Baryons in AdS/QCD

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Based on a work with
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Plan

1. Spin ½ baryons in QCD
   with $N_F = 2$
   - anomaly matching

2. AdS/QCD
   - chiral symmetry and mesons

3. Including baryons in AdS/QCD
   - AdS/CFT with fermions
   - Chiral symmetry

4. Numerical results
   - parity doubling spectrum
   - couplings to pions

Nucleon in QCD with $N_F = 2$

we assume vanishing bare quark mass

	$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$
$SU(2)_L$	2	1
$SU(2)_R$	1	2
$U(1)_B$	$1/3$	$1/3$
$SU(3)_c$	3	3

operators that create nucleons

$$\mathcal{O}_L = \underset{\substack{\uparrow \\ SU(3)_c}}{\varepsilon_{\alpha\beta\gamma}} \underbrace{q_L^\alpha q_L^\beta q_L^\gamma}_{\substack{\uparrow \\ \text{(flavor, spin)-index} \\ \text{totally symmetric}}} \sim (\underset{4}{\square\square\square}, \underset{s=3/2}{\square\square\square}) + (\underset{2}{\square}, \underset{s=1/2}{\square})$$

$\uparrow$ $SU(2)_L$ $\qquad$ $\uparrow$ spin

$$(\square, \square) = (2, \text{spin}=1/2) \rightarrow \boxed{\begin{pmatrix} p_L \\ n_L \end{pmatrix} = N_L}$$

Similarly, $\mathcal{O}_R = \varepsilon_{\alpha\beta\gamma} q_R^\alpha q_R^\beta q_R^\gamma \sim \begin{pmatrix} p_R \\ n_R \end{pmatrix}$

Charges

$N_F = 2$

	$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$	$N_L = \begin{pmatrix} p_L \\ n_L \end{pmatrix}$	$N_R = \begin{pmatrix} p_R \\ n_R \end{pmatrix}$
$SU(2)_L$	2	1	2	1
$SU(2)_R$	1	2	1	2
$U(1)_B$	$1/3$	$1/3$	1	1

UV (first two columns) IR (last two columns)

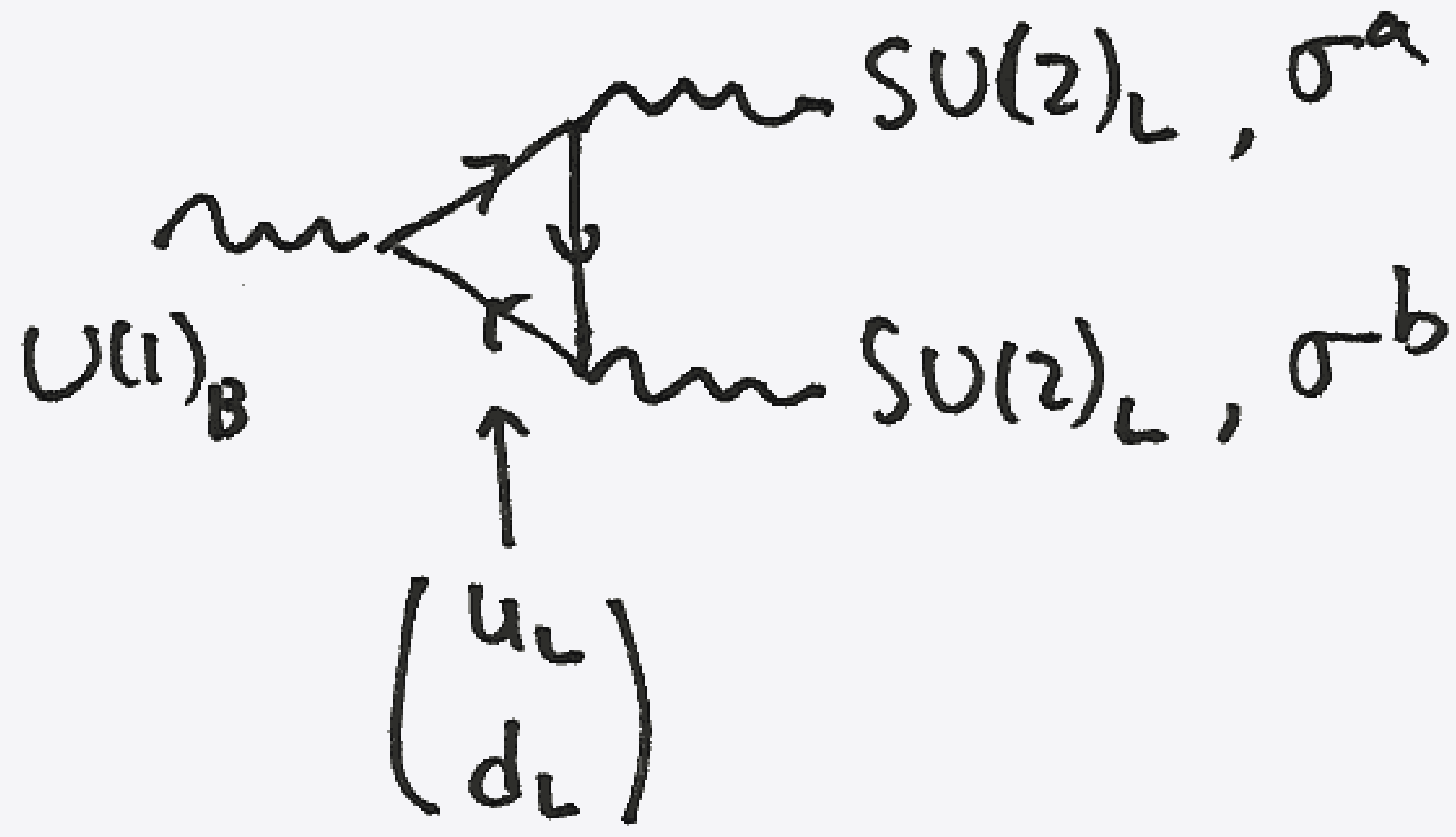
In chiral limit, if $SU(2)_L \times SU(2)_R \times U(1)_B$ is not spontaneously broken to $SU(2)_I \times U(1)_B$,

