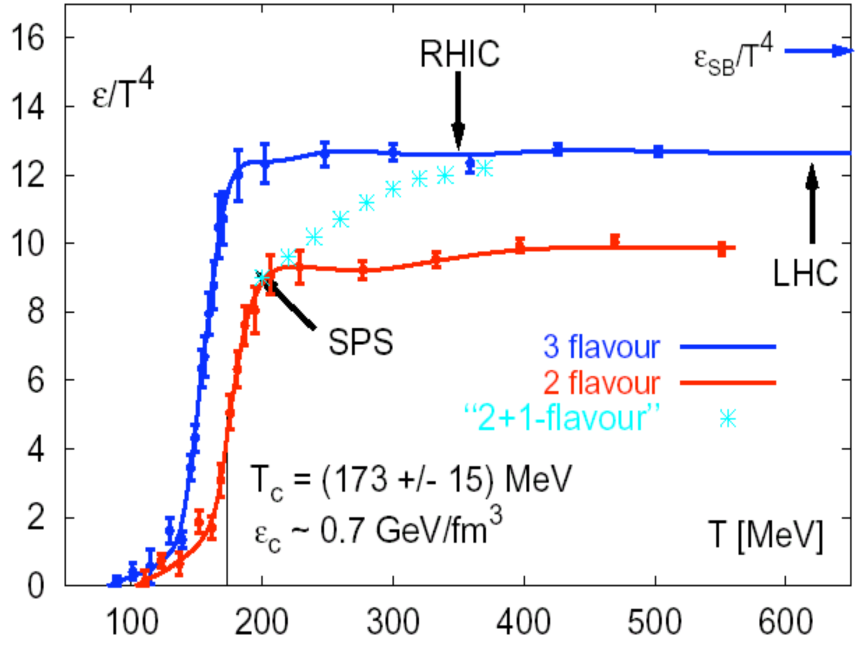
High temperature

High density

QGP

asymptotic freedom

•Equation of State (Lattice QCD)



- Clear evidence of the QCD phase transition

Karsch, Laermann, Peikert, PLB478(2000)447

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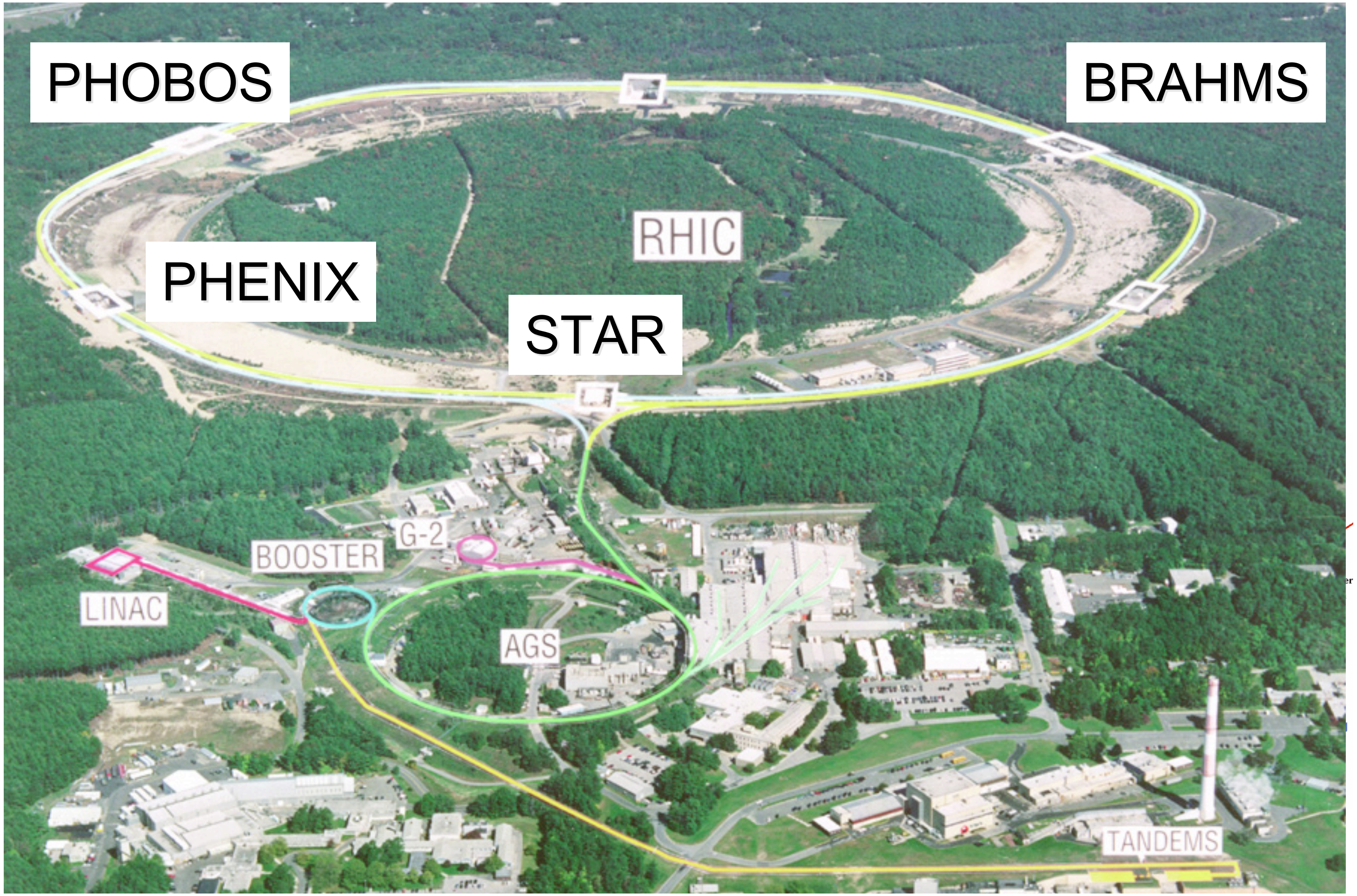
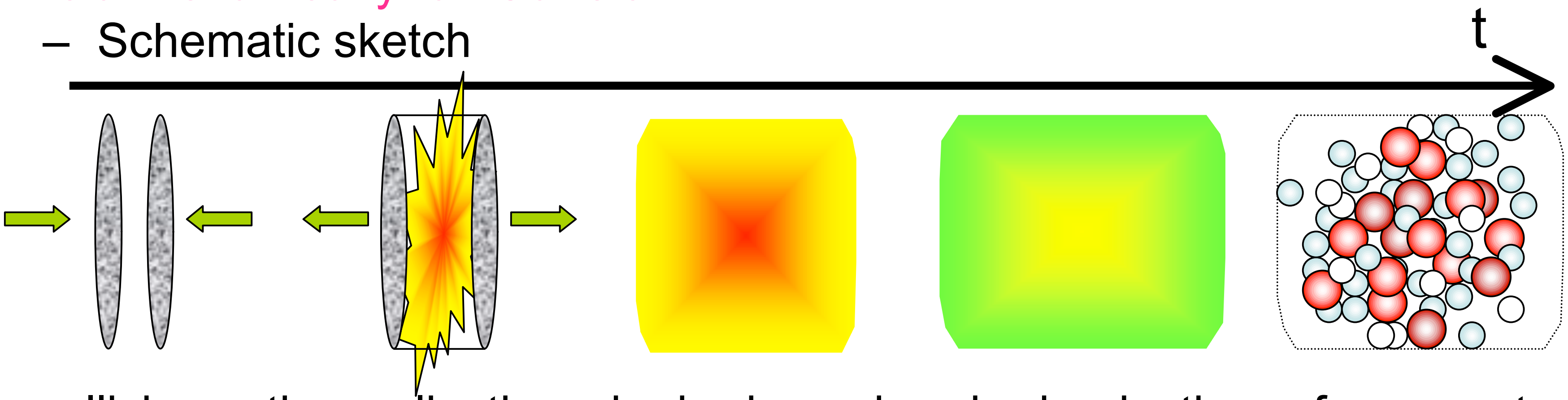
Relativistic Heavy Ion Collider

Au + Au $\sqrt{s_{NN}} =$ 200 A GeV at Brookhaven National Laboratory



Phenomenology at RHIC

- Relativistic Heavy Ion Collision
  - Schematic sketch



collision thermalization hydrodynamic expansion hadronization freeze-out

- Difficulties

•Complicated process

initial state
hydrodynamic expansion
hadronization
freeze-out

•QGP signature ?

hadron spectra
two particle correlations
flow (elliptic, direct)
fluctuation (charge, multiplicity)
electromagnetic probes…..

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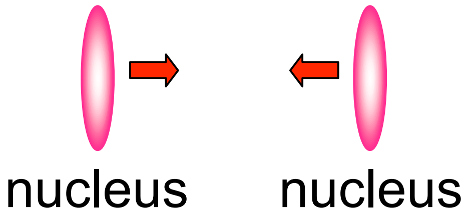
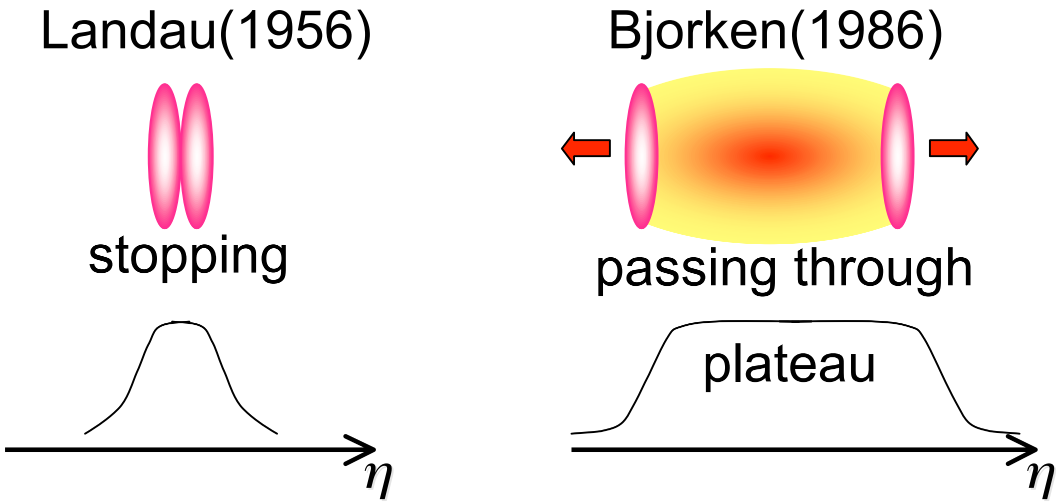
Hydrodynamic Models

- Assumptions (for multiple particle production)
  - Local thermalization
  - Mean free path ~ 0
- Established model





- Equation of State
- SPS at CERN (Pb+Pb $\sqrt{s_{NN}} = 17$ GeV)
  - One of phenomenological models
    transverse momentum spectra ○,
    Hanbury Brown - Twiss (HBT) ○, elliptic flow ✖
    ex.hadron base event generator (RQMD, URASiMA….)

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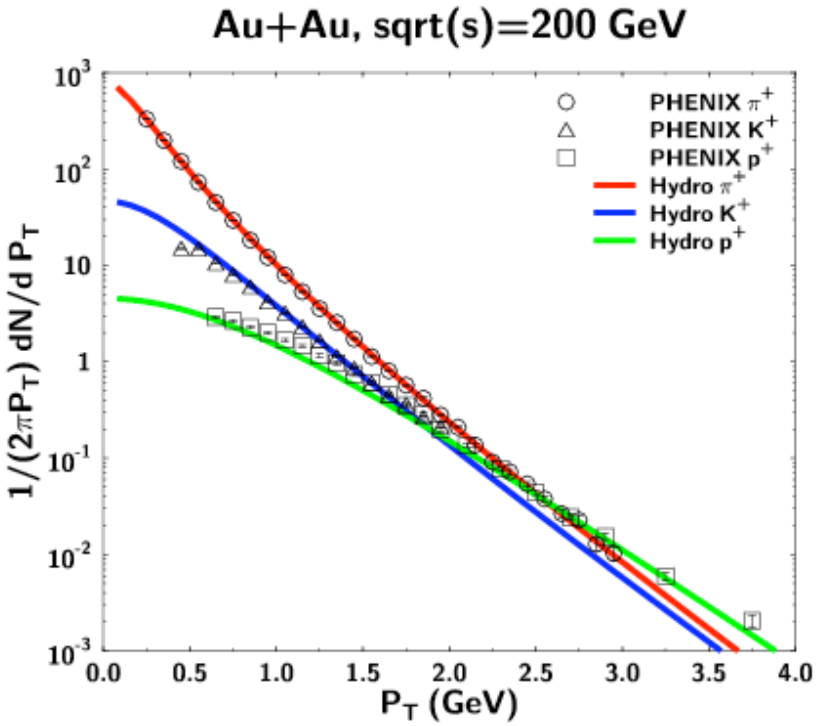
Perfect Fluid at RHIC

- Success of Ideal Hydrodynamic Models at RHIC

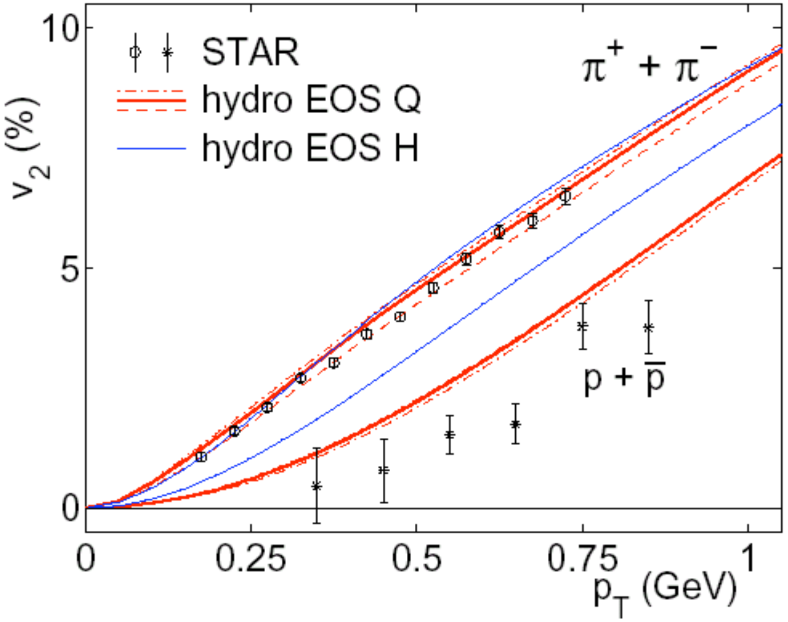
–Single particle spectra
P_T spectra up to ~ 2GeV
Huovinen, Kolb, Heinz, Hirano, Teaney,
Shuryak, Hama, Morita, …….

– Strong elliptic flow
• strong coupled QGP

$$\frac{dN}{d\varphi} \approx v_0(1+2v_1\cos(\varphi)+2v_2\cos(2\varphi))$$



Nonaka and Bass



at mid rapidity

Huovinen et.al, PLB503

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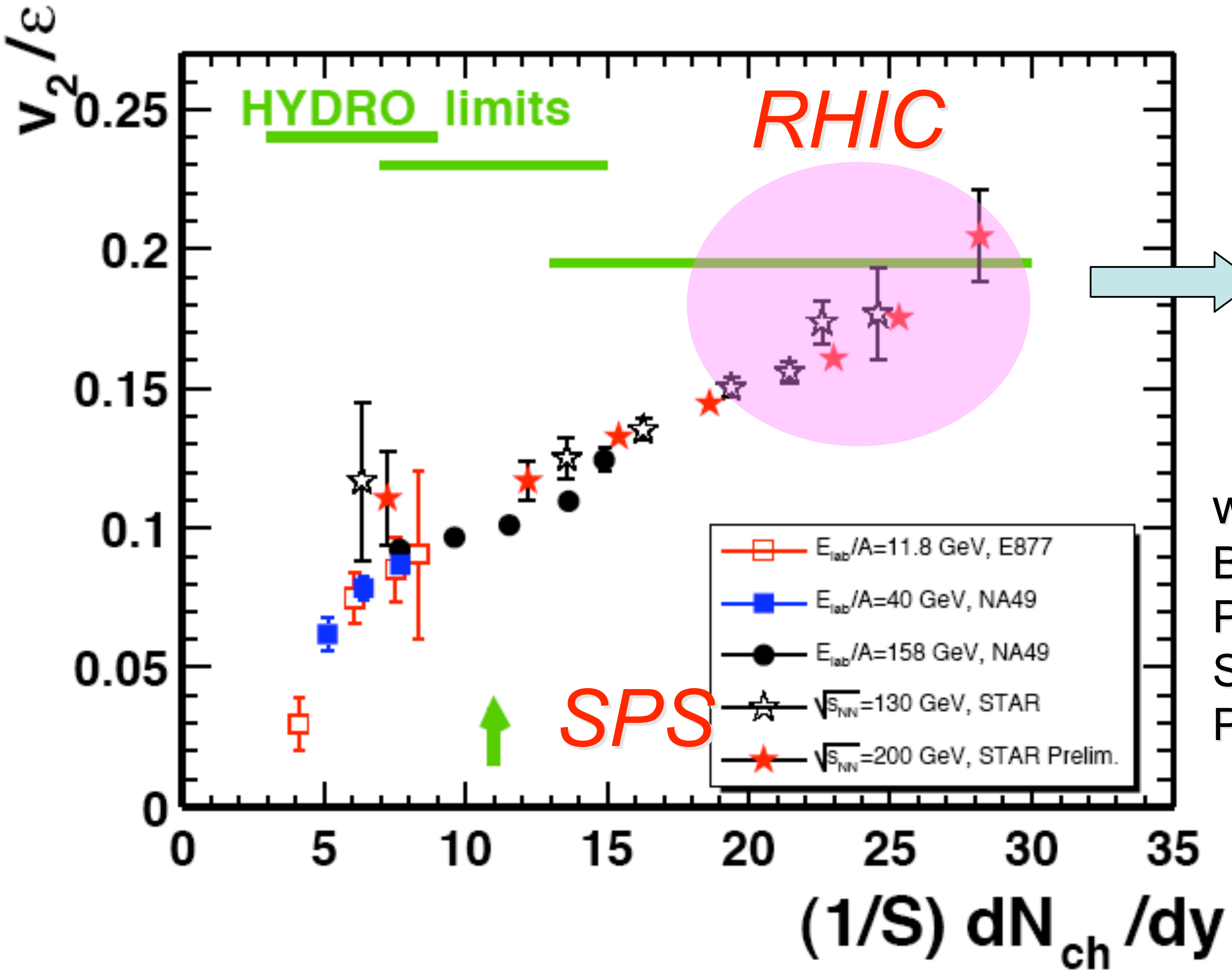
V2 vs multiplicity



NA49:PRC68,034903(2003)

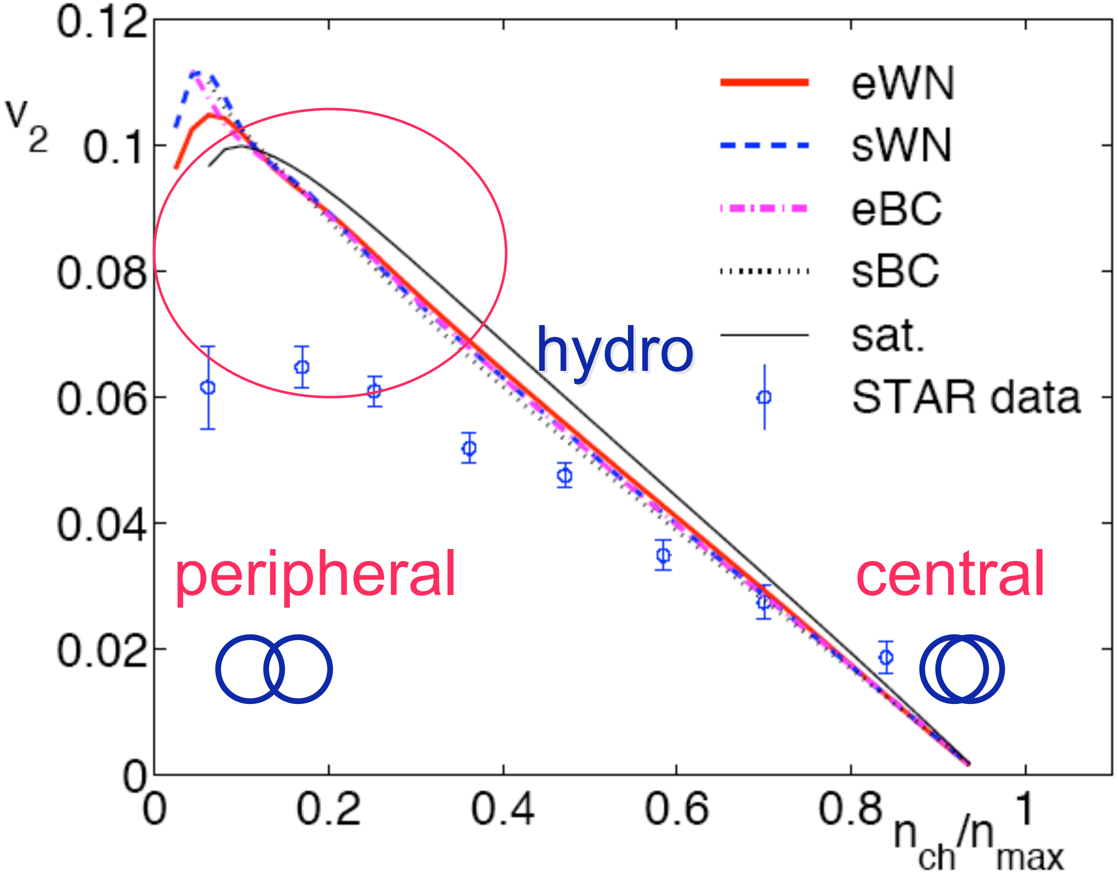
→ *sQGP*

Strongly coupled / interacting QGP

white papers
BRHAMS:NPA757,1(2005)
PHOBOS:NPA757,28(2005)
STAR:NPA757,102(2005)
PHENIX:NPA757,184(2005)

Perfect Fluid at RHIC?