$\rightarrow$ Massless chiral nucleons $\begin{pmatrix} p_L \\ n_L \end{pmatrix}$, $\begin{pmatrix} p_R \\ n_R \end{pmatrix}$ ~~are~~ required to match anomaly

Anomaly matching ('t Hooft)

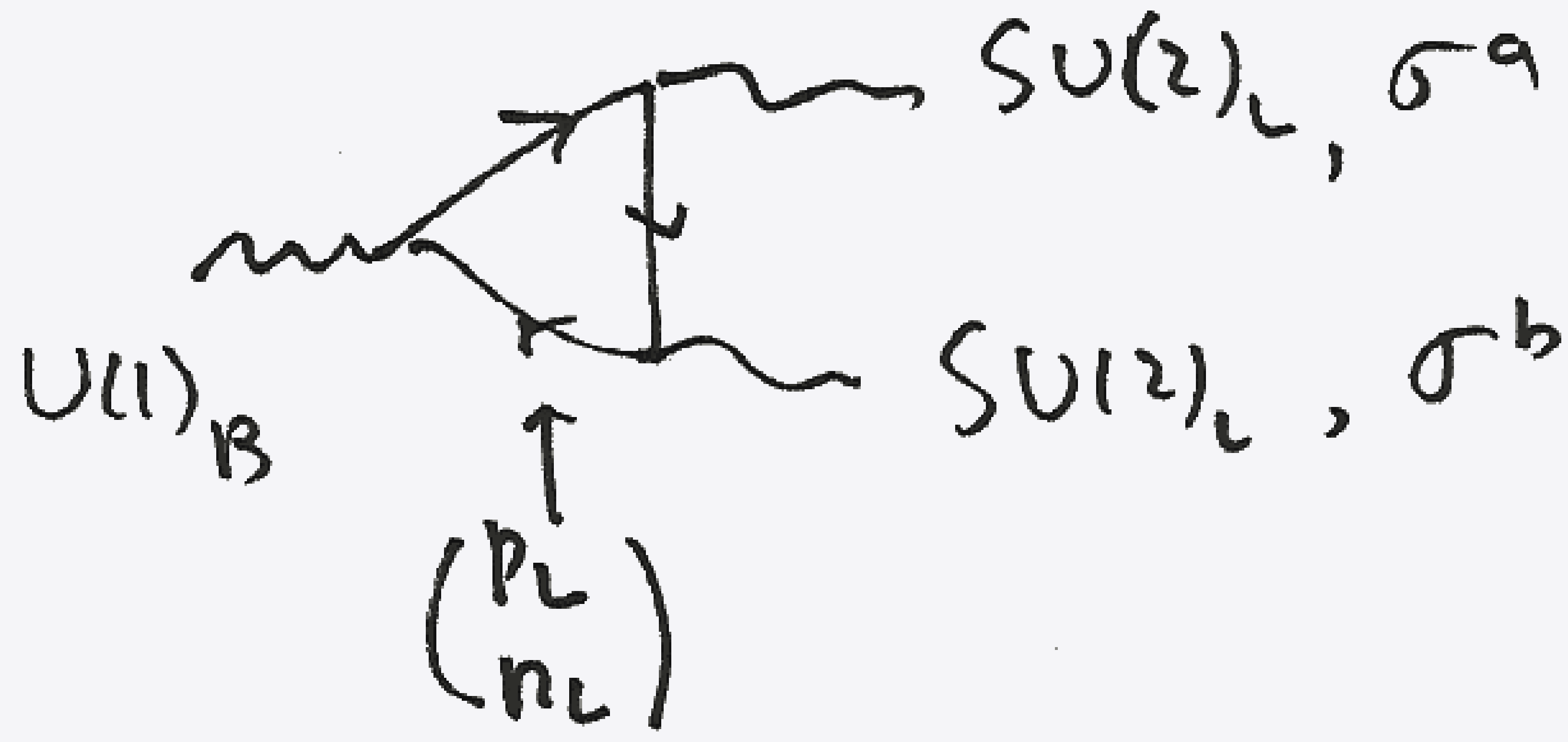
$U(1)_B \times [SU(2)_L]^2$ - anomaly

UV



$$A_{UV} \sim \underset{\substack{\uparrow \\ U(1)_B}}{\frac{1}{3}} \times \underset{\substack{\uparrow \\ \text{QCD triplet}}}{3} \times \mathrm{tr}(\sigma^a \sigma^b)$$

IR



$$A_{IR} \sim \underset{\substack{\uparrow \\ U(1)_B}}{1} \times \underset{\substack{\uparrow \\ \text{QCD singlet}}}{1} \times \mathrm{tr}(\sigma^a \sigma^b)$$

$$\boxed{A_{UV} = A_{IR}}$$

- $U(1)_B \times [SU(2)_R]^2$ - anomaly

matches with $\begin{pmatrix} u_R \\ d_R \end{pmatrix}$ and $\begin{pmatrix} p_R \\ n_R \end{pmatrix}$

$$\mathcal{A} = \underset{\uparrow}{(-1)}\, tr(\sigma^a \sigma^b)$$

Right handed

- $[U(1)_B]^3$ - anomaly is absent

- Does not work for $N_F \neq 2$

$\Rightarrow$ Chiral symmetry must be broken spontaneously

Coupling to pions

$$U = e^{i\pi^a\sigma^a/\sqrt{f_\pi}} \in SU(2)_A$$

is $(2,\bar{2})$ under $SU(2)_L \times SU(2)_R$

and parity for π is odd.

Chiral symmetry invariant coupling

$$\mathcal{L}_{\pi NN} = g(\bar{N}_L U N_R + \bar{N}_R U^\dagger N_L)$$

$$N_L = \begin{pmatrix} p_L \\ n_L \end{pmatrix}, \quad N_R = \begin{pmatrix} p_R \\ n_R \end{pmatrix}$$

Expanding $U \sim 1 + \frac{i\pi^a\sigma^a}{\sqrt{f_\pi}} + \cdots$

$$m_N = |g|$$

$$\mathcal{L}_{\pi NN} \sim \frac{ig}{\sqrt{f_\pi}} \bar{N}(\pi^a\sigma^a)\gamma^5 N = i g_{\pi NN} \bar{N}(\pi^a\sigma^a)\gamma^5 N$$