- Centrality Dependence



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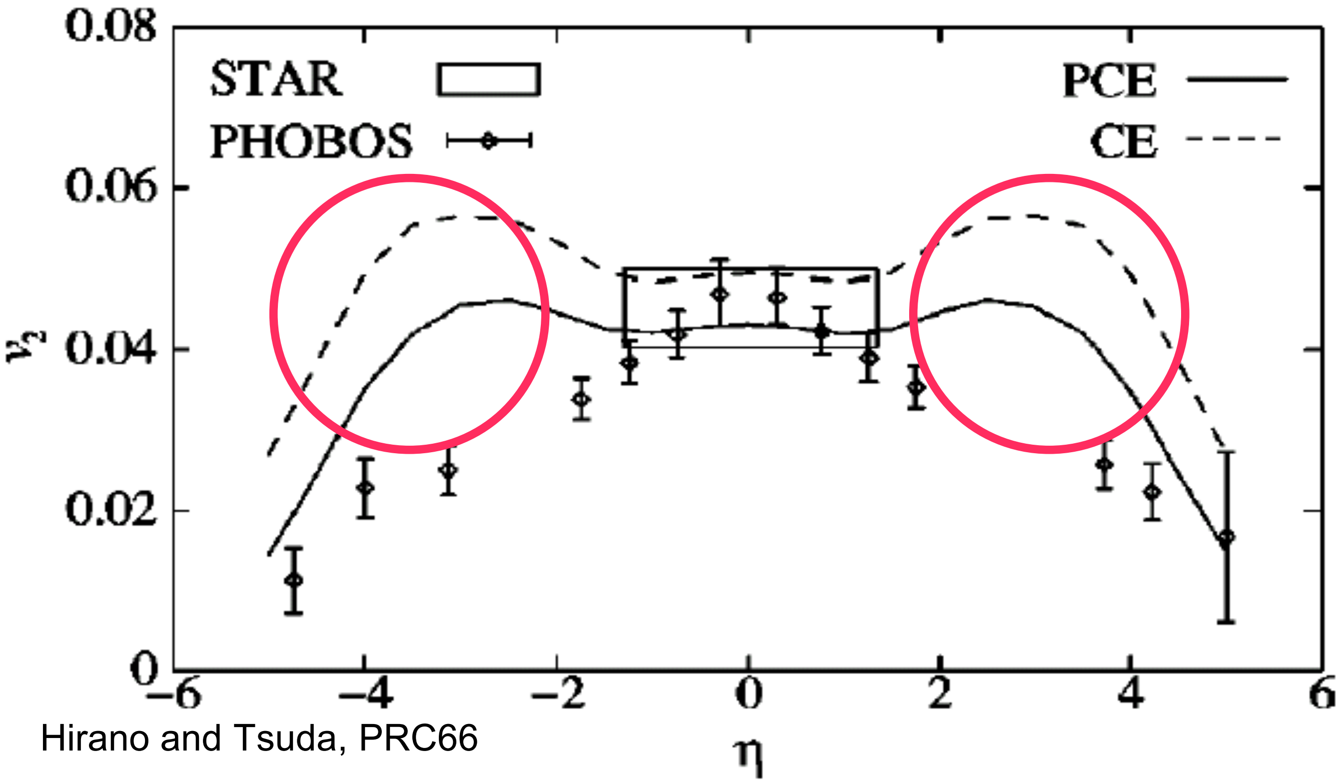
Perfect Fluid at RHIC?

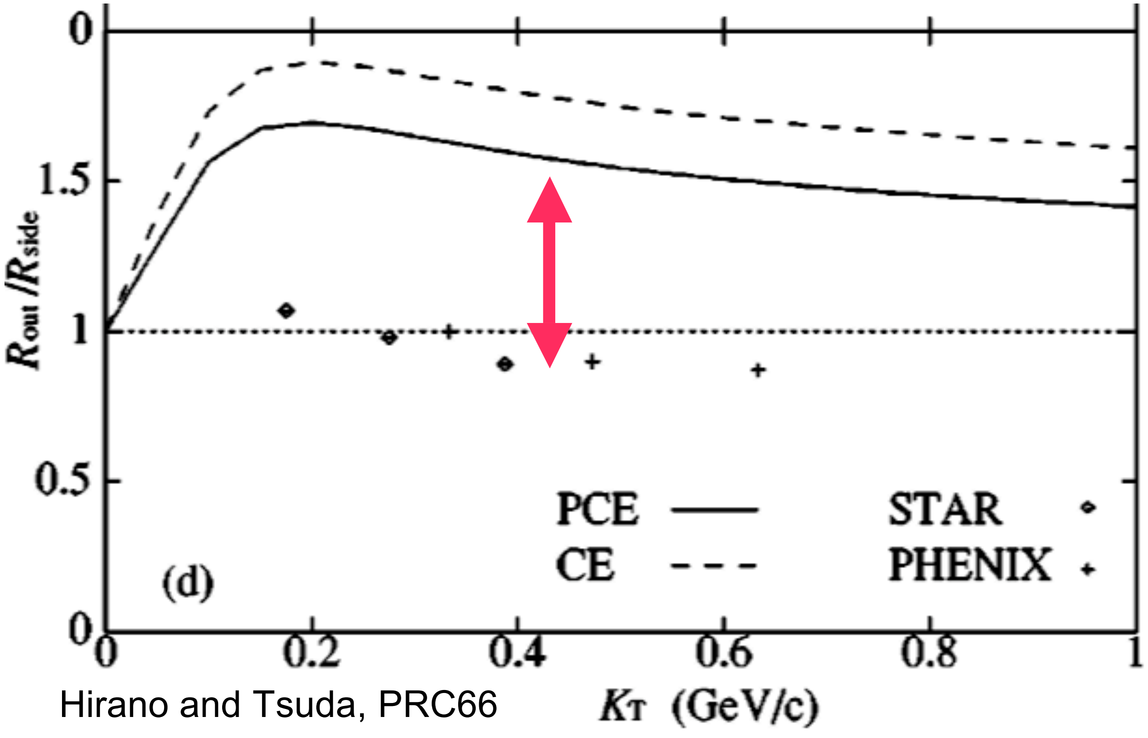
- Elliptic flow as a function of pseudorapidity



Hirano and Tsuda, PRC66

Forward/backward rapidity

HBT puzzle

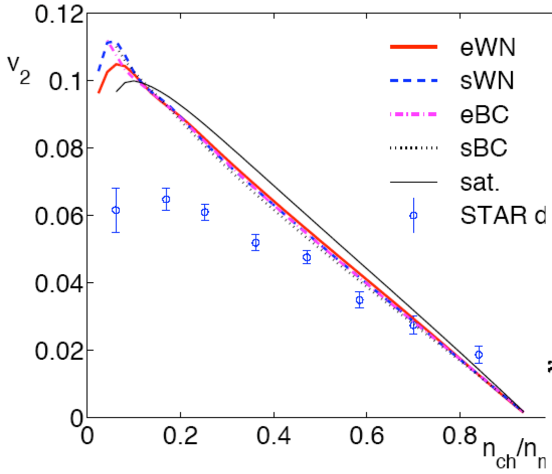


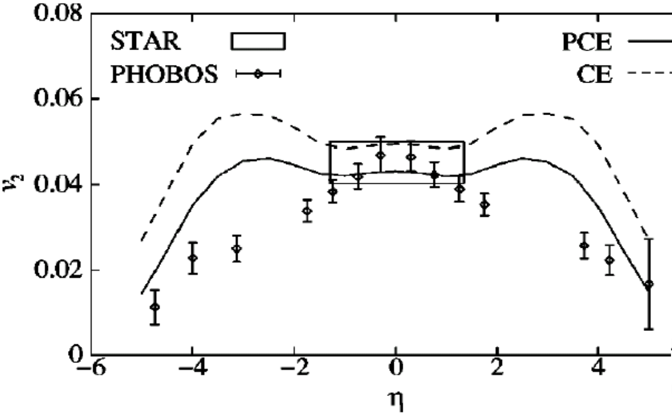
Hirano and Tsuda, PRC66

Short life time?

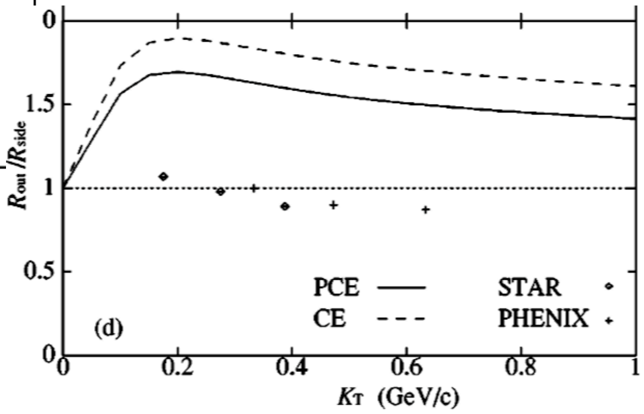
Perfect Fluid at RHIC?

- Elliptic Flow





- HBT



- **Perfect Fluid?**
  - –Peripheral collisions
  - –Forward/Backward rapidity

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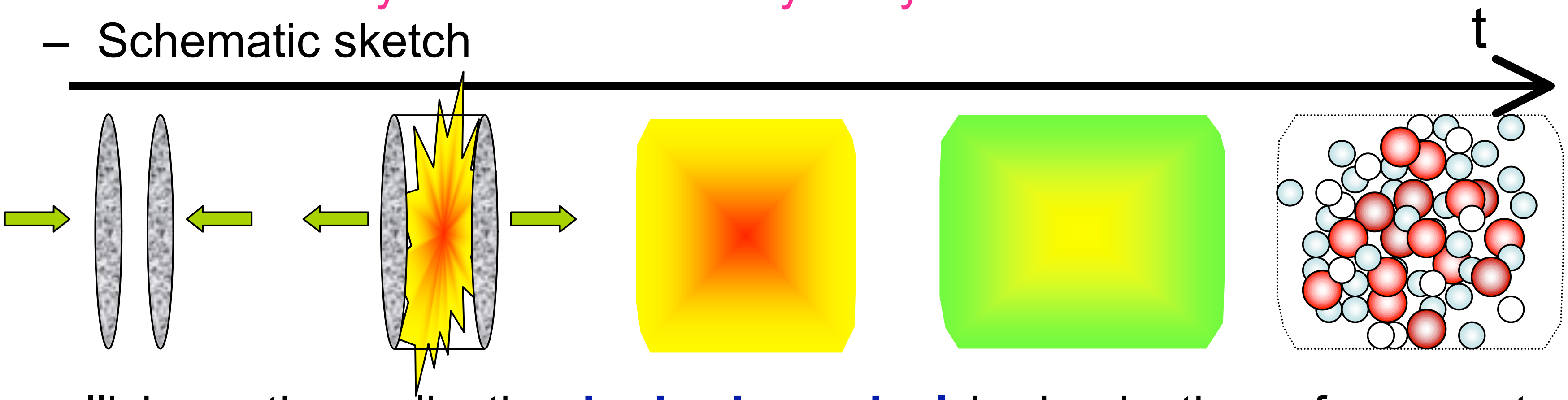
Hydrodynamic Model at RHIC

- Relativistic Heavy Ion Collision & Hydrodynamic models
  - Schematic sketch



collision thermalization **hydrodynamical expansion** hadronization freeze-out

QGP production → hadron phase

phase transition

Hydro Model

initial conditions
- parametrization
- color glass condensate…

equation of states
- bag model
- lattice QCD…

freezeout process
- chemical equilibrium
- partial chemical equilibrium
- cascade model…

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Hydrodynamic Models at RHIC

- Useful tool for experimental data
  - P_T, η -distributions, radial flow, anisotropic flow, HBT, jets
- Check List
  - Initial conditions, equation of states, freezeout process
  - Initial conditions
    - Glauber type, Huovinen, Kolb, Heinz, Teaney, Shuryak
    - Event Generator, CN, Socolwski, Grassi, Hama, Kodama…
    - Color Glass Condensate, Hirano, Nara…
    - pQCD + saturation model, Eskola, Honkanen, Niemi, Ruuskanen, Rasanen
    - String tube - color flux tube, Csernai

Standard model for the initial transverse density profile

Initial energy density or entropy is taken from Wounded nucleon model:

number of participants or collision scaling.

$$n_{part}(\boldsymbol{x}_\perp,\boldsymbol{b})=T_A(\boldsymbol{x}_\perp+\boldsymbol{b}/2)\left(1-(1-\sigma_{NN}^{inel}T_B(\boldsymbol{x}_\perp-\boldsymbol{b}/2)/B)^B\right)$$
$$+T_B(\boldsymbol{x}_\perp-\boldsymbol{b}/2)\left(1-(1-\sigma_{NN}^{inel}T_A(\boldsymbol{x}_\perp+\boldsymbol{b}/2)/A)^A\right)$$

$$n_{coll}(\boldsymbol{x}_\perp,\boldsymbol{b})=\sigma_{NN}^{inel}T_A(\boldsymbol{x}_\perp+\boldsymbol{b}/2)T_B(\boldsymbol{x}_\perp-\boldsymbol{b}/2)$$

$$T_A(\boldsymbol{x}_\perp)=\int dz\,\rho_A(\boldsymbol{x}_\perp,z)\qquad \rho_A(\boldsymbol{r})=\frac{\rho_0}{1+\exp[(r-R_0)/a]}$$

sWN, eWN, sBC, wBC

Nara

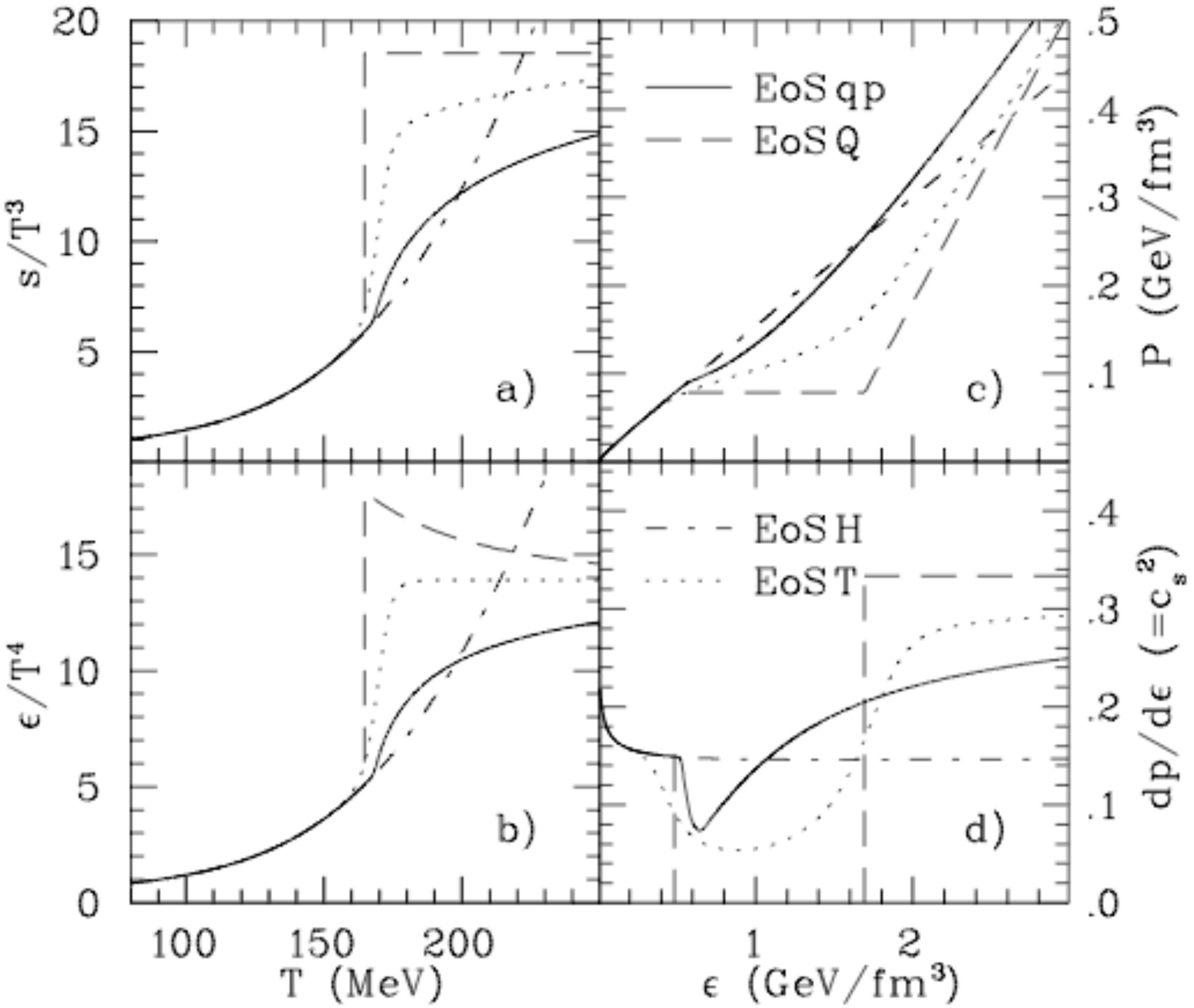
Hydrodynamic Models at RHIC

- Useful tool for experimental data
  - P_T, η -distributions, radial flow, anisotropic flow, HBT, jets
- Check List
  - Initial conditions, equation of states, freezeout process
  - EoS dependence, Huovinen, Kolb, Heinz, Teaney, Shuryak…
    - Hadron gas
    - 1st order phase transition, Bag Model
    - Parametrized EoS based on Lattice QCD
    - Quasiparticle Model

EOS Dependence

Huovinen nucl-th/0505036

- Equation of states



EoS qp:
Quasi particle
inspired by lattice QCD
EoS Q:
1st order phase transition
EoS H:
resonance hadron gas
EoS T:
crossover (parametrization)

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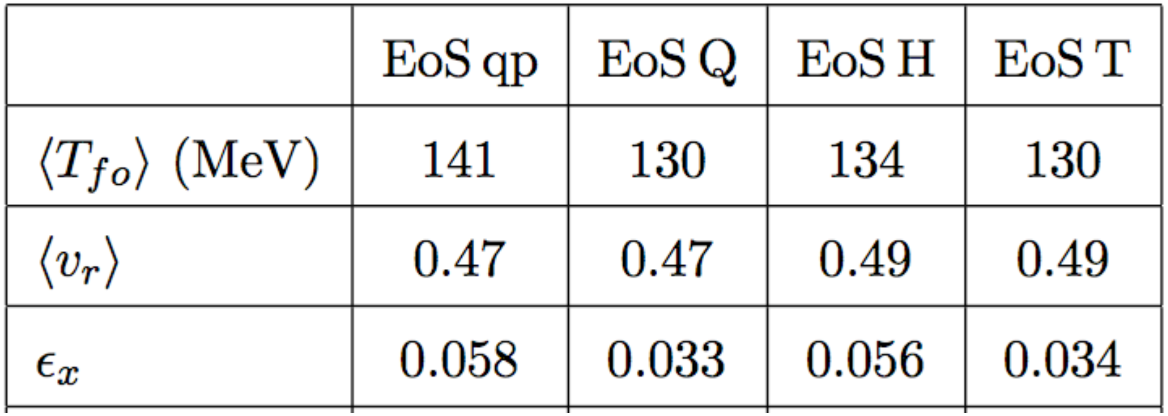
EOS Dependence

Huovinen nucl-th/0505036

- Initial conditions are fixed.
  - Glauber Type: 0.75*sWN + 0.25*sBC
  - τ_0=0.6 fm
- Freezeout conditions:chemical equilibrium
  - They are fixed from P_T spectra



	EoS qp	EoS Q	EoS H	EoS T
$\langle T_{fo} \rangle$ (MeV)	141	130	134	130
$\langle v_r \rangle$	0.47	0.47	0.49	0.49
ϵ_x	0.058	0.033	0.056	0.034

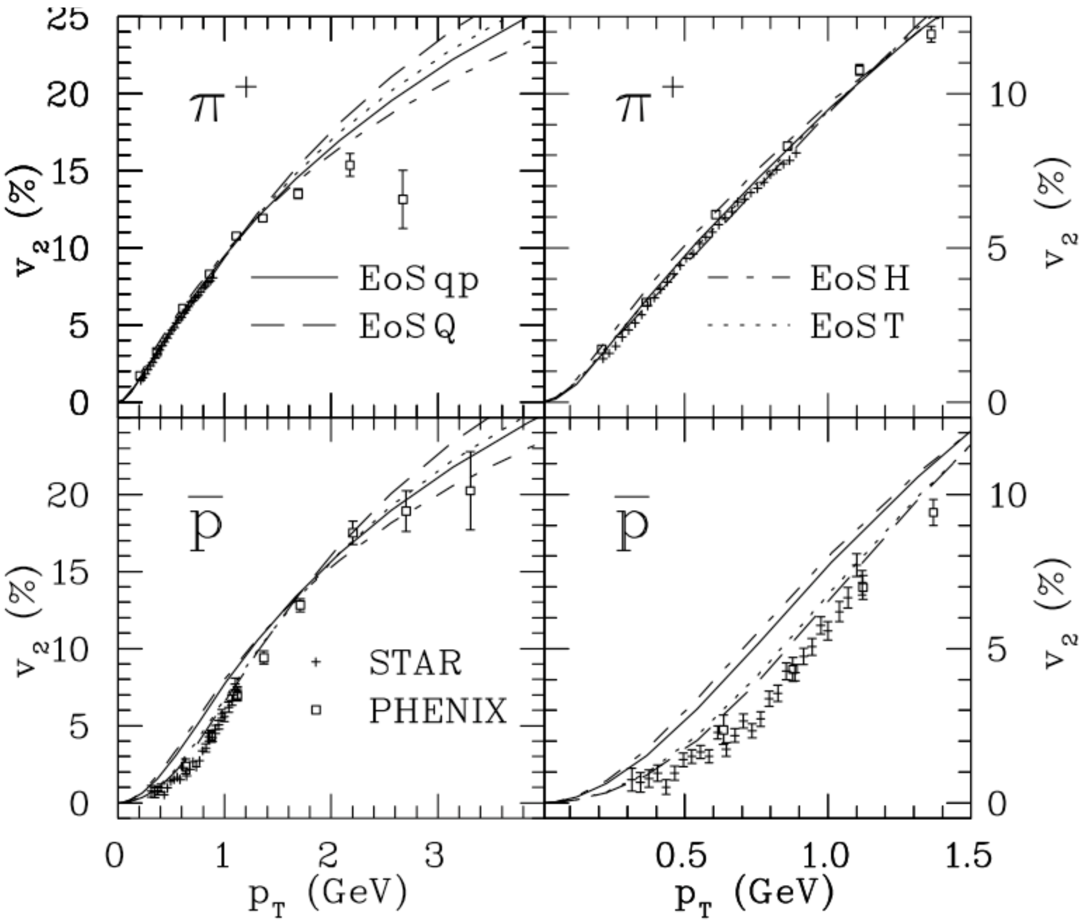
← input

} output

EOS Dependence

Huovinen nucl-th/0505036

	EoS qp	EoS Q	EoS H	EoS T
$\langle T_{fo} \rangle$ (MeV)	141	130	134	130



- Difference is small.
- Lattice QCD EoS ?

EOS dependence

Initial conditions and freezeout process should be fixed.

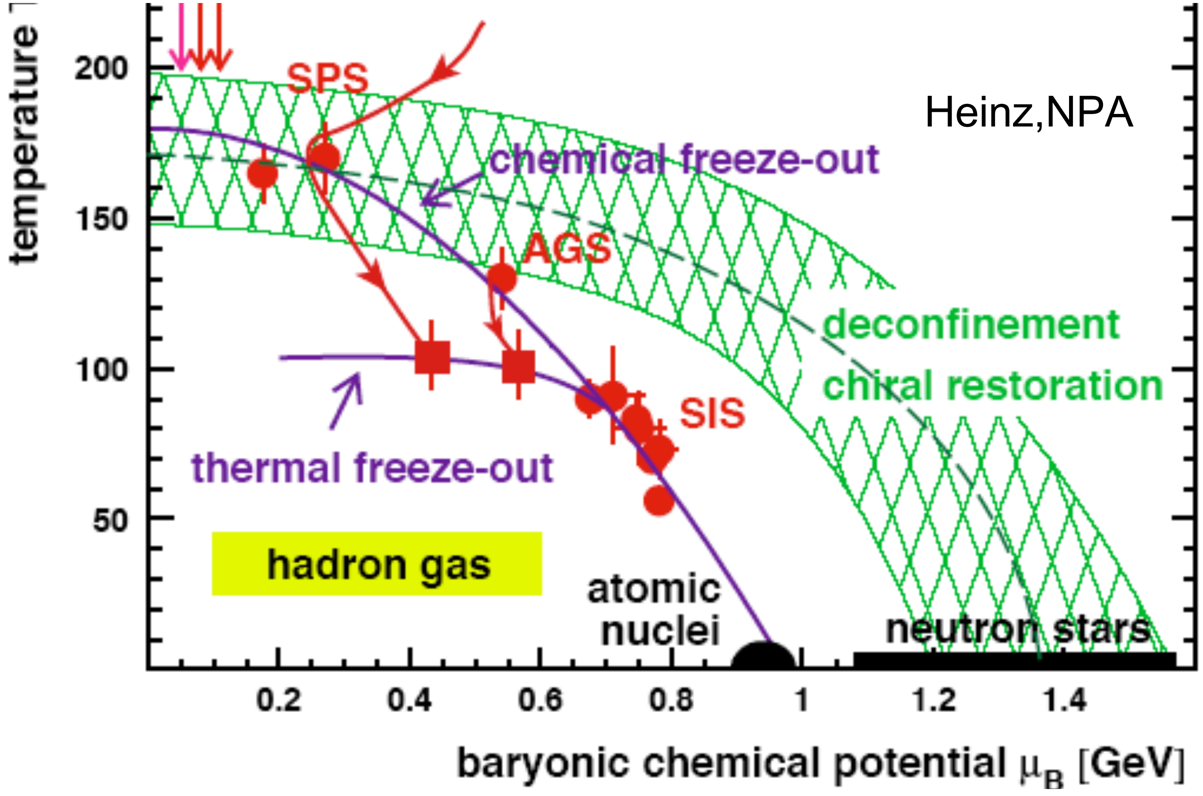
Hydrodynamic Models at RHIC

- Useful tool for experimental data
  - P_T, η -distributions, radial flow, anisotropic flow, HBT, jets
- Check List
  - Initial conditions, equation of states, freezeout process
  - Freezeout process

    Chemical freezeout, kinetic freezeout, final state interactions
    - Chemical equilibrium, Huovinen, Kolb, Heinz…
    - Partial chemical equilibrium, Hirano, Tsuda, Kolb, Rapp
    - Continuous Emission Model, Socolwski, Grassi, Hama, Kodama
    - Cascade model, Bass, Dumitru, Teaney, Shuryak, Nonaka, Hirano, Nara, Heinz…

Freezeout process in Hydro

- Single freezeout temperature?
  - Difference between chemical and thermal freezeout



Heinz,NPA

- chemical freezeout
  statistical model
  T_{ch} ~170 MeV
  hadron ratio

- thermal freezeout
  hydro
  T_f ~ 110~140 MeV

Possible solutions:

- Partial chemical equilibrium (PCE)
  Hirano, Teaney, Kolb, Rapp, Brazil group
- Hydro + Micro Model
  Bass, Dumitru, Teaney, Shuryak

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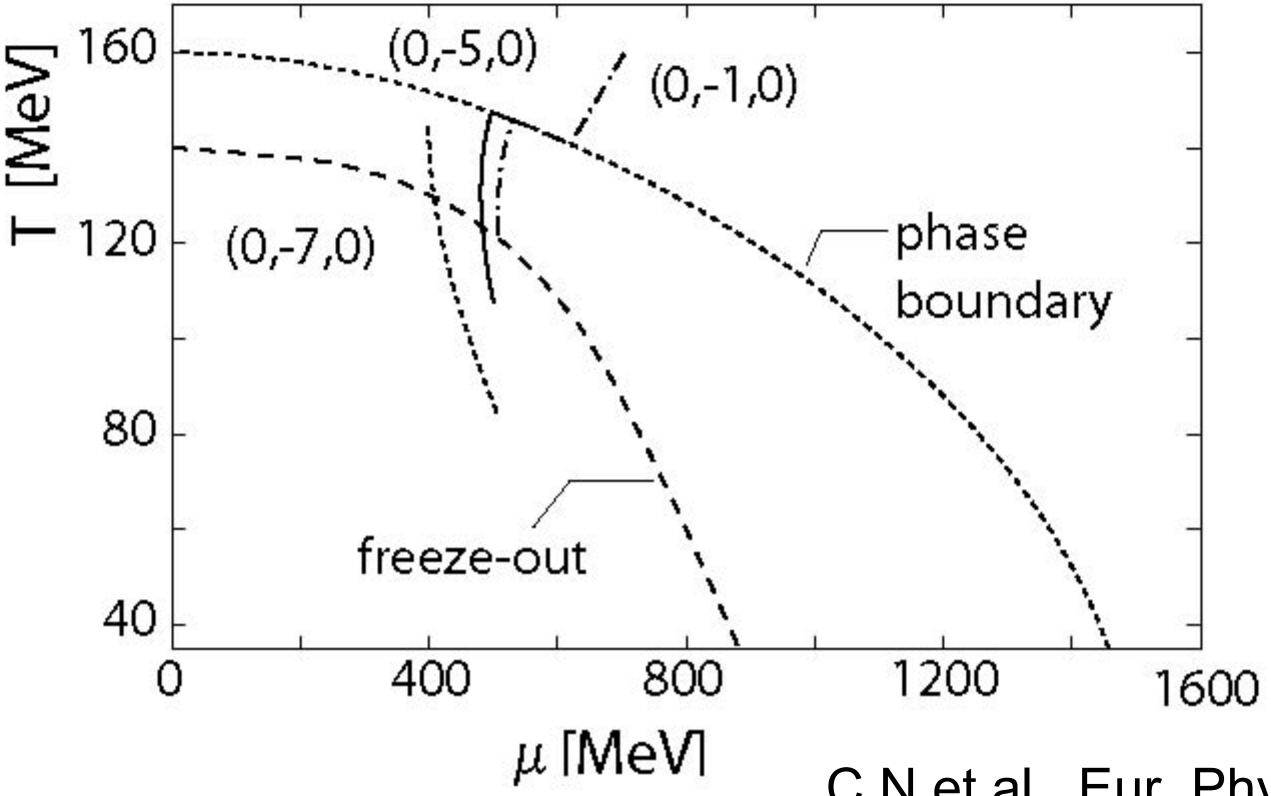
Trajectories on the phase diagram

temperature and chemical potential of volume element of fluid

n_B/s=const



C.N et al., Eur. Phys.J C17,663(2000)

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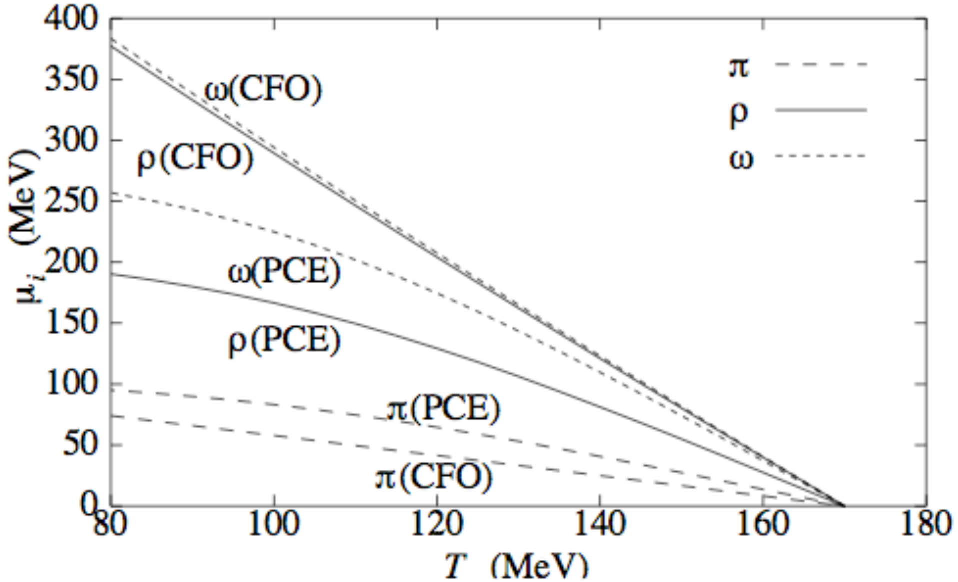
Partial Chemical Equlibrium

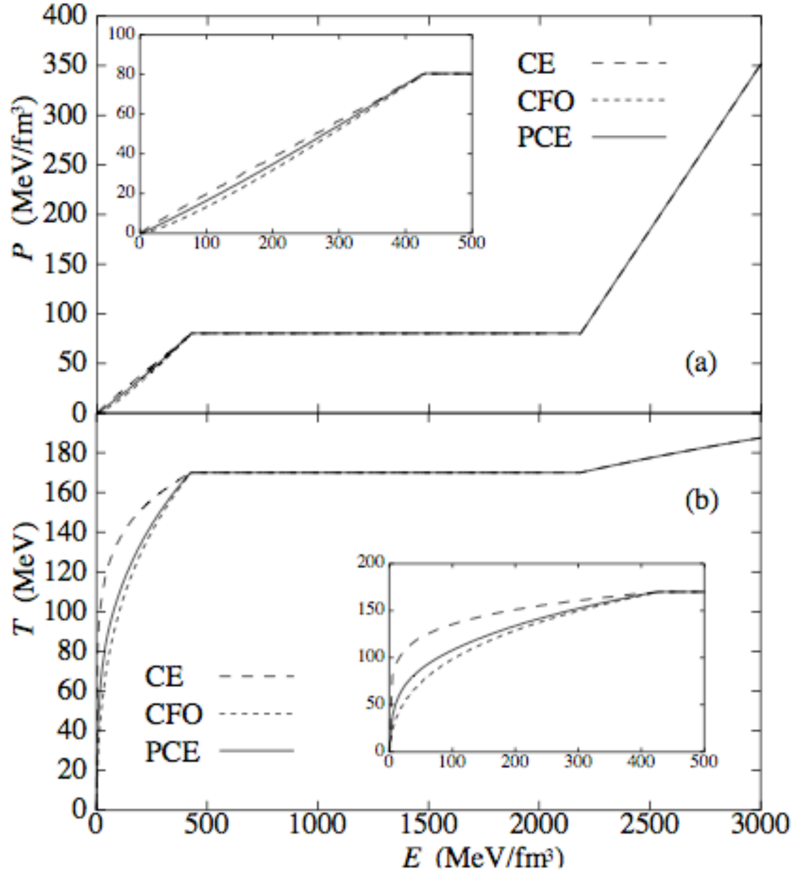
Hirano and Tsuda nucl-th/0205043

- Introducing chemical potential for each particle below T_{ch}

$$\frac{\bar{n}_i(T,\mu_i)}{s(T,\{\mu_i\})} = \frac{\bar{n}_i(T_{\rm ch},\mu_i=0)}{s(T_{\rm ch},\{\mu_i\}=0)}$$

$n_B/s=0$





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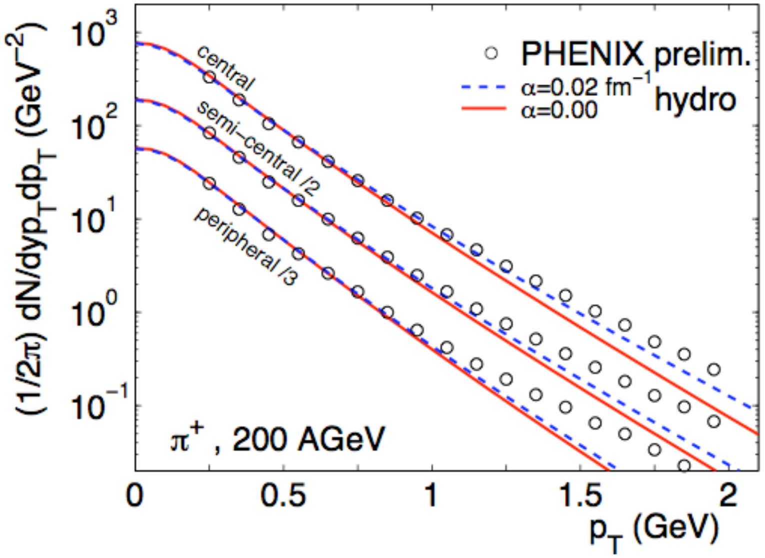
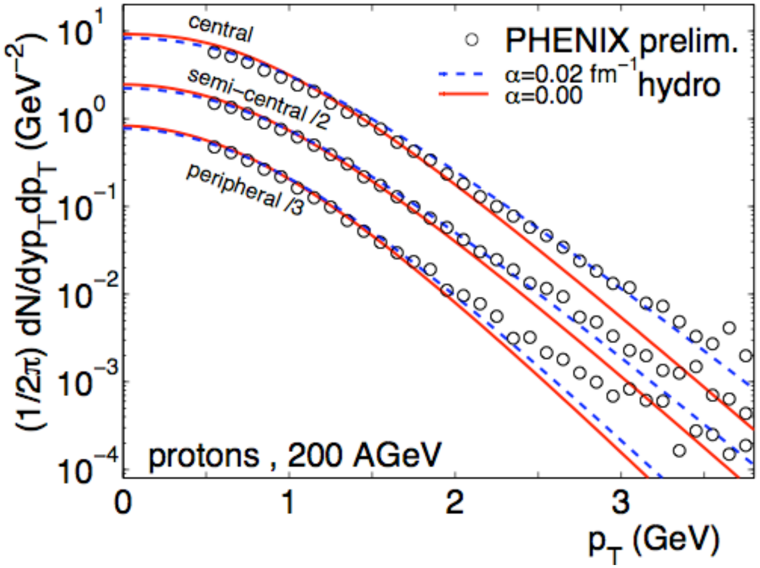
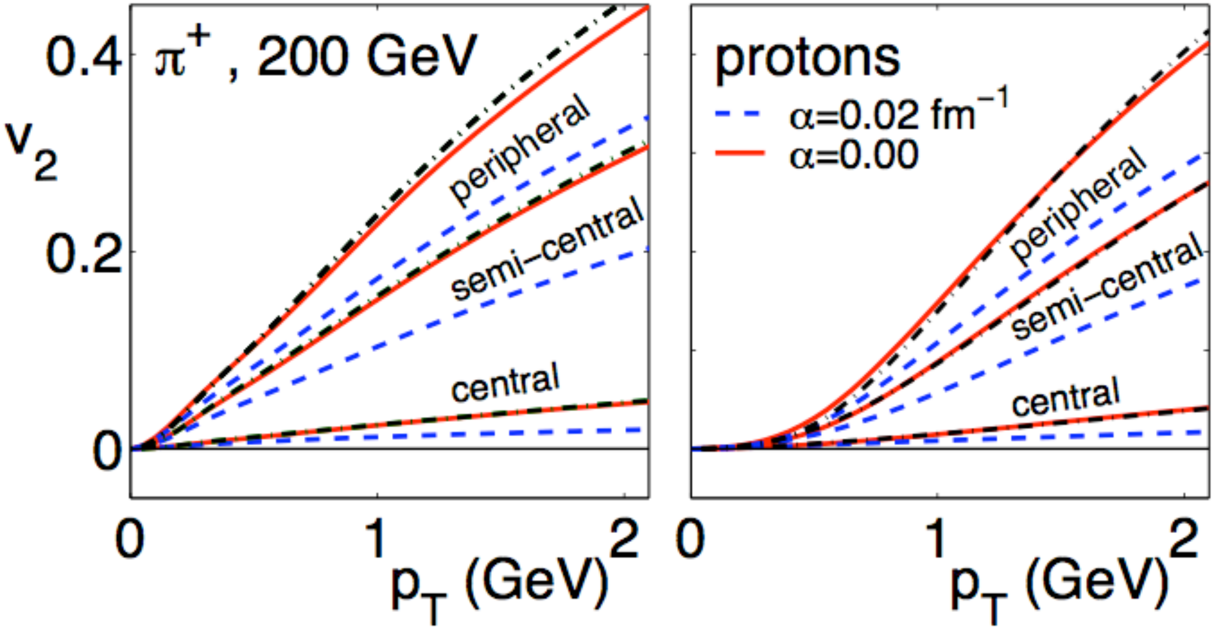
PCE with Initial v_T *Kolb and Rapp nuch-th/0210222*