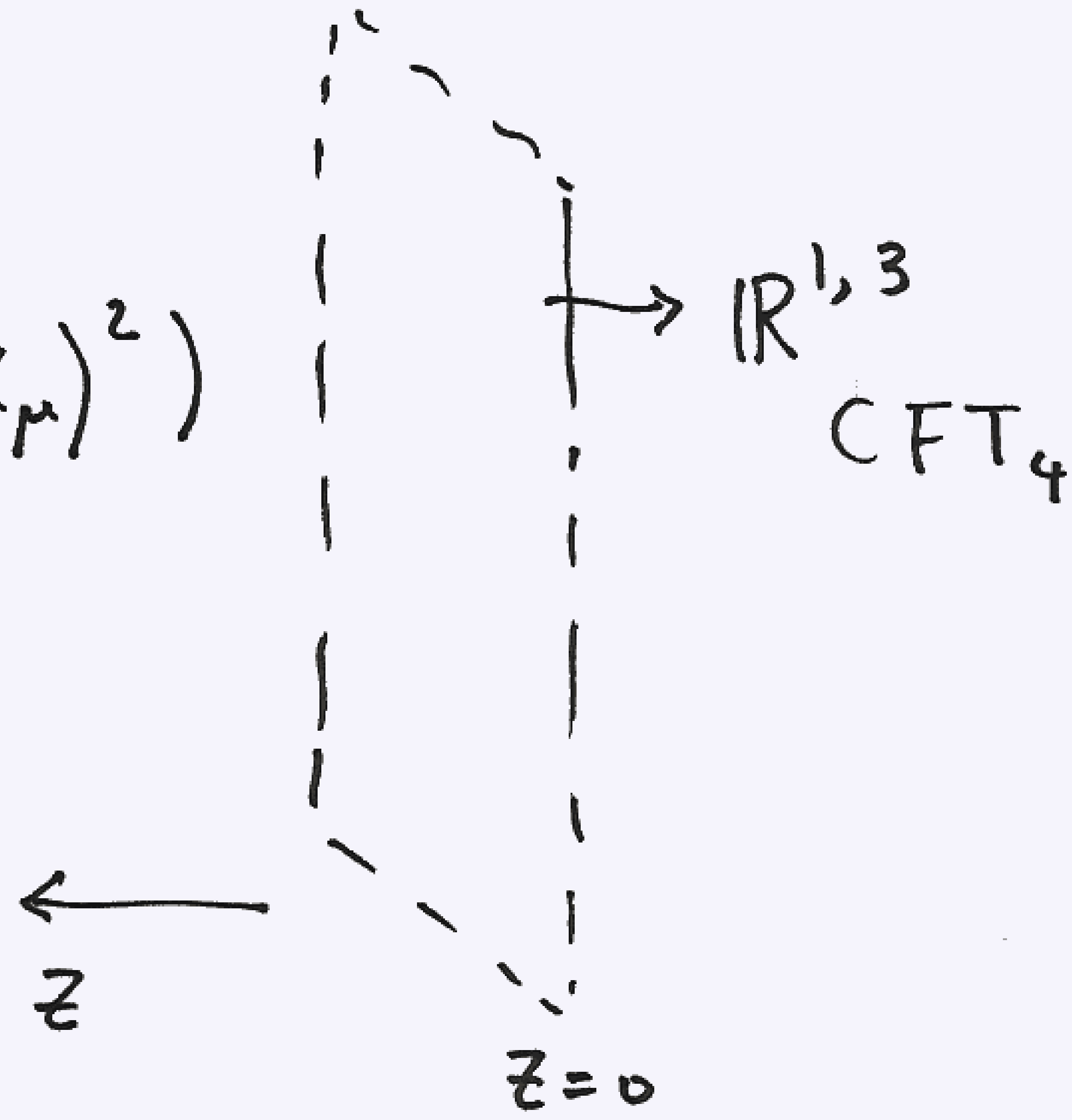
$$\Rightarrow \boxed{g_{\pi NN} = \frac{m_N}{\sqrt{f_\pi}}} \simeq \frac{0.94}{0.094} \simeq \underline{\underline{10}}$$