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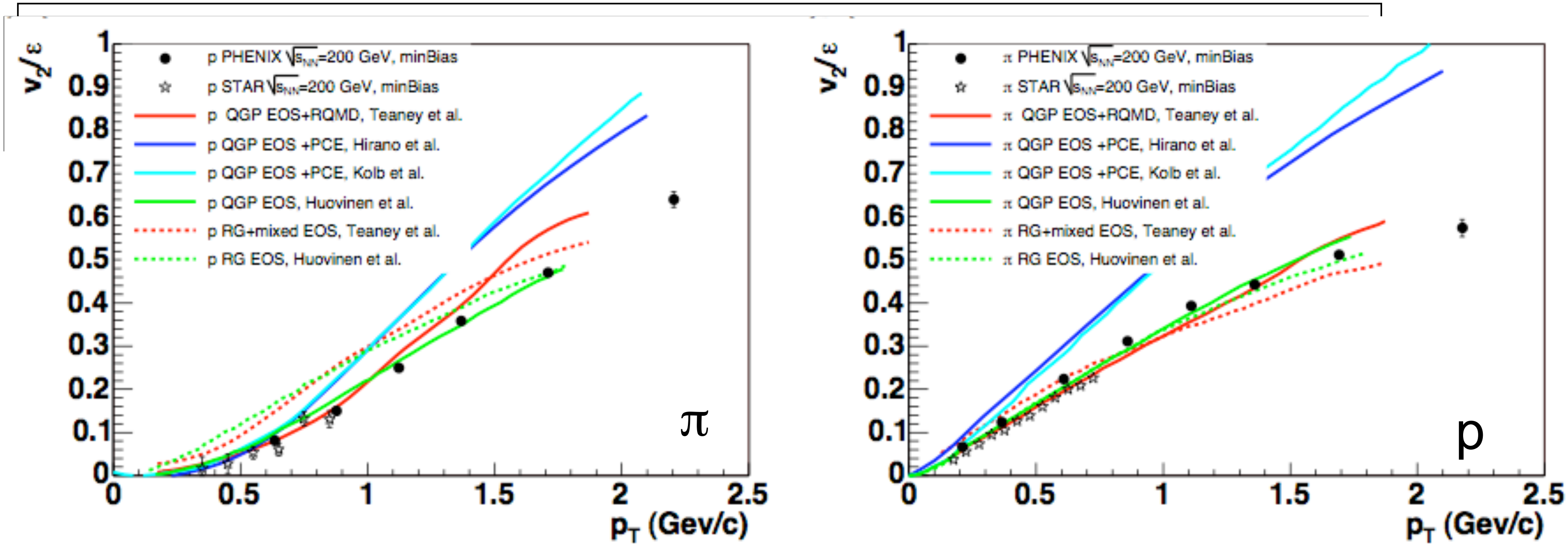
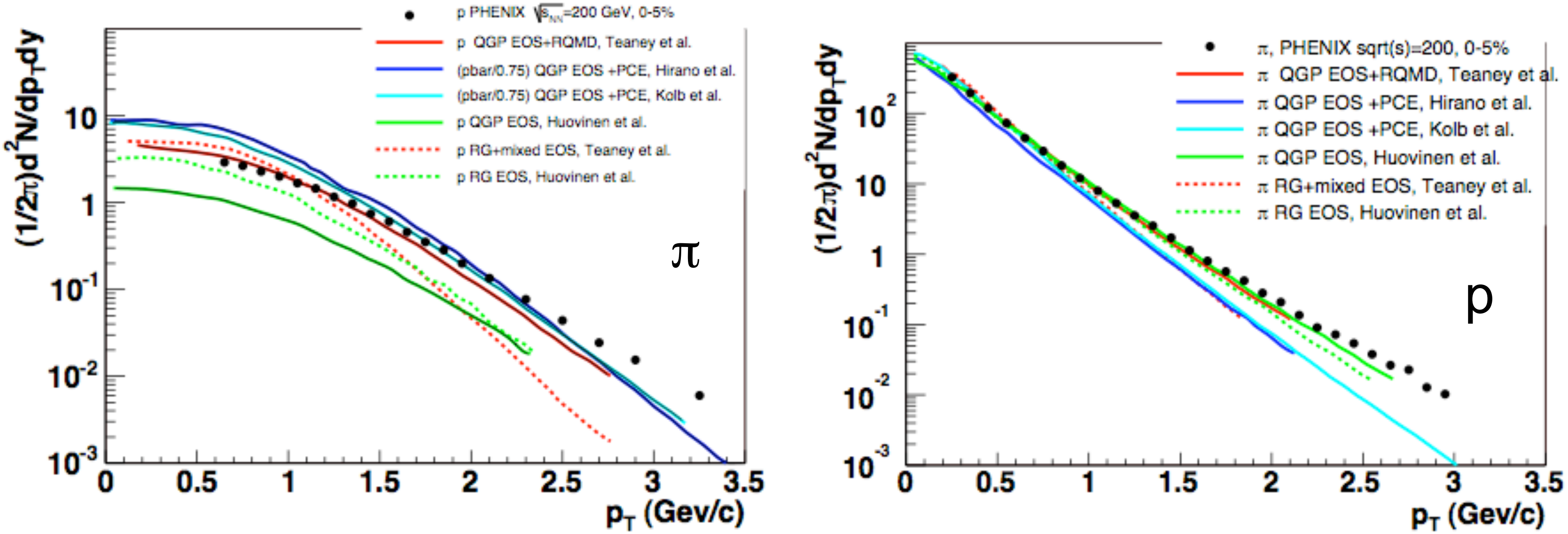


White paper by PHENIX nucl-ex/0410003



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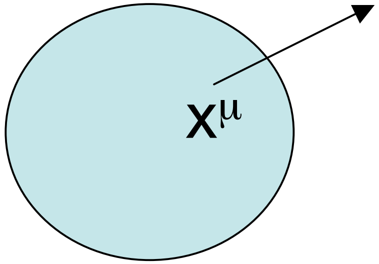
Continuous Emission Model

Socolowski,Grassi,Hama,Kodama: hep-ph/0405181

- Escaping probability

$$\mathcal{P}(x,k) = \exp\left[-\int_{\tau}^{\infty} \rho\sigma v d\tau'\right]$$



- Distribution function

$$f(x,k) = f_{free}(x,k) + f_{int}(x,k)$$

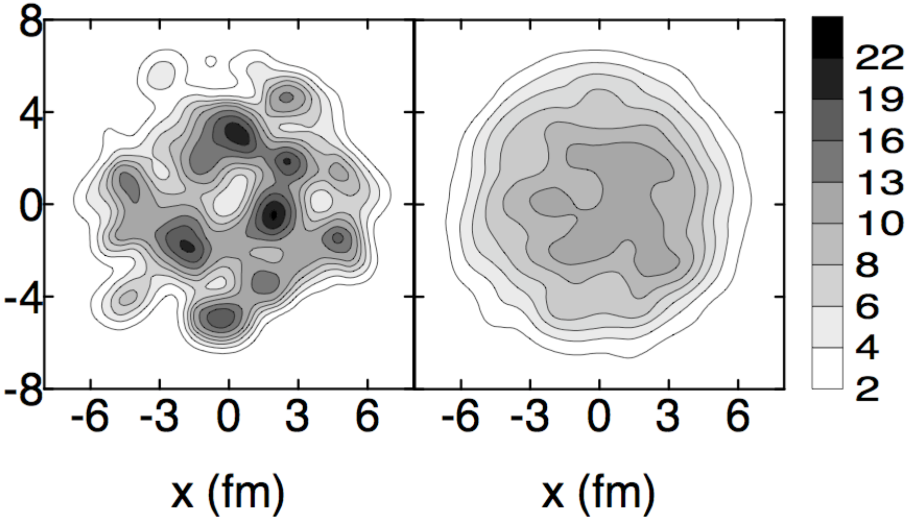
Interacting part

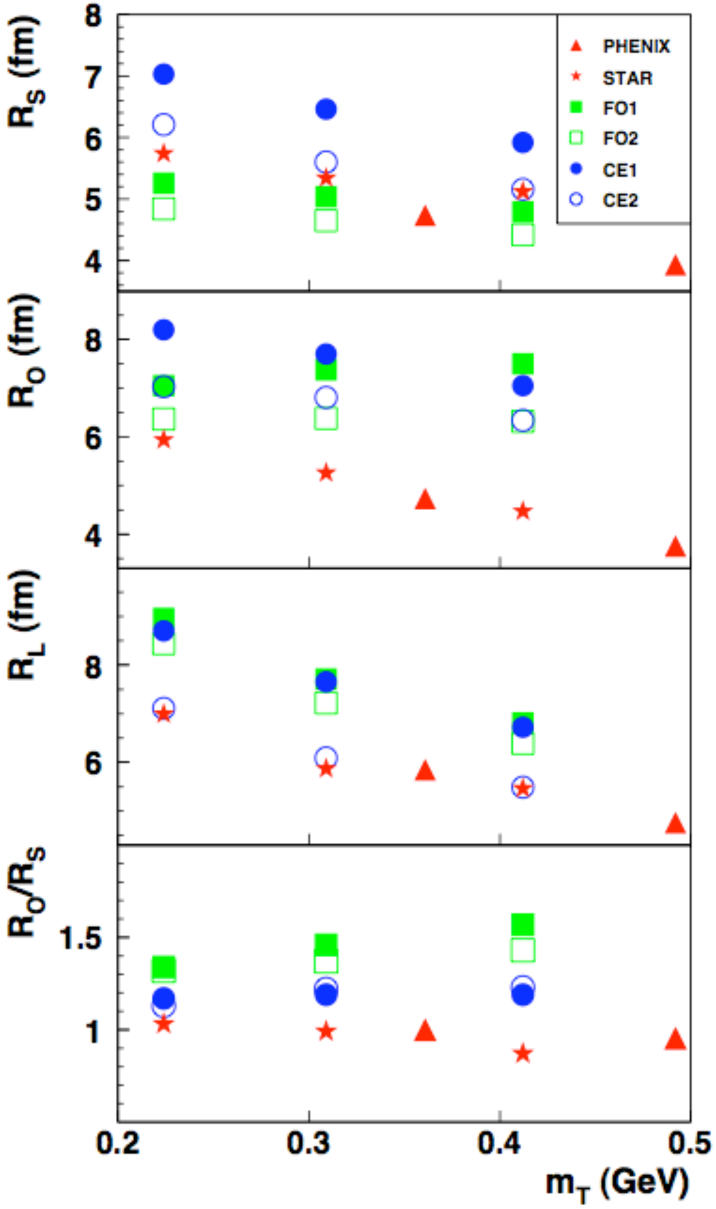
$$f_{free}(x,k) = \mathcal{P} f(x,k)$$

fluctuation in initial conditions + continuous emission model
➔HBT

Continuous Emission Model

Initial energy density
from NeXus





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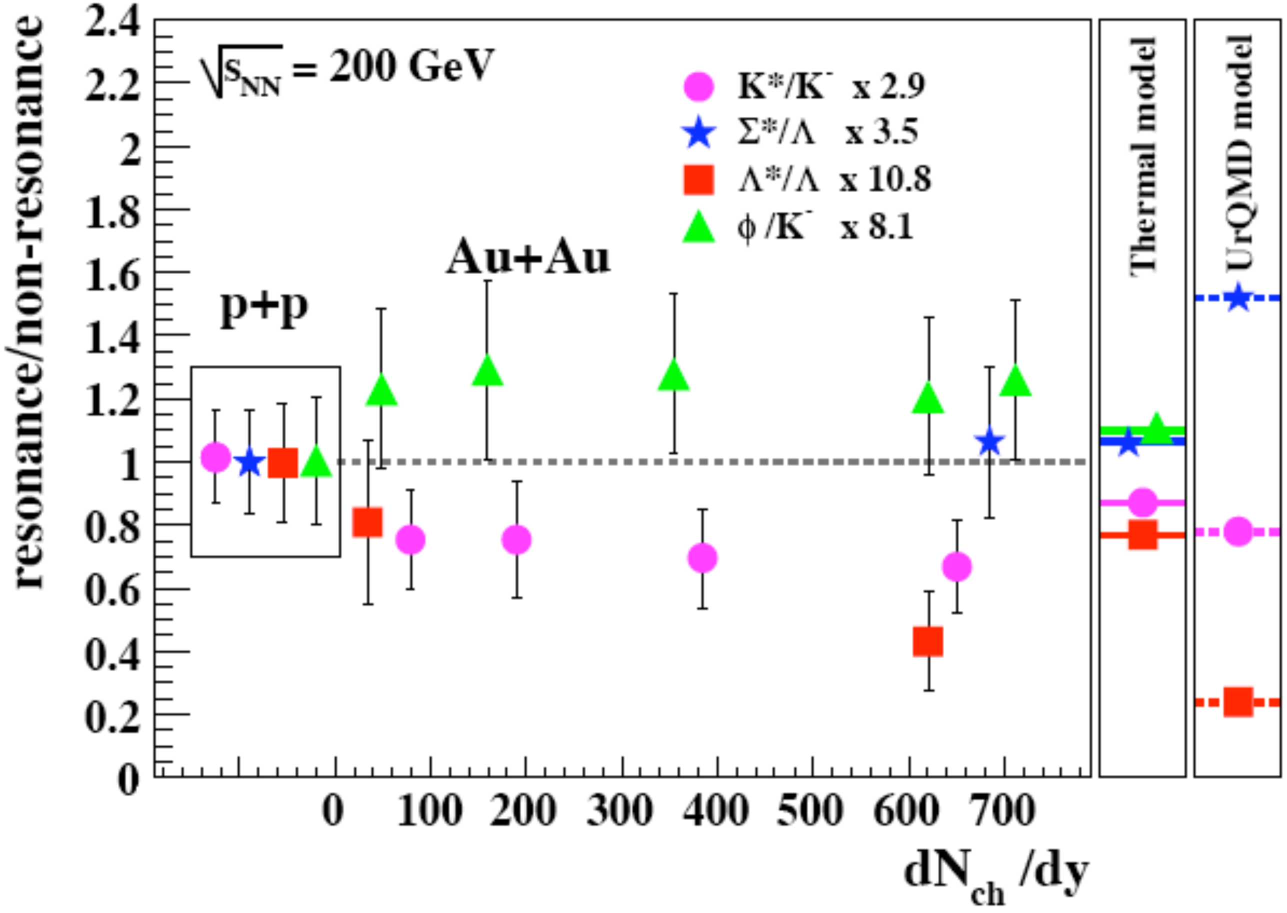
Final State Interactions

Ex. In UrQMD final state interactions are included correctly.



3D-Hydro +UrQMD

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Hydrodynamic Models

- Useful tool for experimental data
  - P_T , η -distributions, radial flow, anisotropic flow, HBT, jets
- Check List

	Initial Conditions	Freezeout Process	P_T distribution	Anisotropic Flow
[1]	**Glauber Type**	**Chemical Equilibrium**	☹ **ex. proton renormalized**	☹ **Accidentally?** by Hirano and Gyulassy
[2]		**Partial CE**	**Initial transverse flow?**	
[3][4][5]		**Cascade**	**Promising?**	**Viscosity**
[4]	**CGC**	**Partial CE**	☺	😐
[4]		**Cascade**	**Viscosity in early stage ?**	

[1]Huovinen, Kolb, Heinz…[2]Hirano, Tsuda,Kolb,Rapp [3]Bass, Dumitru,[4]Hirano, Nara, Heinz…[5]CN, Bass

Realistic Hydro Model

- Hydro + Cascade Model

  Bass, Dumitru, Teaney, Shuryak, Hirano, Nara, CN, Heinz….

  – Viscosity in hadron phase

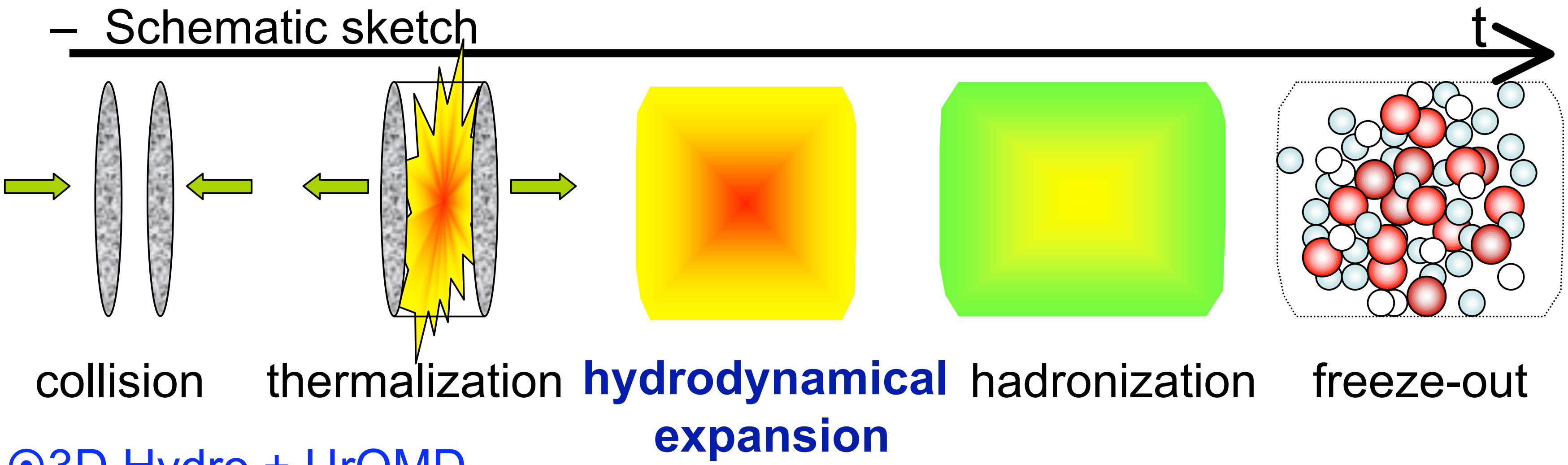
  – Realistic freezeout process

  – Final state interactions

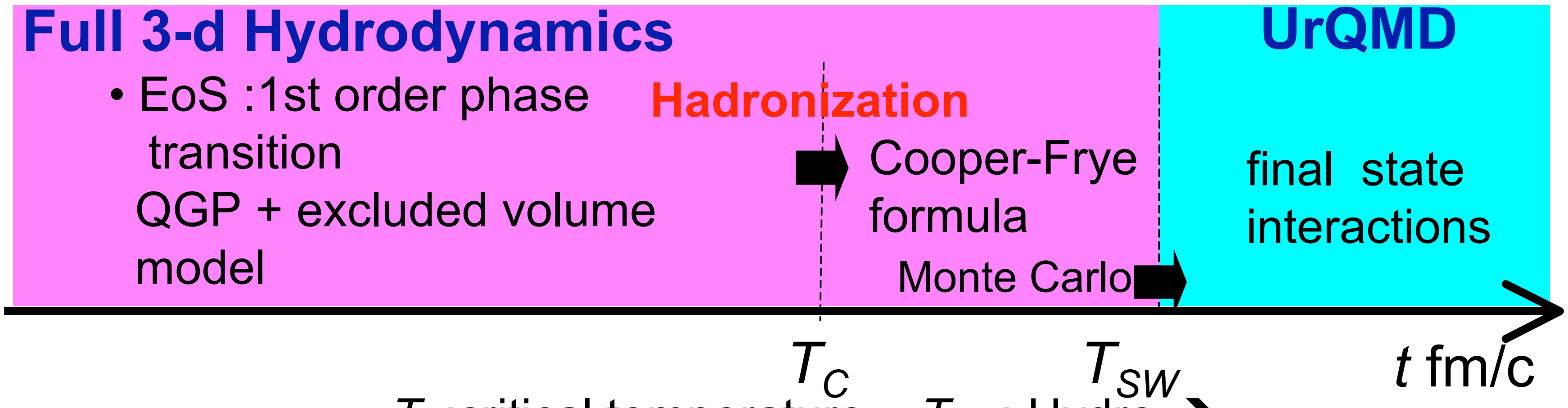
3D Hydro+UrQMD

- Relativistic Heavy Ion Collision
  - Schematic sketch

Nonaka and Bass PRC75:014902(2007)



⊙3D Hydro + UrQMD



Full 3-d Hydrodynamics

- EoS :1st order phase transition
  QGP + excluded volume model

Hadronization

Cooper-Frye formula

Monte Carlo

UrQMD

final state interactions

T_C T_{SW} t fm/c

T_C:critical temperature > T_{SW}: Hydro ➔ UrQMD

3-D Hydrodynamic Model

- Relativistic hydrodynamic equation

  $$\partial_\mu T^{\mu\nu} = 0 \qquad T^{\mu\nu}$$: energy momentum tensor

  - Baryon number conservation

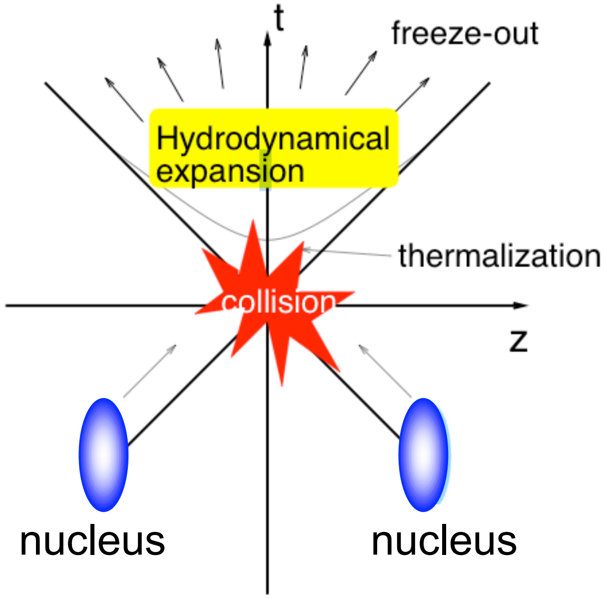
    $$\partial_\mu (n_B(T,\mu)) = 0$$

- Coordinates

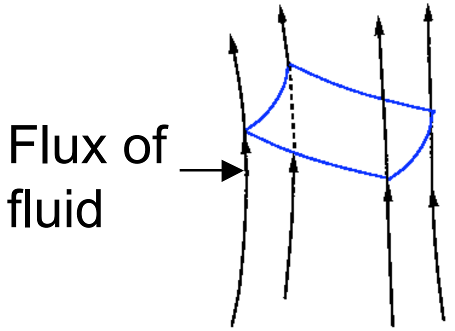
  $$(\tau, x, y, \eta) : \tau = \sqrt{t^2 - z^2}, \eta = \tanh^{-1}\left(\frac{z}{t}\right)$$

- Lagrangian hydrodynamics
  - Tracing the adiabatic path of each volume element
  - Effects of phase transition on observables
  - Computational time
  - Easy application to LHC
- Algorithm
  - Focusing on the conservation law

    $$\partial_\mu (s(T,\mu)u^\mu) = 0,\ \partial_\mu (n_B(T,\mu)u^\mu) = 0$$



Lagrangian hydrodynamics



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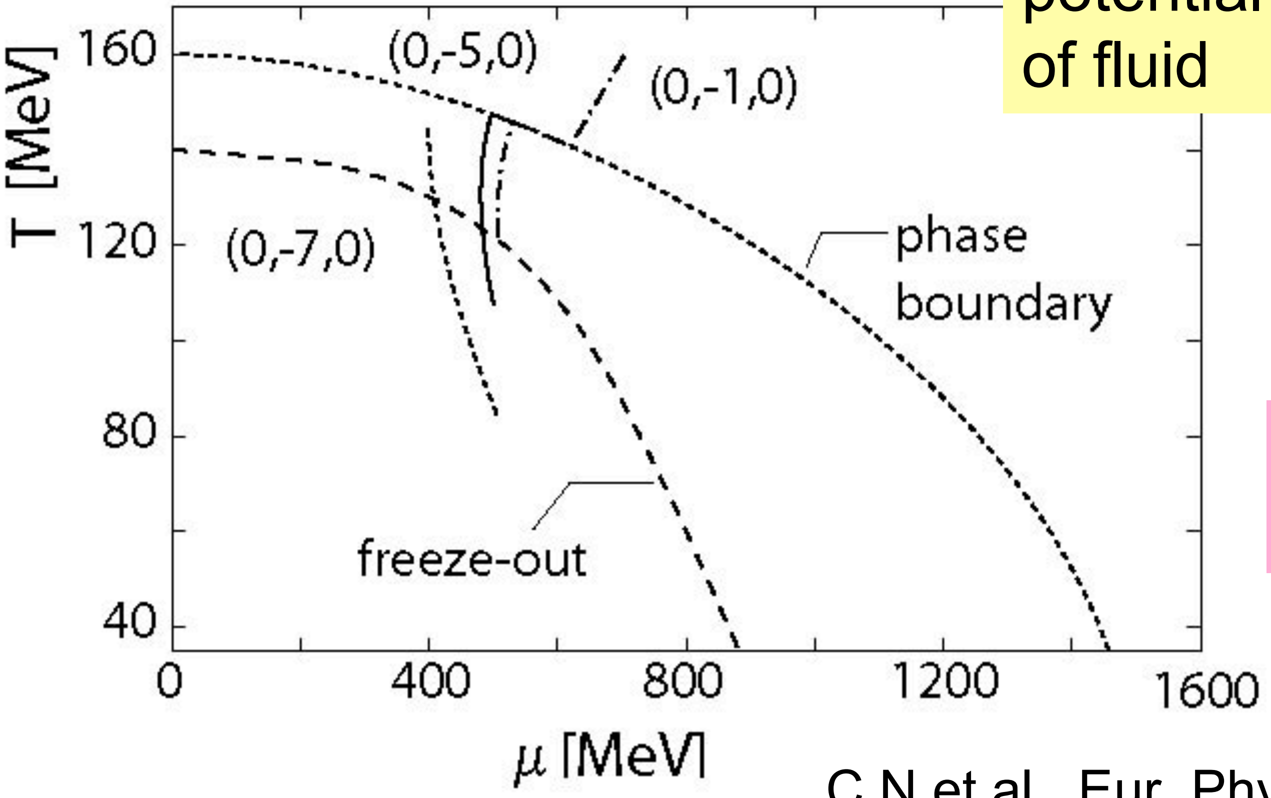
Trajectories on the phase diagram

- Lagrangian hydrodynamics



temperature and chemical potential of volume element of fluid

Effect of phase transition

C.N et al., Eur. Phys.J C17,663(2000)

Parameters

- Initial Conditions
  - Energy density

    $\varepsilon(x,y,\eta) = \varepsilon_{\max} W(x,y;b)H(\eta)$
  - Baryon number density

    $n_B(x,y,\eta) = n_{B\max} W(x,y;b)H(\eta)$
  - Parameters

    τ_0=0.6 fm/c

    $\varepsilon_{\max}$=40 GeV/fm^3, $n_{B\max}$=0.15 fm^{-3}

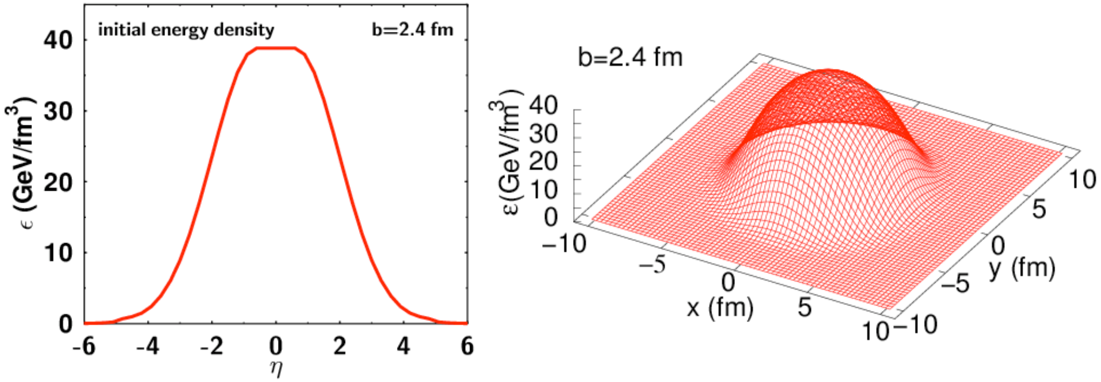
    η_0=0.5 σ_η=1.5
  - Flow

    $v_L=\eta$ (Bjorken's solution); $v_T=0$
- Equation of State

  1st order phase transition, T_c=160 MeV
- Switching temperature

  T_{SW}=150 [MeV]

•longitudinal direction •transverse plane



Different from Pure Hydro !

	hydro	Hydro+ UrQMD
τ_0(fm)	0.6	0.6
$\varepsilon_{\max}$(GeV/fm^3)	55	40
$n_{B\max}$(fm^{-3})	0.15	0.15
η_0,σ_η	0.5, 1.5	0.5, 1.5

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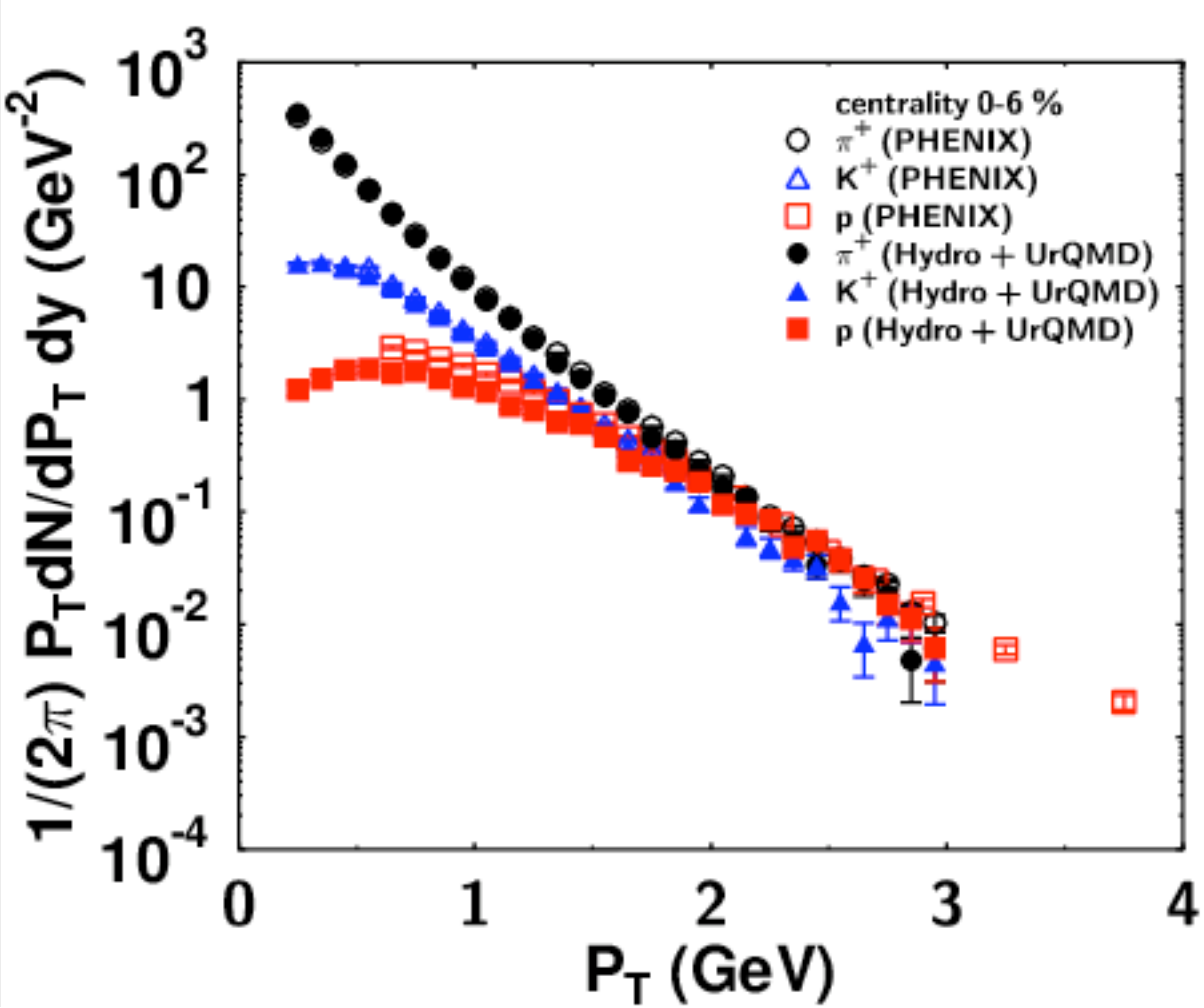
P_T spectra

- *P_T* spectra at central collisions



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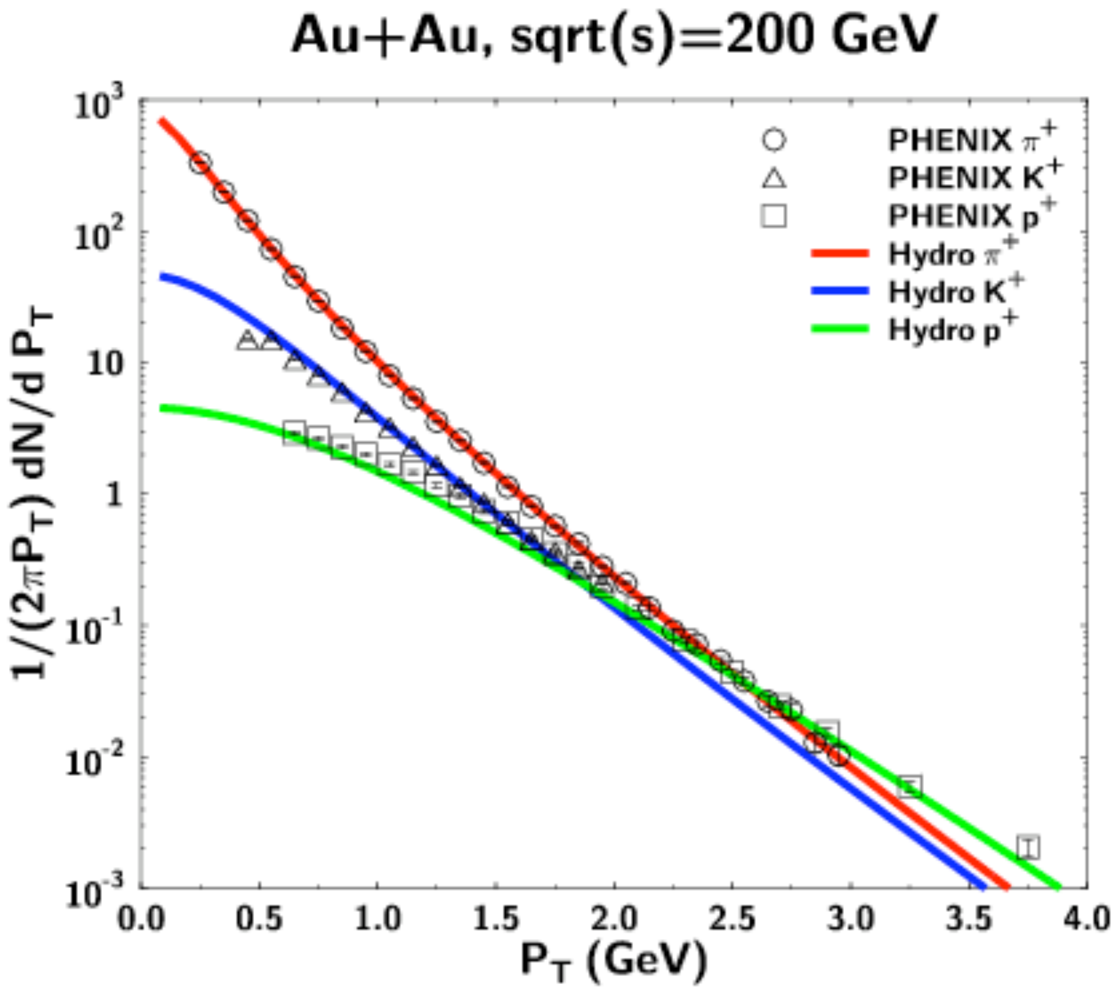
P_T Spectra (Pure Hydro)



T_f=110 MeV
normalization of p:
ratio at T_{chem}
Heinz and Kolb, hep-ph/0204061

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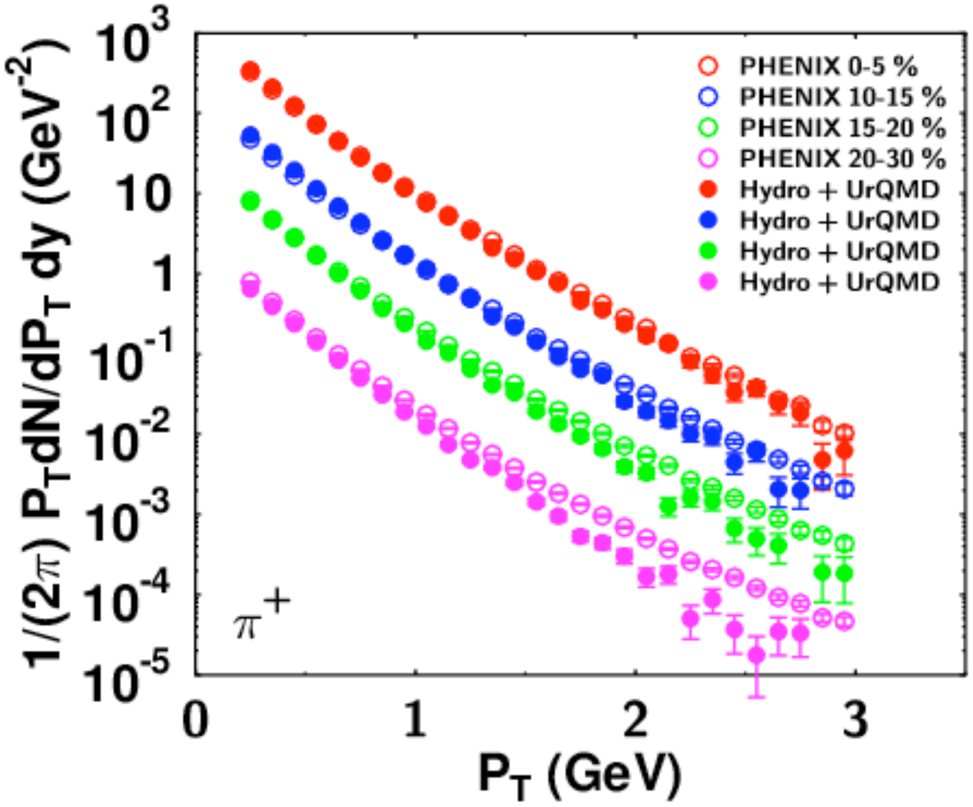
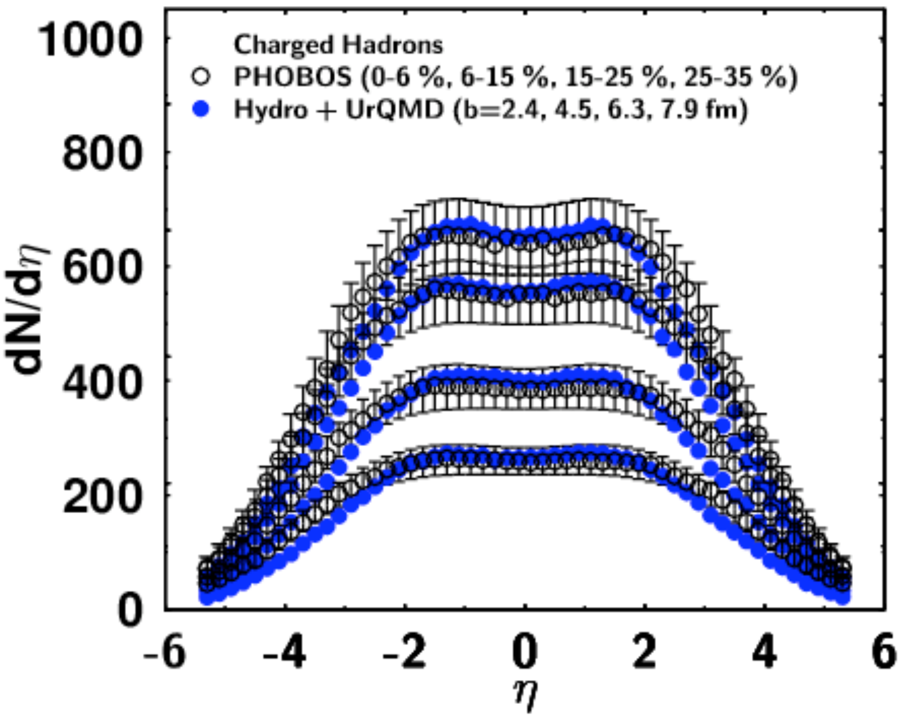
Centrality Dependence





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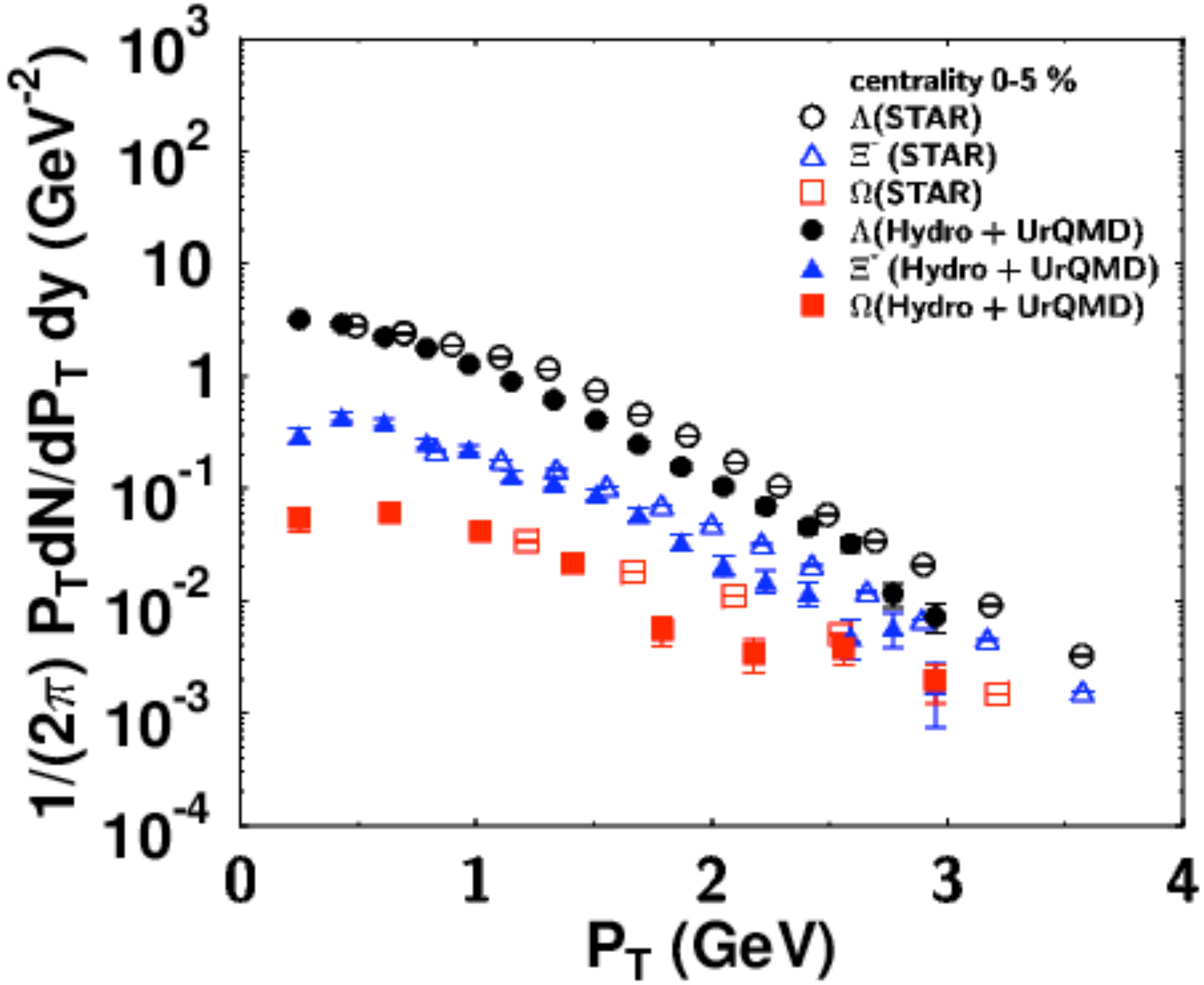
P_T Spectra for Strange Particles

Normalization is perfect!