$$g_{\pi NN}^{exp} \simeq 13$$

AdS/CFT correspondence

AdS_5

$$ds^2 = \frac{1}{z^2}\left(dz^2 + (dx_\mu)^2\right)$$



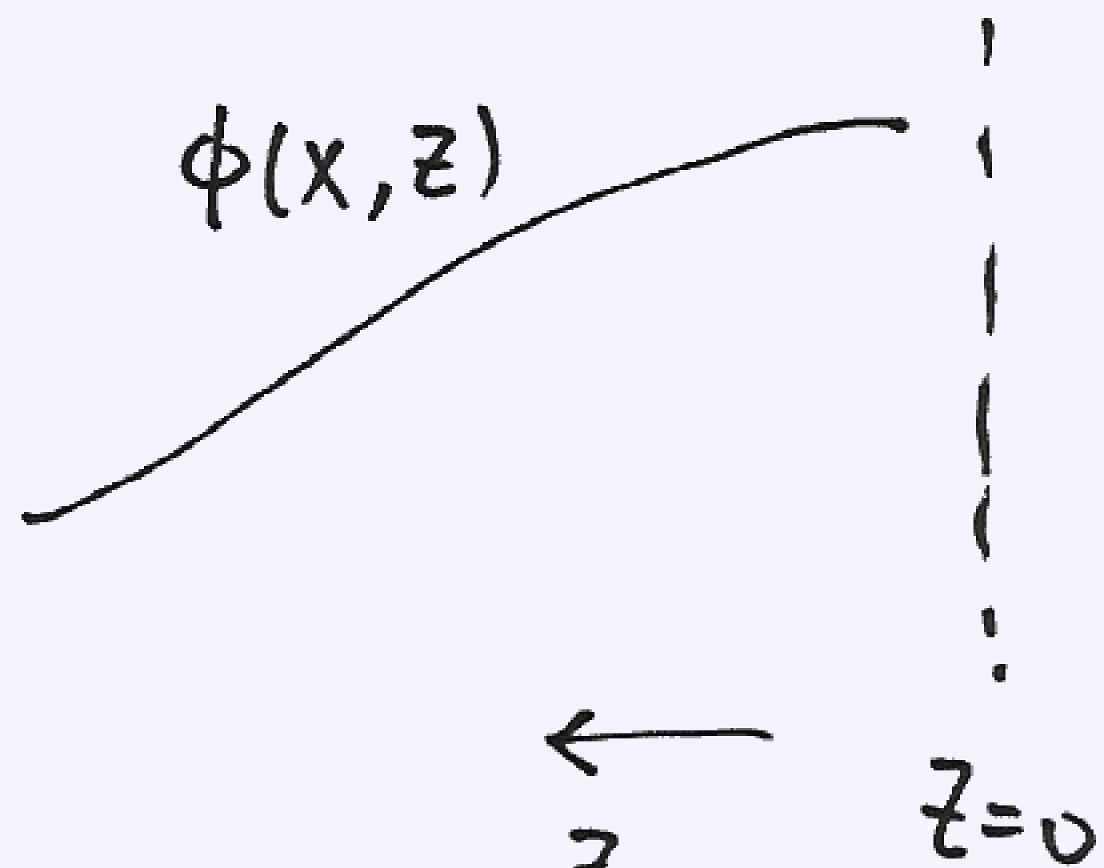
Dictionary

AdS_5	CFT_4
gravity (background)	strong (gauge) dynamics
$\frac{1}{z}$	energy scale μ
fields	operators
mass of fields	scaling dimension
values at $z=0$	source for operators
gauge symmetry	global symmetry
normalizable modes	physical spectrum
weak coupling $g_s \to 0$	$1/N$ - expansion, $N \to \infty$

Two examples

1.

AdS_5	CFT_4
Scalar field ϕ with mass m_5	Scalar operator $\mathcal{O}$ with $\Delta(4-\Delta) = -m_5^2$



$$\boxed{e^{S_{cl}(\phi)}\Big|_{\phi(0,x)=\text{fixed}} = \left\langle e^{\int d^4x\, \phi(x,0)\, \mathcal{O}(x)} \right\rangle_{CFT}}$$

2.

AdS_5	CFT_4
gauge field A_M	global symmetry with current J^μ

$$\boxed{e^{S_{cl}(A)}\Big|_{A_\mu(x,0)=\text{fixed}} = \left\langle e^{\int d^4x\, A_\mu(x,0)\, J^\mu(x)} \right\rangle_{CFT}}$$

AdS/QCD meson sector

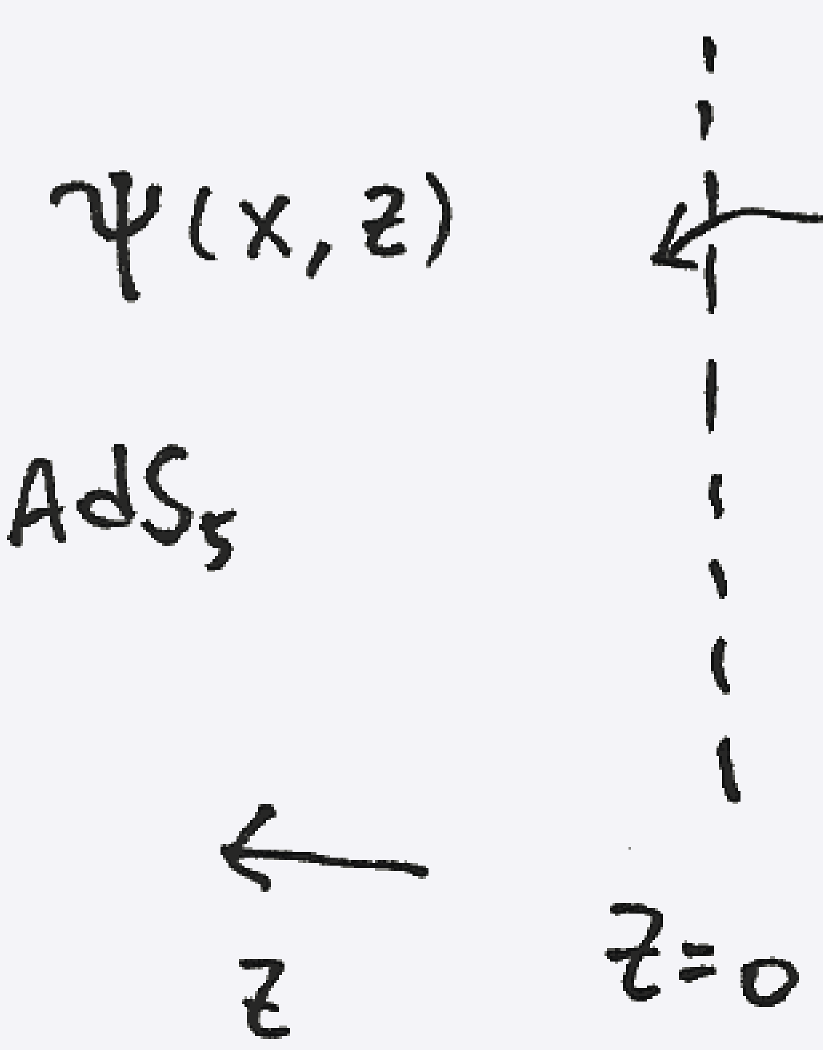
AdS_5	QCD
IR cutoff z_m	Λ_{QCD}
$SU(2)_L \times SU(2)_R$ gauge fields A_L, A_R	$SU(2)_L \times SU(2)_R$ global chiral symmetry
scalar field X^{ij}, $m_5^2 = -3$ bifund. under $SU(2)_L \times SU(2)_R$	$\mathcal{O}^{ij} = \bar{q}_L{}^i q_R{}^j$ with $\Delta = 3$
$X \neq 0$ $SU(2)_L \times SU(2)_R \to SU(2)_I$	$\langle \bar{q}_L q_R \rangle \neq 0$ $SU(2)_L \times SU(2)_R \to SU(2)_I$
Massless fields from Wilson lines	Goldstone pions
KK-spectrum from $A_{L\mu}, A_{R\mu}$	excited vector mesons
KK-spectrum from X & A_{L5}, A_{R5}	excited scalar mesons

Nucleons in AdS/QCD

In QCD, we have spin-½ operators

N_L, N_R in $\langle \bar{q} q \rangle = 0$ limit

AdS_5 Dirac spinor Ψ with mass m_5



When $\underline{m_5 > 0}$, only right handed component survives with $\Psi \sim \Psi_R \sim z^{2-m_5}$ as $z \to 0$.