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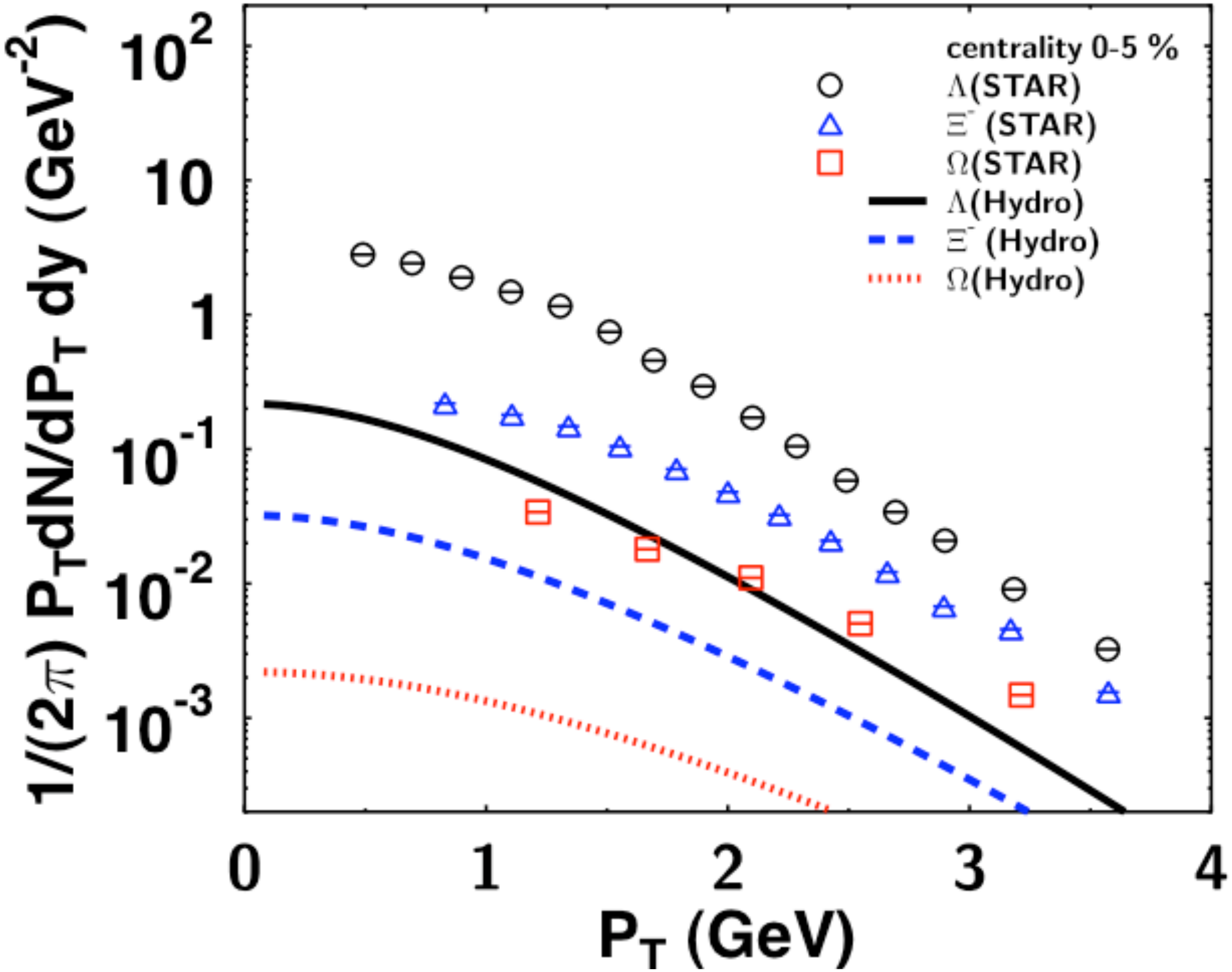
P_T Spectra for Strange Particles

Hydro…..



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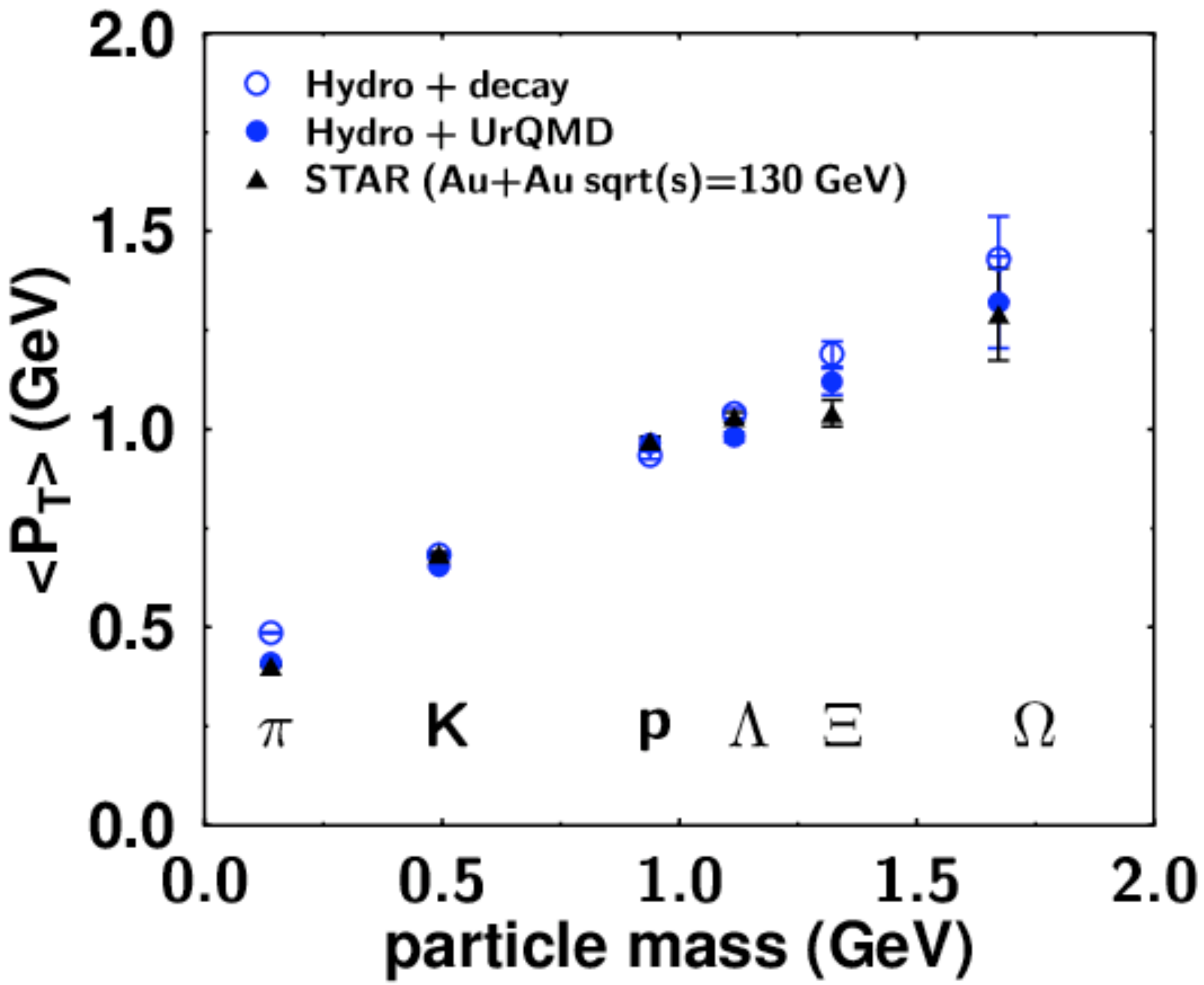
$<P_T>$ vs mass

- At mid rapidity



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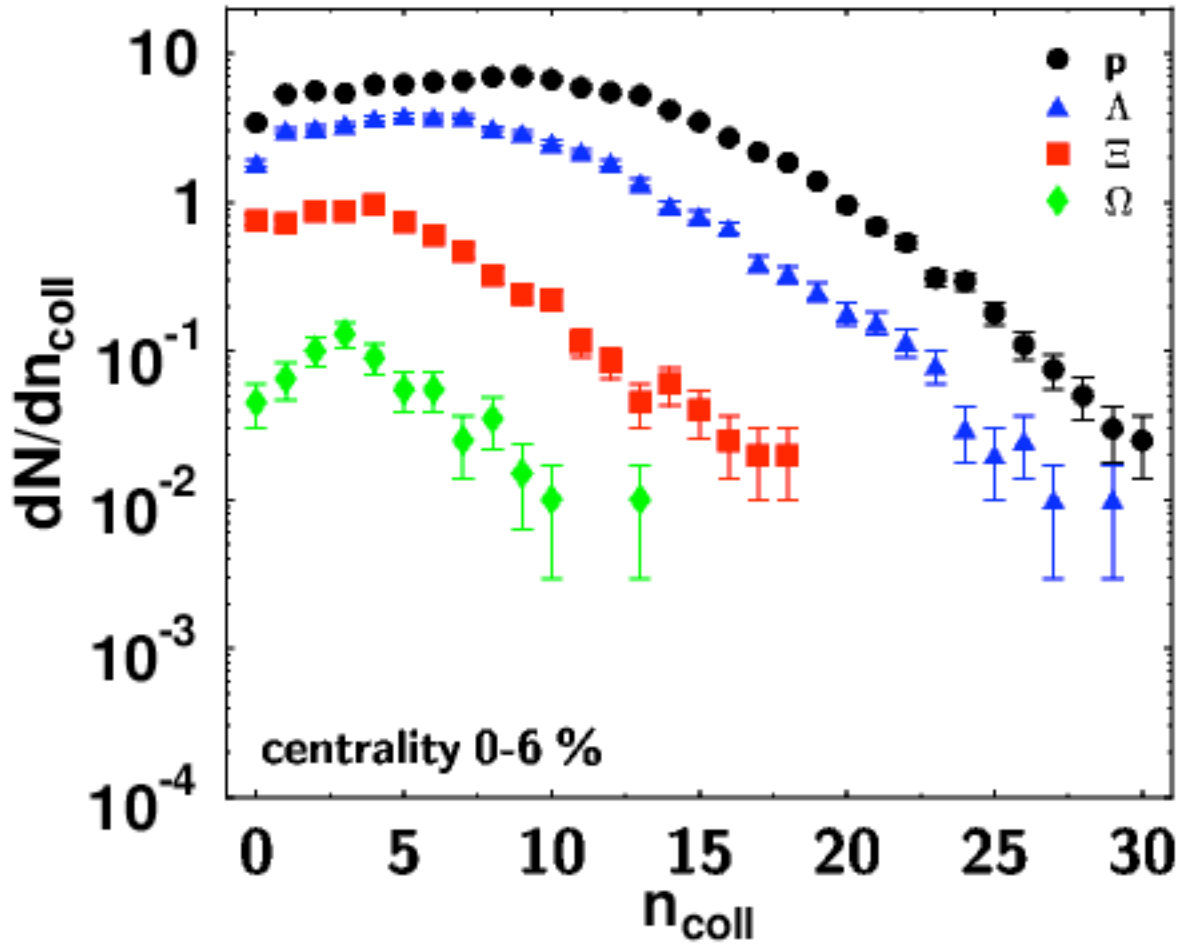
Distribution of # of Collisions



- p, Λ broad
  p ~ 10 times, Λ ~ 7 times
- Ξ, Ω   small cross section

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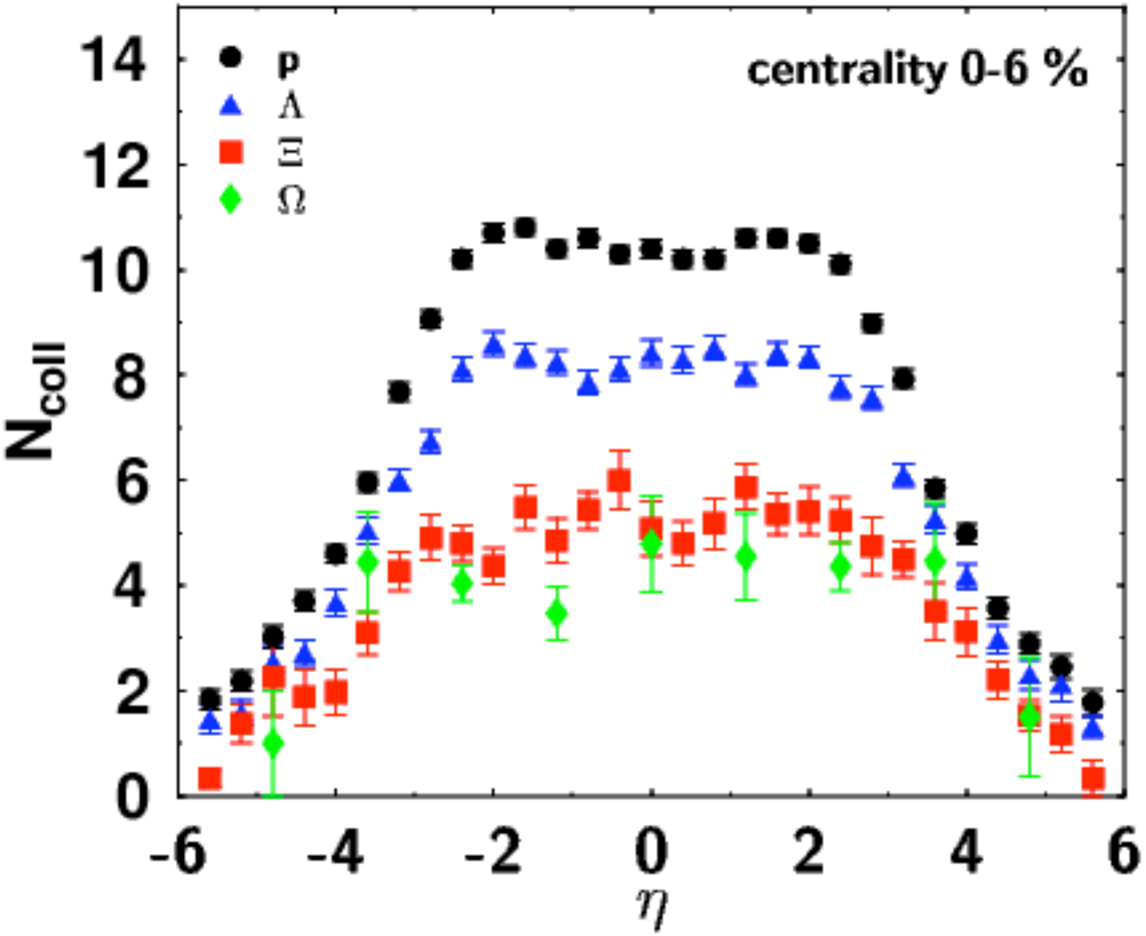
Distribution of # of Collisions



- plateau

  $-3 < \eta < 3$
- flavor difference

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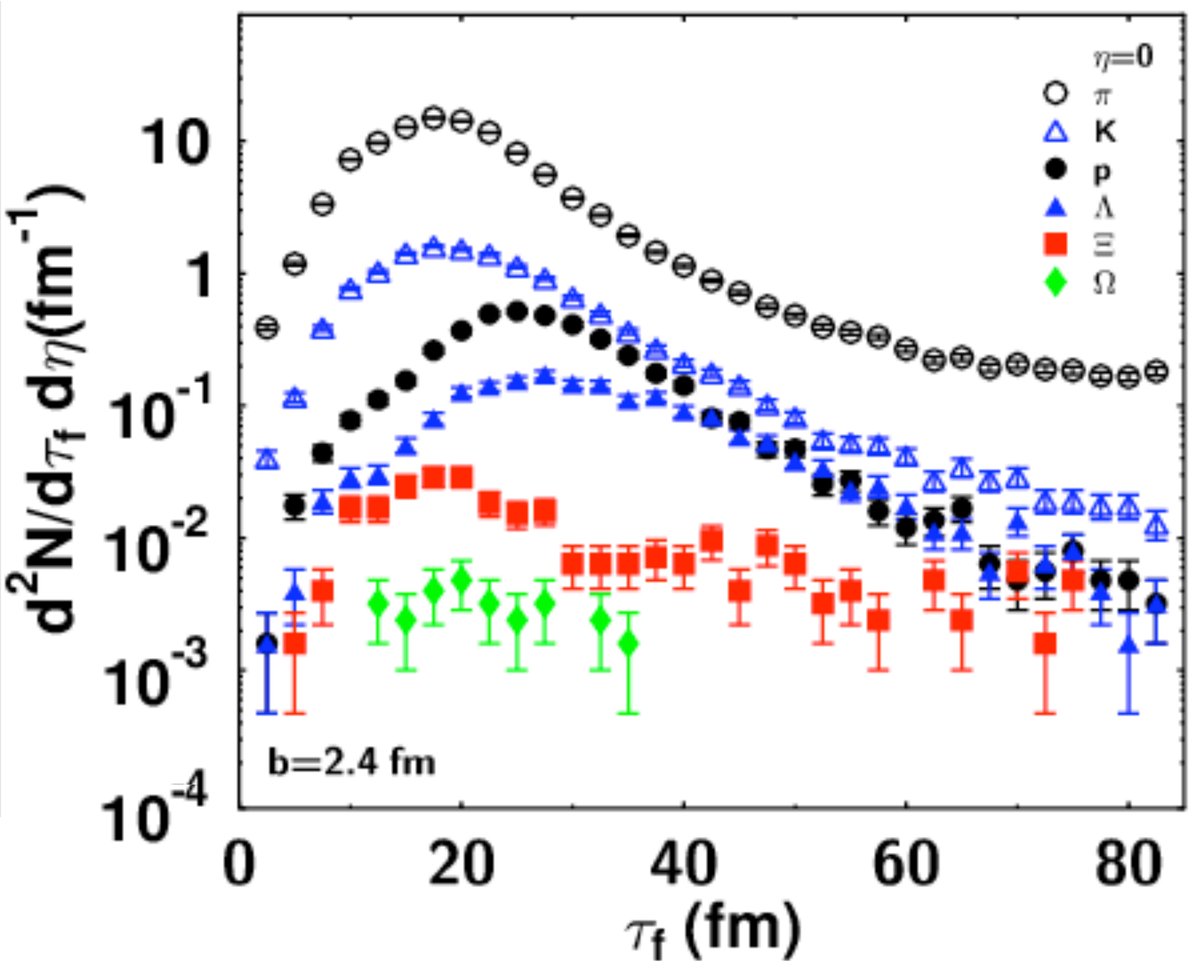
τ_f Distribution



- π, K: peak ~ 19 fm
- p, Λ: peak ~ 22 fm
- Multistrange particles small

•Species dependence
•Broad distribution

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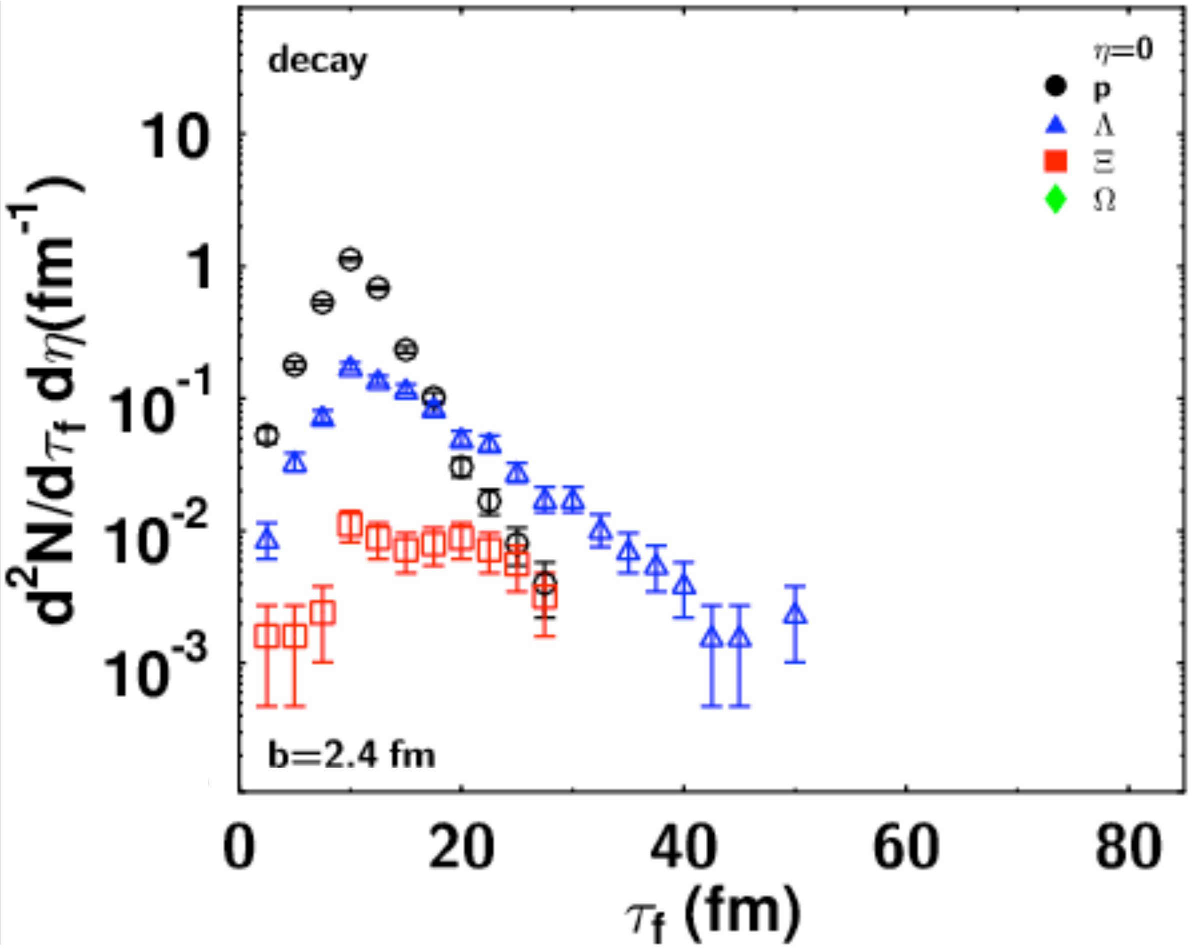
τ_f Distribution of B



- Final state interactions

↓

Duration of freezeout process

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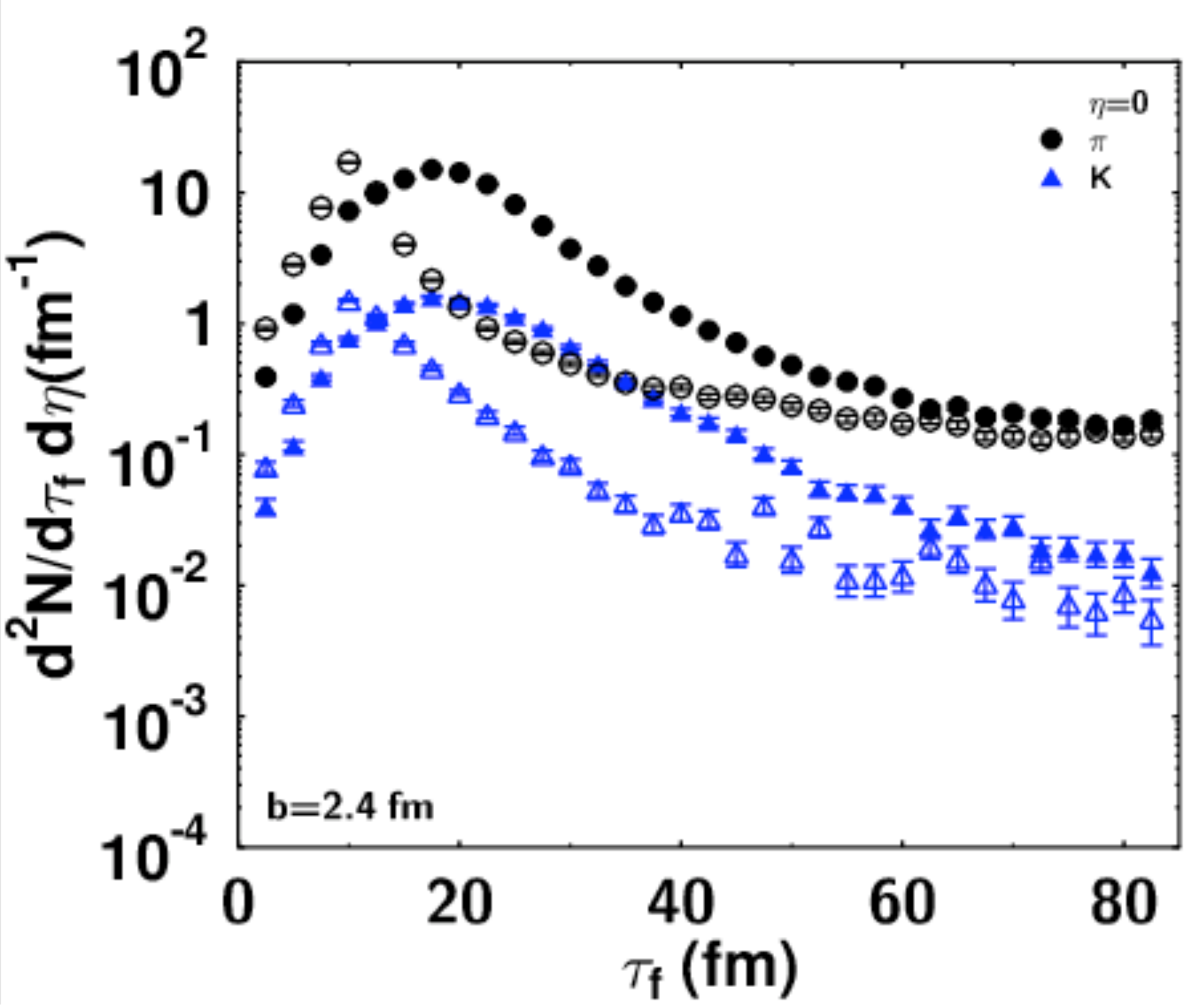
τ_f Distribution of M



- Final state interactions

↓

Duration of
freezeout process

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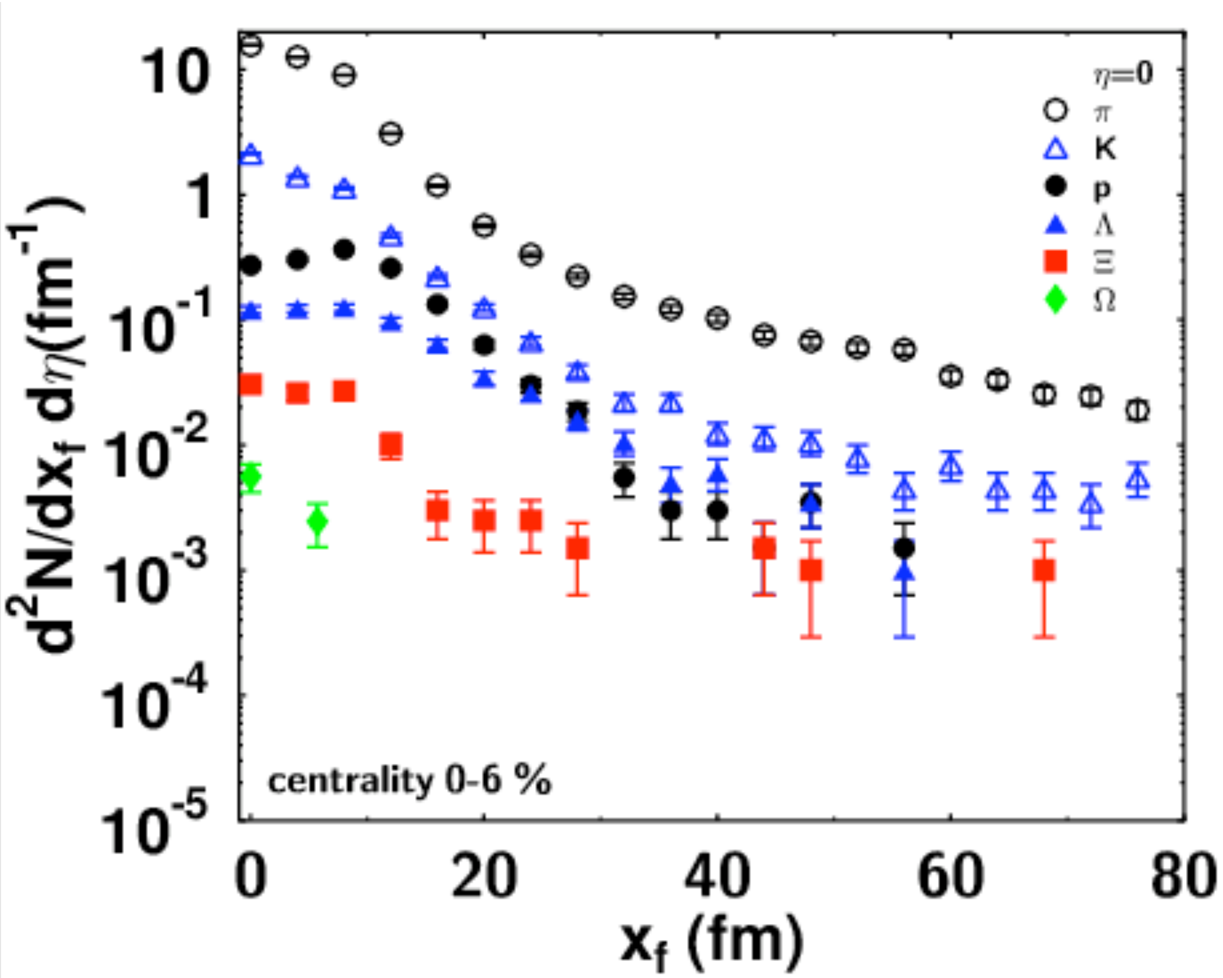
x_f Distribution



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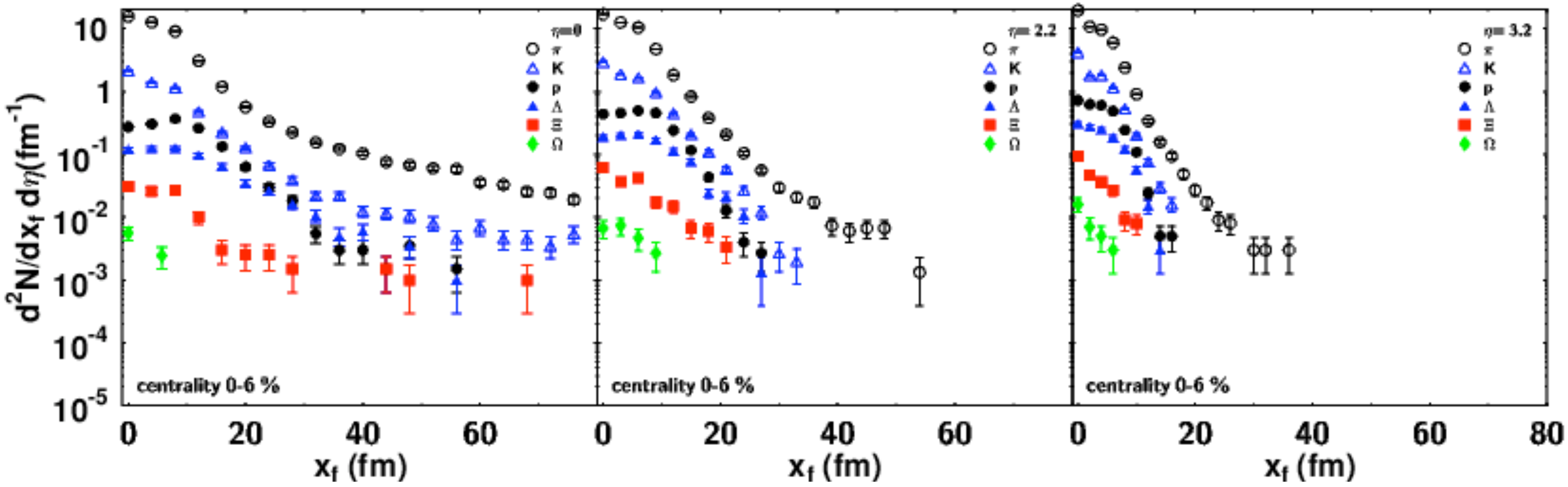
x_f Distribution II



→ *HBT analyses*

work in progress

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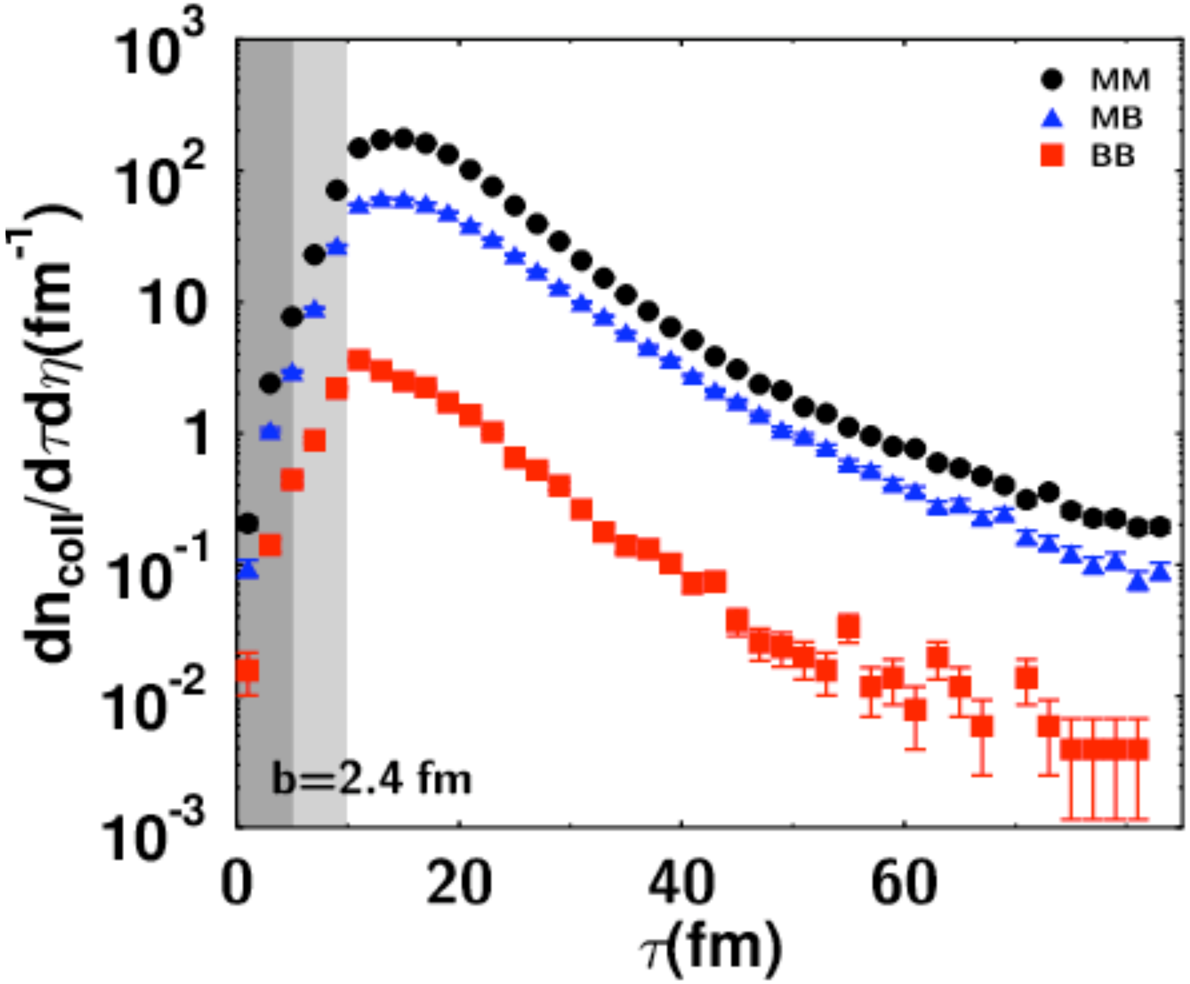
Collision Rate



- MM is dominant.

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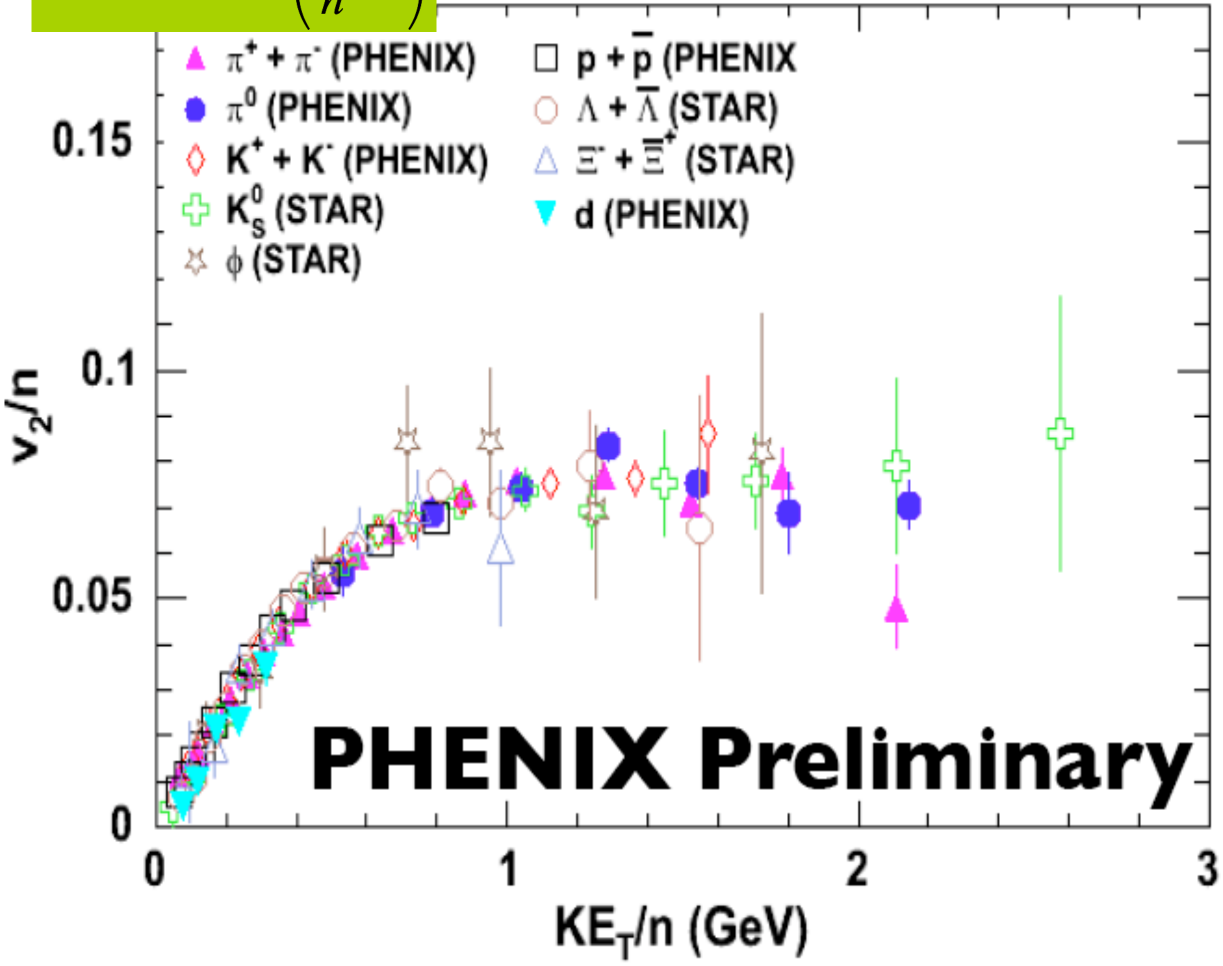
v_2 in hadron phase ?