It couples to $\mathcal{O}_L$ in CFT by $\sim \int \bar{\Psi}_R \mathcal{O}_L$ with

$$\boxed{\dim \mathcal{O}_L = 4 - (2 - m_5) = 2 + m_5}$$

For $\underline{m_5 < 0}$, $\Psi \sim \Psi_L \sim z^{2+m_5}$ near $z \to 0$ and couples to $\mathcal{O}_R$ in CFT by $\sim \int \bar{\Psi}_L \mathcal{O}_R$ with

$$\boxed{\dim \mathcal{O}_R = 2 - m_5}$$

Summary of AdS/CFT with spin $1/2$

AdS_5	CFT_4
Ψ with $m_5 > 0$ IR B.C. $\Psi_R(z_m) = 0$	$\mathcal{O}_L$, spin $1/2$ with $\Delta = 2 + m_5$ massless Left-handed states
Ψ with $m_5 < 0$ IR B.C. $\Psi_L(z_m) = 0$	$\mathcal{O}_R$ with $\Delta = 2 - m_5$ massless right-handed state

For nucleons in QCD with $\Delta = 9/2$

AdS_5-slice	QCD
N_1 with $m_5 = 5/2$ with $N_{1R}(z_m) = 0$	$N_L = \begin{pmatrix} p_L \\ n_L \end{pmatrix}$ with $\Delta = 9/2$ and massless state (LH) in $\langle \bar{q} q \rangle = 0$ limit
N_2 with $m_5 = -5/2$ with $N_{2L}(z_m) = 0$	$N_R = \begin{pmatrix} p_R \\ n_R \end{pmatrix}$ with $\Delta = 9/2$ and massless (RH)
$\boxed{\mathcal{L}_{5D} \sim g \bar{N}_1 X N_2 + h.c}$	$\boxed{\mathcal{L} \sim g \bar{N}_L U N_R + h.c.}$

Holographic model for spin 1/2 baryons

AdS_5 slice with $0 \lesssim z \lesssim z_m$ and two Dirac spinors N_1 with $m_5 = 5/2$, N_2 with $m_5 = -5/2$ with IR B.C. $N_{1R}(z_m) = 0$, $N_{2L}(z_m) = 0$ with the coupling $\mathcal{L} \sim g \bar{N}_1 X N_2 + h.c.$

where $X = \frac{1}{2} \sigma z^3$, $\sigma \propto \langle \bar{q} q \rangle$

given by meson sector

Parameters : z_m & g

Strategy

Fit the lowest and 1^{st} excited masses as $m_N \sim 0.94$ GeV, $m_{N(1440)} \sim 1.44$ GeV by tuning z_m & g. Then, other physical quantities are predictions of our model

Spectrum

$z_m(\mathrm{GeV}^{-1})$	g	(p,n)(GeV)	N(1440)	N(1535)	3rd	4th	5th	6th
$(0.33)^{-1*}$	8.67	0.94*	2.14	2.24	3.25	3.30	4.35	4.36
$(0.205)^{-1}$	14.4	0.94*	1.44*	1.50	2.08	2.12	2.72	2.75

TABLE I: Numerical result for spin-$\frac{1}{2}$ baryon spectrum. $*$ indicates an input and we used $\sigma = \frac{\sqrt{2}\xi}{g_5\, z_m^3}$ with $3.4 \le \xi \le 4$.

$\boxed{\text{Parity doubling pattern is one of our prediction!}}$

Coupling to pions

From $i\bar{N}_1 \not{\partial} N_1 + i\bar{N}_2 \not{\partial} N_2 + g\bar{N}_1 X N_2 + \text{h.c.}$

and pions $\sim$ linear mixture of A_5 & X

$$\mathcal{L} \sim i\, g_{\pi NN}\, \bar{N}\, \pi^a \sigma^a \gamma^5 N$$

$z_m(\mathrm{GeV}^{-1})$	ξ	$g_{\pi NN}$	$g_{\pi NN_{1440}}$	$g_{\pi NN_{1535}}$
$(0.205)^{-1}$	3.4	7.34	-0.025	2.04
$(0.205)^{-1}$	3.7	6.52	0.30	1.88
$(0.205)^{-1}$	4.0	5.84	0.56	1.74
$(0.33)^{-1}$	3.4	4.35	-	-
$(0.33)^{-1}$	3.7	3.78	-	-
$(0.33)^{-1}$	4.0	3.30	-	-

TABLE II: Numerical result for pion-nucleon-nucleon couplings.

Outlook

- Relax $\Delta = 9/2$ to incorporate anomalous dimension
- Modify IR boundary including dilatons, etc.
- Spin-$3/2$ baryons with Rarita-Schwinger fields
- Finite temperature and finite density with baryons