- Quark number scaling of v_2

$$v_2^h(P_T) \approx n v_2\left(\frac{1}{n}P_T\right)$$



v_2: early stage
of expansion

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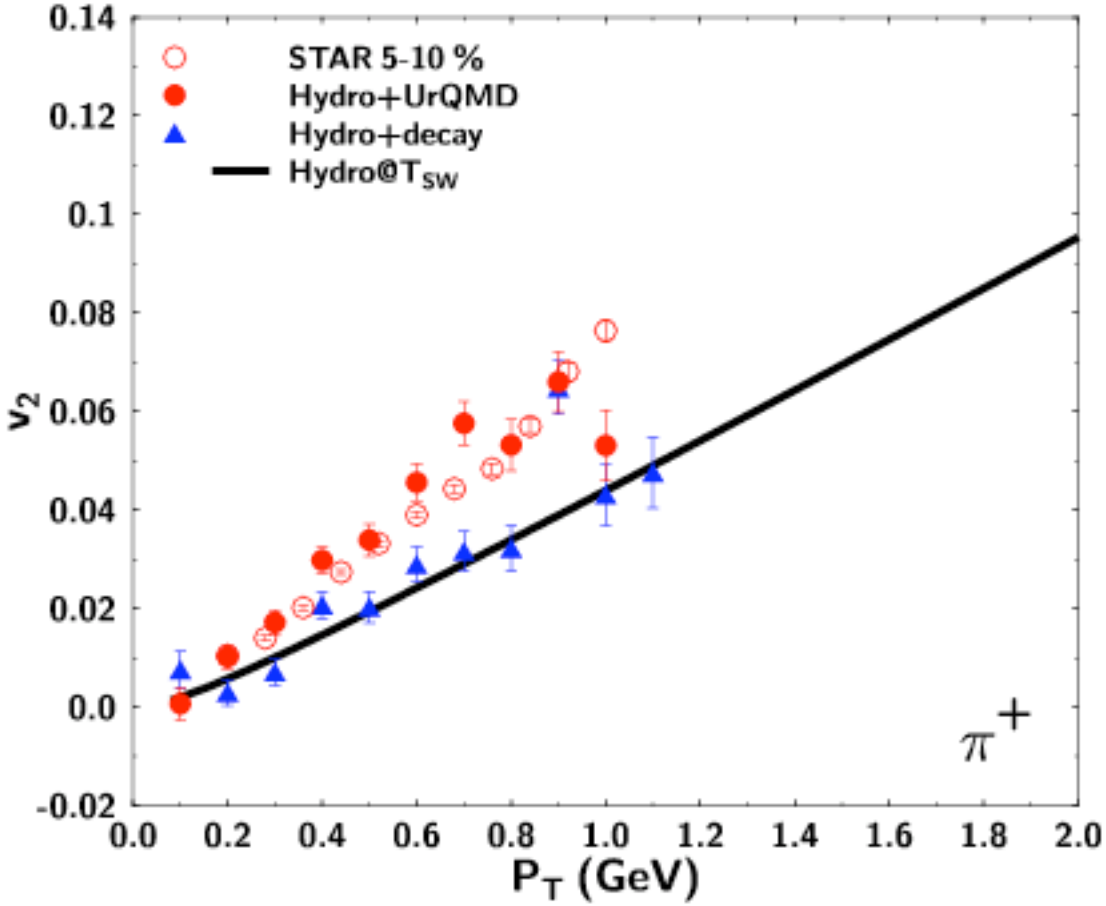
Reaction Dynamics in v_2 I



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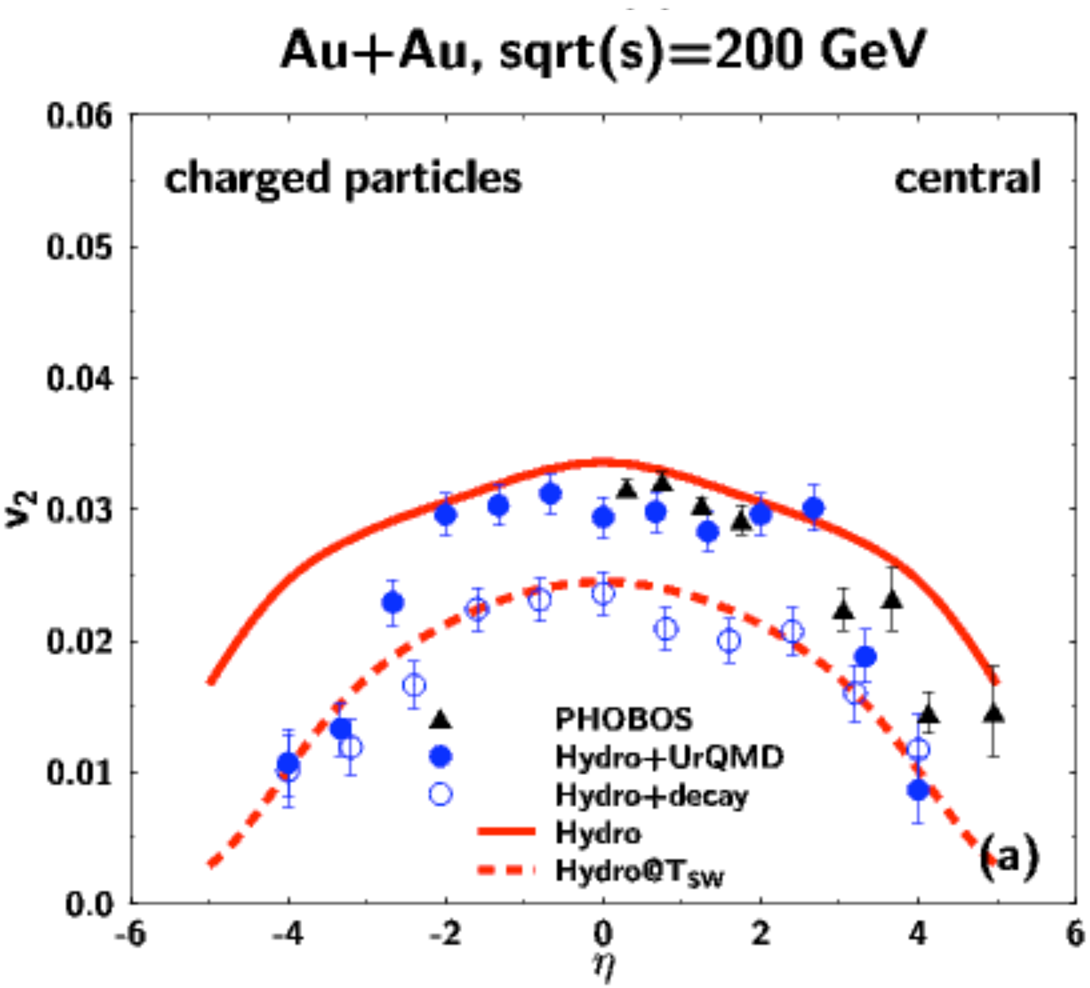
Reaction Dynamics in v_2 II



- •Hydro+decay ~ Hydro@T_{SW}
- •v_2 grows in hadron phase a bit.
- •v_2 builds up in QGP phase.

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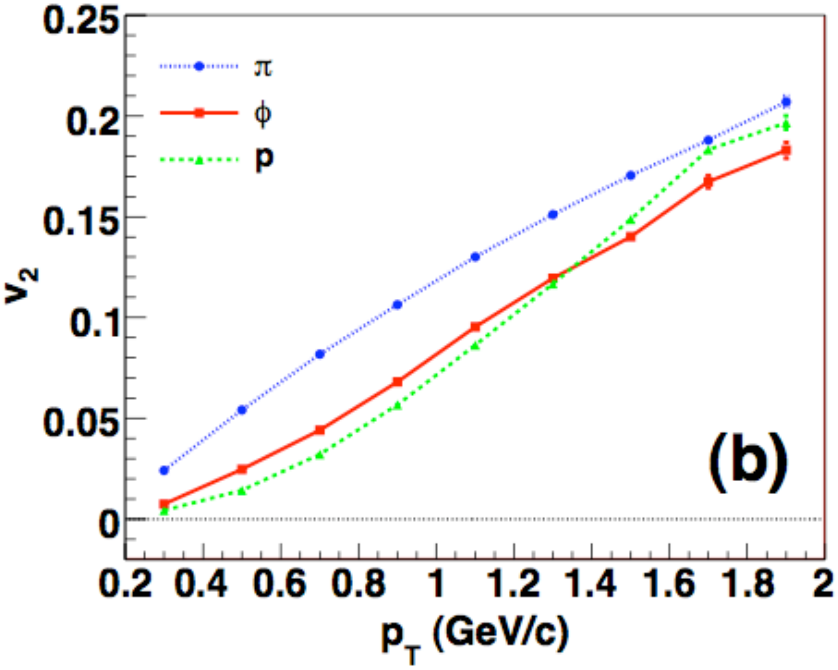
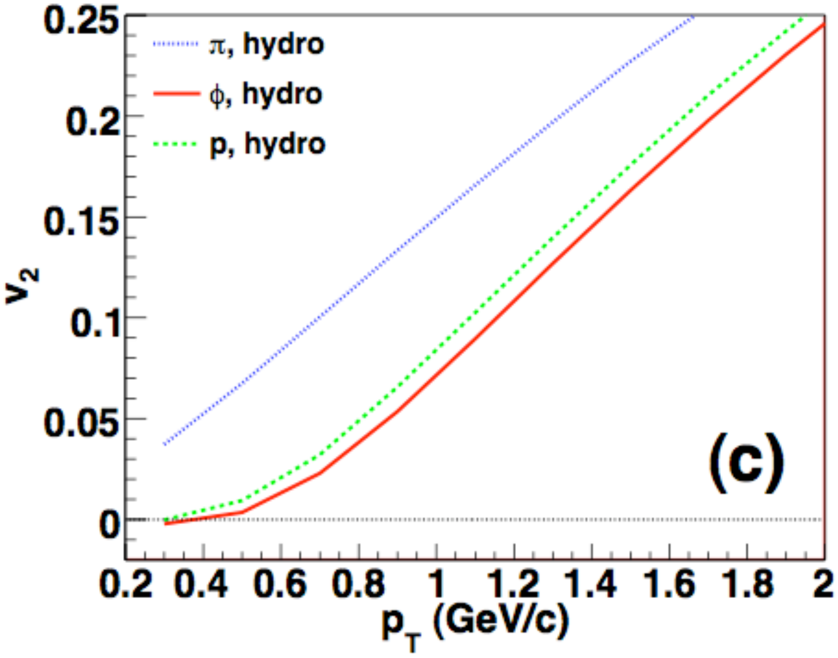
Mass Ordering in Elliptic Flow

Hirano et at. arXiv:0710.5795



Hydro + Cascade



Ideal Hydro

Viscosity in Hydro

- Viscous Relativistic Hydrodynamics

  Teaney,Muronga,Heinz,Song,Chaudhuri,Baier,Romatschke,Wiedemann, Kodama, Koide

  - Necessity of introduction of second order in entropy current

    Kunihiro-san's talk

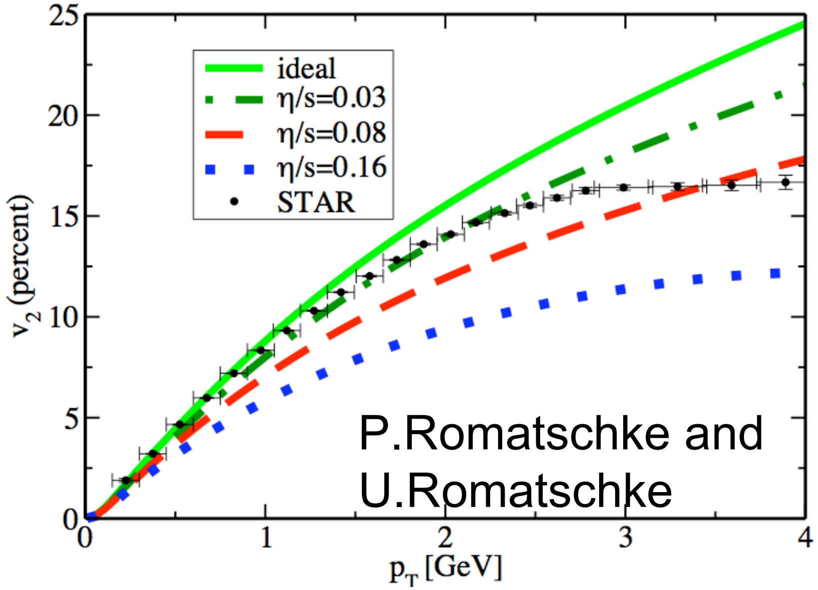
    stable 1st order relativistic dissipative hydrodynamics

  - Numerical calculation: shear viscosity

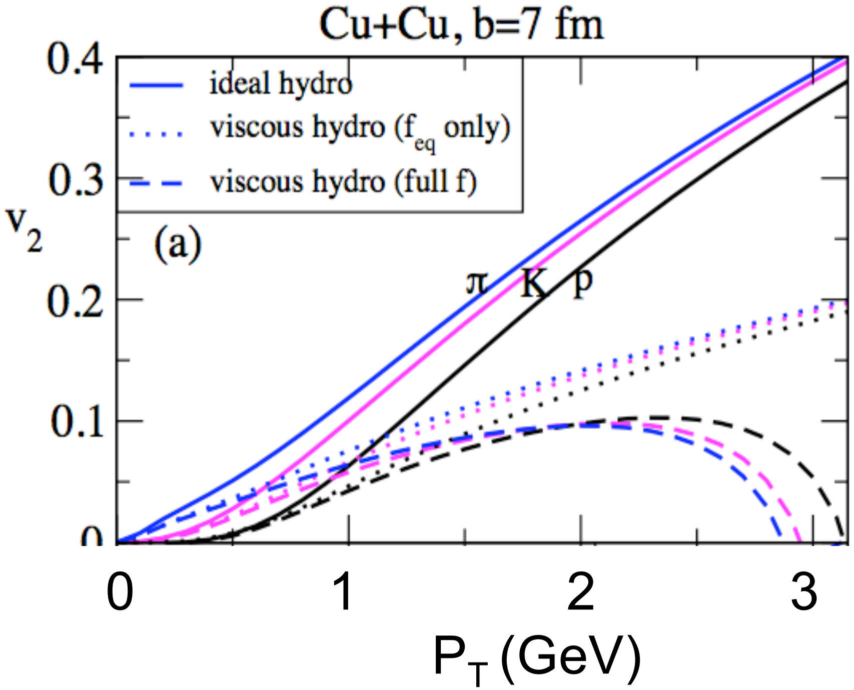
    Causal Israel-Stewart formalism

    P_T spectra, elliptic flow, HBT …

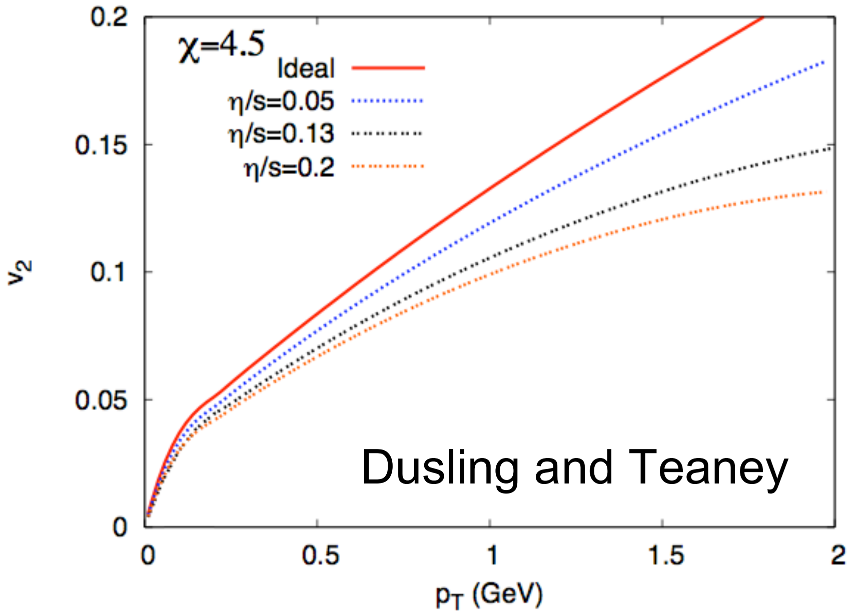
Viscous Hydro



P.Romatschke and U.Romatschke



Song and Heinz



Dusling and Teaney

They use different
- initial conditions
- EoSs
- freezeout conditions…

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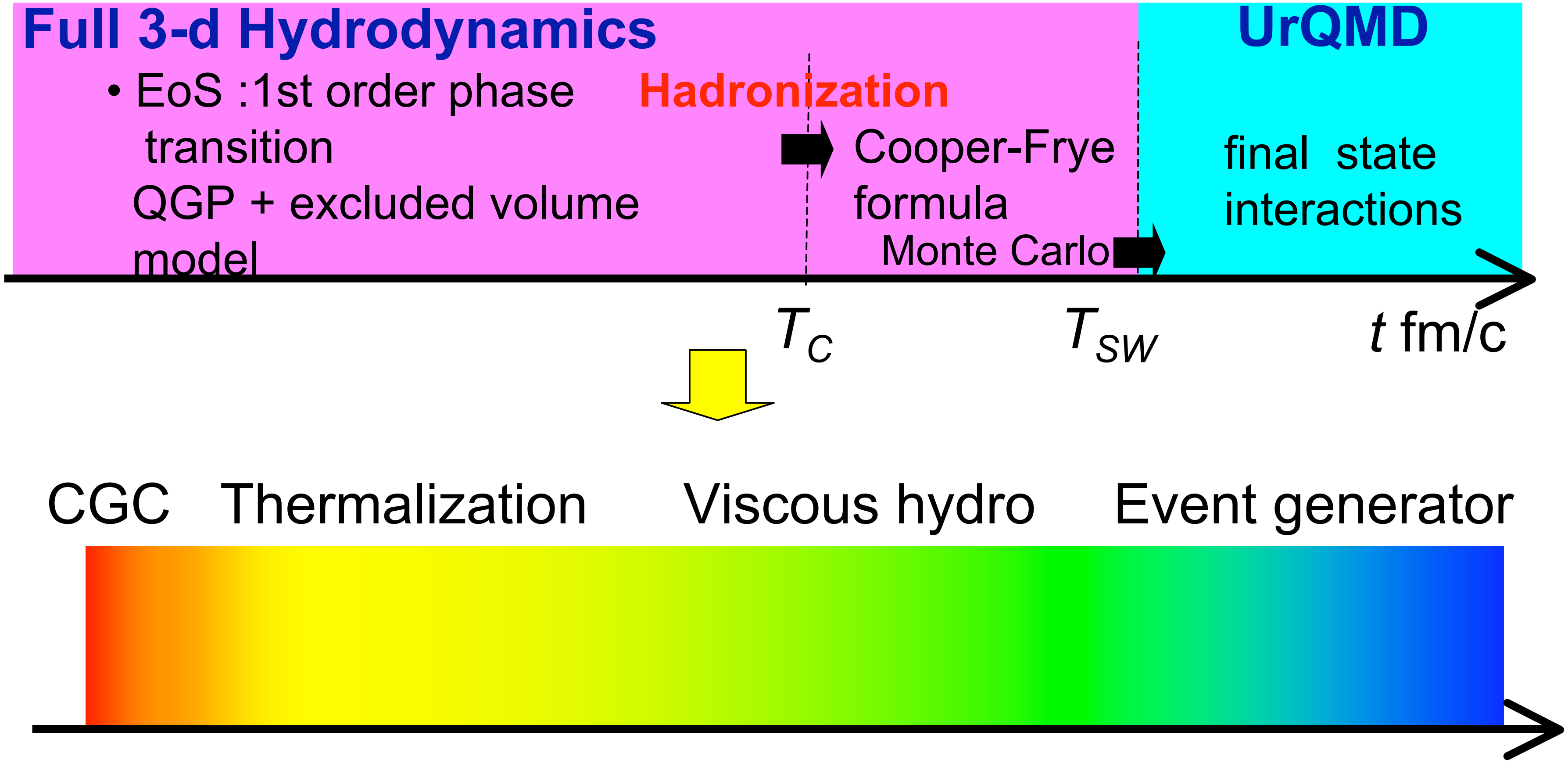
Summary -- Model



Full 3-d Hydrodynamics

- EoS :1st order phase transition QGP + excluded volume model

Hadronization

Cooper-Frye formula

Monte Carlo

UrQMD

final state interactions

CGC Thermalization Viscous hydro Event generator

- perfect fluid? Viscous hydro
- fluctuation in initial conditions ex. elliptic flow
- equation of state ex. QCD critical point
- freezeout process: recombination?

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Summary -- Applications

- Jets in Medium
  - Mach cone, ridge
- QCD critical point search

- LHC
  - Hydrodynamics works?

Hydrodynamics is a useful tool for understanding heavy ion physics!